GENERAL PHYSICS I

by

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Course Necessities

- It is aimed that the students to learn and apply basic concepts of mechanics.
- Each week the first course day is going to be assigned for lecturing and the second for recitation.
- Grading will be done according to following criteria: 40% of Mid-Term + 60% of Final
- Do not hesitate to ask question during the class. It might be helpful to understand subject better not only for you but also for your peers.
- Each student should have a functional calculator (since it will be used in the coming years preferably CASIO)
- Attendance to 70% of lecture is required
- Office # 350

Course Content

- Physical Units and Metric Prefixes
- Vectors and Vector Operations
- One / Two Dimensional Motion
- Newton’s Laws of Motion
- Work-Energy Principle and Conservation of Energy
- Momentum
- Angular momentum
- Static Equilibrium

Textbooks

- Physics for Scientists and Engineers/ John W. Jewett ; Raymond A. Serway
- Fundamentals of Physics/ David Halliday ; Robert Resnick, Jearl Walker.
- Physics : Principles with Applications/ Douglas C. Giancoli.
- University Physics/ Hugh D. Young ; Roger A. Freedman.

PHYSICAL UNITS

&

METRIC PREFIXES

Physical Units

Mechanics is the branch of physics in which the basic physical units are developed. The logical sequence is from the description of motion to the causes of motion (forces and torques) and then to the action of forces and torques. The basic mechanical units are those of

MASS, LENGTH, & TIME

All mechanical quantities can be expressed in terms of these three quantities. The standard units are the Systeme Internationale or SI units. The primary SI units for mechanics are the kilogram (mass), the meter (length) and the second (time). However if the units for these quantities in any consistent set of units are denoted by M, L, and T, then the scheme of mechanical relationships can be sketched out.

Base quantity	Examples of units
Length	**metre (m),** centimetre (cm), kilometre (km), foot (ft), inch (in)
Mass	**kilogram (kg)**, gram (g)
Time	**second (s)**, minute (min), Hour (hr)

(meter - kilogram - second = MKS system)

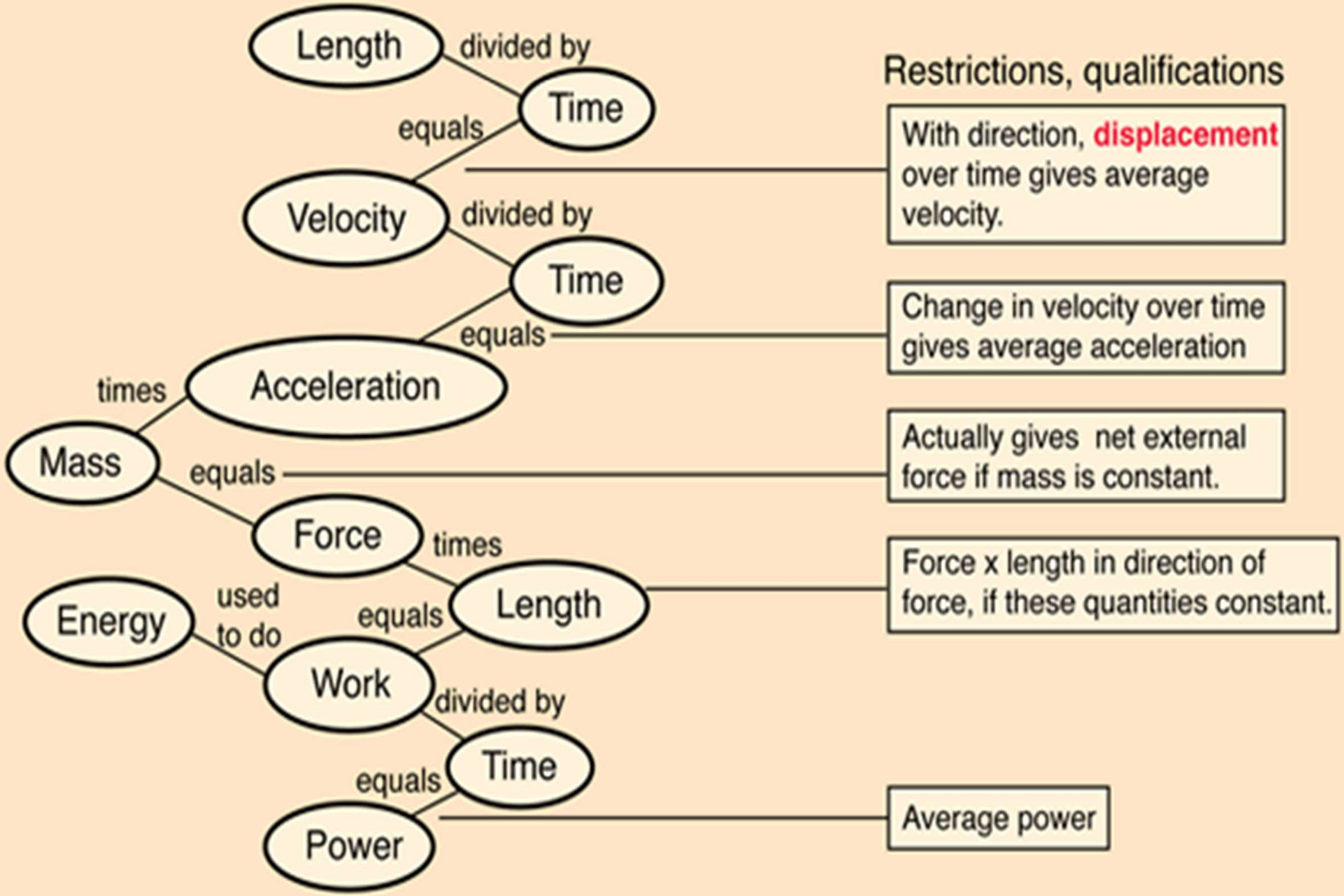
Conversion factors:

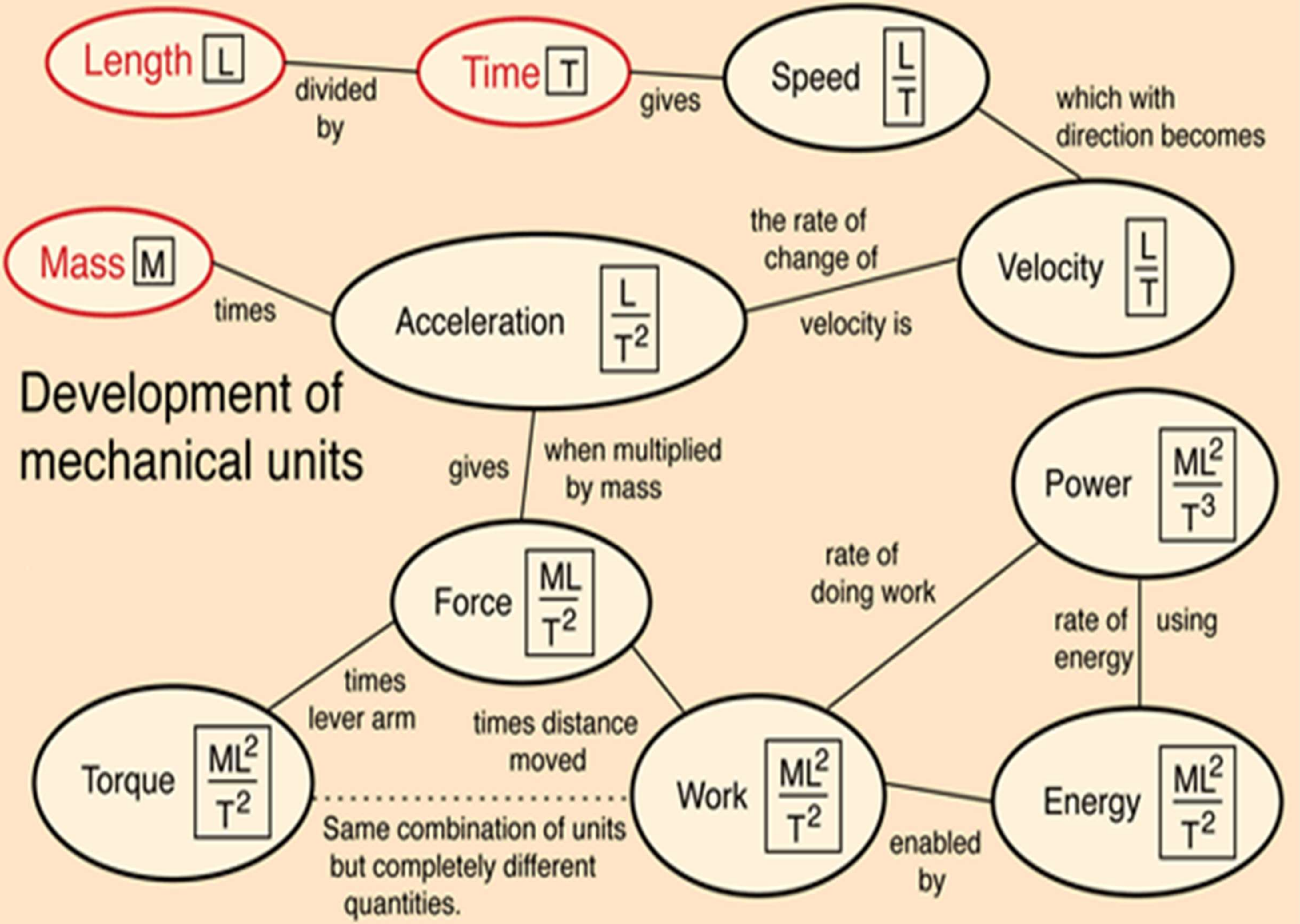
Length
1m =100 cm = 0.001 km = 3.28 ft = 39.37 in

Mass
1 kg =1000 g

Time
1 hr = 60 min = 3600 s

The Chain of Mechanical Quantities




Length L
divided by
Time T
gives
Speed L/T
which with direction becomes
Velocity L/T
the rate of change of velocity is
Mass M
times
Acceleration L/T²
Development of mechanical units
gives
when multiplied by mass
Power ML²/T³
Force ML/T²
rate of doing work
rate of energy
using
times lever arm
times distance moved
Torque ML²/T²
Work ML²/T²
Energy ML²/T²
Same combination of units but completely different quantities.
enabled by

Basic Mechanical Units

	SI Units (MKS)	(CGS)	U.S. Common
Length (L)	meter (m)	centimeter (cm)	foot (ft)
Time (T)	second (s)	second (s)	second (s)
Mass (M)	kilogram (kg)	gram (gm)	slug
Velocity (L/T)	m/s	cm/s	ft/s
Acceleration (L/T^2)	m/s^2	cm/s^2	ft/s^2
Force (ML/T^2)	kg m/s^2=Newton(N)	gm cm/s^2= dyne	slug ft/s^2=pound(lb)
Work (ML^2/T^2)	N m = joule (j)	dyne cm = erg	lb ft = ft lb
Energy (ML^2/T^2)	joule	erg	ft lb
Power (ML^2/T^3)	j/s = watt (W)	erg/s	ft lb/s

Dimensional Analysis

Having the same units on both sides of an equation does not gaurantee that the equation is correct, but having different units on the two sides of an equation certainly gaurantees that it is wrong! So it is good practice to reconcile units in problem solving as one check on the consistency of the work. Units obey the same algebraic rules as numbers, so they can serve as one diagnostic tool to check your problem solutions.

For example, in the solution for distance in constant acceleration motion, the distance is set equal to an expression involving combinations of distance, time, velocity and acceleration. But the combination of the units in each of the terms must yield just the unit of distance, since the left hand side of the equation has the dimension of distance.

$$y\ m = y_0\ m + v_0 \frac{m}{s}\ t\ s + \frac{1}{2}\ a \frac{m}{s^2}\ t^2\ s^2$$

$$y\ m = y_0\ m + v_0 \frac{m}{\cancel{s}}\ t\ \cancel{s} + \frac{1}{2}\ a \frac{m}{\cancel{s^2}}\ t^2\ \cancel{s^2}$$

Combinations of units pervade all of physics, and doing some analysis of the units is common practice. For example, in the case of centripetal force, it is not immediately evident that the quantity on the right has the dimensions of force, but it must. Checking it out:

Centripetal force: $$F = m \frac{v^2}{r}$$

Units $$N = kg \frac{(m/s)^2}{m} = kg\ m/s^2$$

Metric prefixes are pretty easy to understand and very handy for metric conversions. You don't have to know the nature of a unit to convert, for example, from *kilo-unit* to *mega-unit*. All metric prefixes are powers of 10. The most commonly used prefixes are highlighted in the table.

Prefix	Symbol	Power	Factor
yotta	Y	10^{24}	1,000,000,000,000,000,000,000,000
zetta	Z	10^{21}	1,000,000,000,000,000,000,000
exa	E	10^{18}	1,000,000,000,000,000,000
peta	P	10^{15}	1,000,000,000,000,000
tera	T	10^{12}	1,000,000,000,000
giga	**G**	10^{9}	**1,000,000,000**
mega	**M**	10^{6}	**1,000,000**
kilo	**k**	10^{3}	**1,000**
hecto	h	10^{2}	100
deka	da	10^{1}	10
deci	**d**	10^{-1}	**0.1**
centi	**c**	10^{-2}	**0.01**
milli	**m**	10^{-3}	**0.001**
micro	**µ**	10^{-6}	**0.000,001**
nano	n	10^{-9}	0.000,000,001
pico	p	10^{-12}	0,000,000,000,001
femto	f	10^{-15}	0.000,000,000,000,001
atto	a	10^{-18}	0.000,000,000,000,000,001
zepto	z	10^{-21}	0.000,000,000,000,000,000,001
yocto	y	10^{-24}	0.000,000,000,000,000,000,000,001

Conversions of area and volume units

Area Units

1m =100 cm

$(1m)^2=(100cm)^2 \longrightarrow 1\ m^2=10000\ cm^2 = 10^4\ cm^2$

1m =1000 mm

$(1m)^2=(1000mm)^2 \longrightarrow 1\ m^2=1000000\ mm^2 = 10^6\ mm^2$

$1cm=10^{-2}m$

$(1cm)^2=(10^{-2}m)^2 \longrightarrow 1\ cm^2=10^{-4}\ m^2$

$1mm=10^{-3}m$

$(1mm)^2=(10^{-3}m)^2 \longrightarrow 1\ mm^2=10^{-6}\ m^2$

That is,

- m^2 to cm^2 multiply by a factor of 10^4
- cm^2 to m^2 multiply by a factor of 10^{-4}
- m^2 to mm^2 multiply by a factor of 10^6
- mm^2 to m^2 multiply by a factor of 10^{-6}

Volume Units

1m =100 cm

$(1m)^3=(100cm)^3$ → $1\ m^3=1000000\ cm^3 = 10^6\ cm^3$

1m =1000 mm

$(1m)^3=(1000mm)^3$ → $1\ m^3=1000000000\ mm^3 = 10^9\ mm^3$

$1cm=10^{-2}m$

$(1cm)^3=(10^{-2}m)^3$ → $1\ cm^3=10^{-6}\ m^3$

$1mm=10^{-3}m$

$(1mm)^3=(10^{-3}m)^3$ → $1\ mm^3=10^{-9}\ m^3$

That is,

- m^3 to cm^3 multiply by a factor of 10^6
- cm^3 to m^3 multiply by a factor of 10^{-6}
- m^3 to mm^3 multiply by a factor of 10^9
- mm^3 to m^3 multiply by a factor of 10^{-9}

Example 1: Convert the following units

1 tera	=	….. mikro
0.3 kilo	=	….. mili
0.7 giga	=	….. Piko
0.5 mega	=	…… nano
0.6 mili	=	…… mikro
1 mikro	=	……. piko

Anwers of Example 1

1 tera	=	$1x10^{18}$ mikro
0.3 kilo	=	$0.3x10^{6}$ mili
0.7 giga	=	$0.7x10^{21}$ Piko
0.5 mega	=	$0.5x10^{15}$ nano
0.6 mili	=	$0.6x10^{3}$ mikro
1 mikro	=	$1x10^{6}$ piko

Example 2: Convert the following units

1 mikro	=	….. tera
0.3 kilo	=	….. mega
0.7 nano	=	….. giga
0.5 nano	=	…… tera
0.6 mili	=	…… tera
1 mikro	=	……. kilo

Answers of Example 2

1 mikro	$= 1x10^{-18}$ tera
0.3 kilo	$= 0.3x10^{-3}$ mega
0.7 nano	$= 0.7x10^{-18}$ giga
0.5 nano	$= 0.5x10^{-21}$ tera
0.6 mili	$= 0.6x10^{-15}$ tera
1 mikro	$= 1x10^{-9}$ kilo

Example 3: Convert the following units

$100\ m^2 = \ldots.\ cm^2$

$100000\ cm^2 = \ldots..\ m^2$

$0.5\ m^2 = \ldots..\ mm^2$

$100000\ mm^2 = \ldots\ldots\ m^2$

$10\ m^3 = \ldots.\ cm^3$

$1000000\ cm^3 = \ldots..\ m^3$

$0.5\ m^3 = \ldots..\ mm^3$

$1000000\ mm^3 = \ldots\ldots\ m^3$

Answers of Example 3

$100\ m^2 = 100 \times 10^4\ cm^2 = 10^6\ cm^2$

$100000\ cm^2 = 100000 \times 10^{-4}\ m^2 = 10\ m^2$

$0.5\ m^2 = 0.5 \times 10^6\ mm^2 = 5 \times 10^5\ mm^2$

$100000\ mm^2 = 100000 \times 10^{-6}\ m^2 = 0.1\ m^2$

$10\ m^3 = 10 \times 10^6\ cm^3 = 10^7\ cm^3$

$1000000\ cm^3 = 1000000 \times 10^{-6}\ m^3 = 1\ m^3$

$0.5\ m^3 = 0.5 \times 10^9\ mm^3 = 5 \times 10^8\ mm^3$

$1000000\ mm^3 = 1000000 \times 10^{-9}\ m^3 = 0.001\ m^3 = 10^{-3}\ m^3$

Conversion of quotient units

Example 4: Convert the following units

100 m/s	= .... cm/s
1 cm/h	= ..... m/s
10 m/s^2	= .... cm/s^2
0.1 cm/h^2	= ..... m/s^2
10 kg/m^3	= .... g/cm^3
0.1 g/cm^3	= ..... kg/m^3

Answers of Example 4

$$100\frac{m}{s}\times\frac{100\ cm}{1\ m}=10000\ \frac{cm}{s}=10^4\ \frac{cm}{s}$$

$$1\frac{cm}{hr}\times\frac{1\,hr}{3600\,s}\times\frac{1\,m}{100\,cm}=2.78\times10^{-6}\ \frac{m}{s}$$

$$10\frac{m}{s^2}\times\frac{100\,cm}{1m}=1000\frac{cm}{s^2}$$

$$0.1\frac{cm}{hr^2}\times\frac{1\,hr}{3600\,s}\times\frac{1\,hr}{3600\,s}\times\frac{1\,m}{100\,cm}=7.72\times10^{-11}\ \frac{cm}{s^2}$$

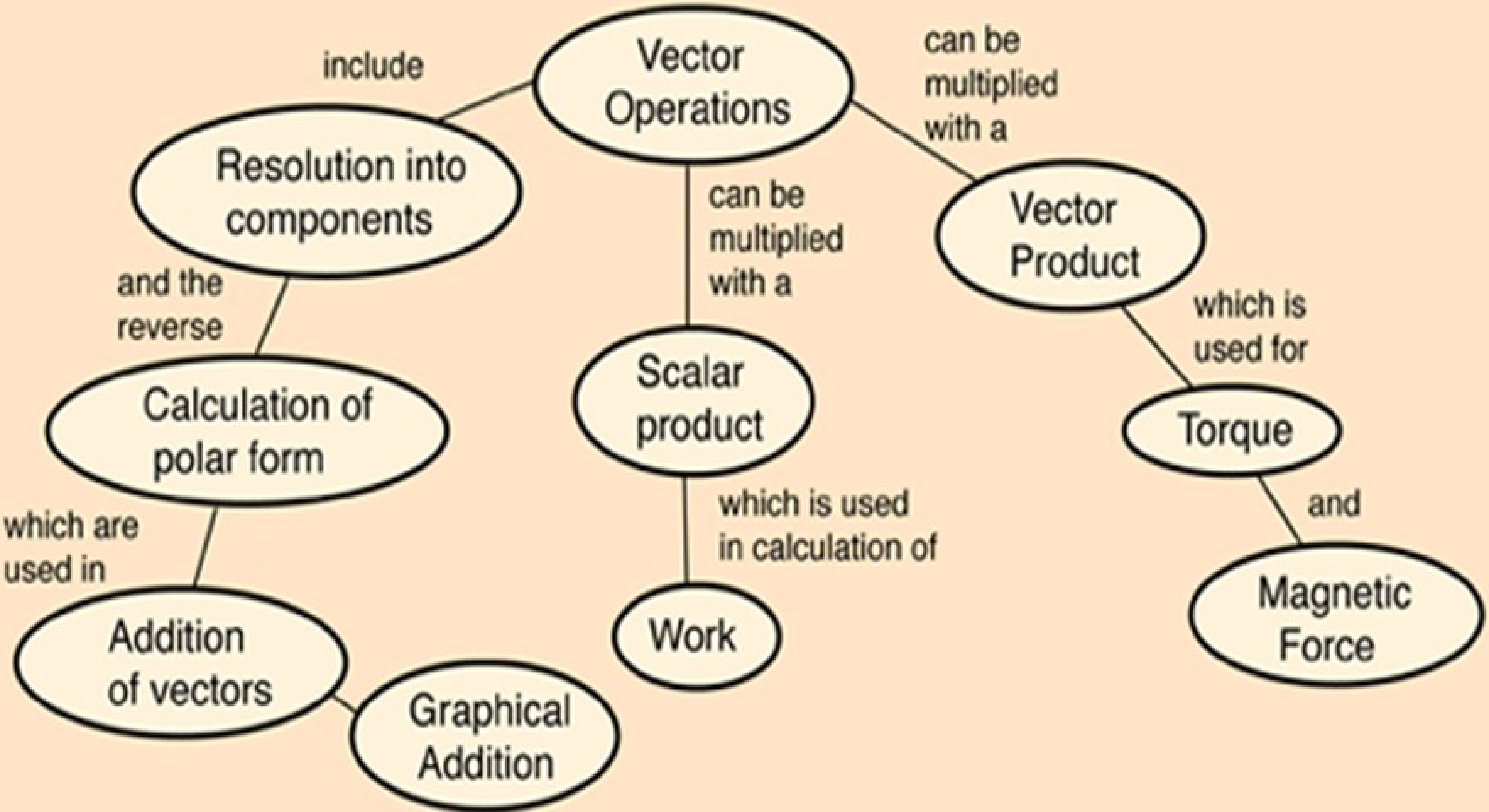
$$10\frac{kg}{m^3}\times\frac{1000g}{1kg}\times\frac{1m^3}{1000000cm^3}=0.01\frac{g}{cm^3}$$

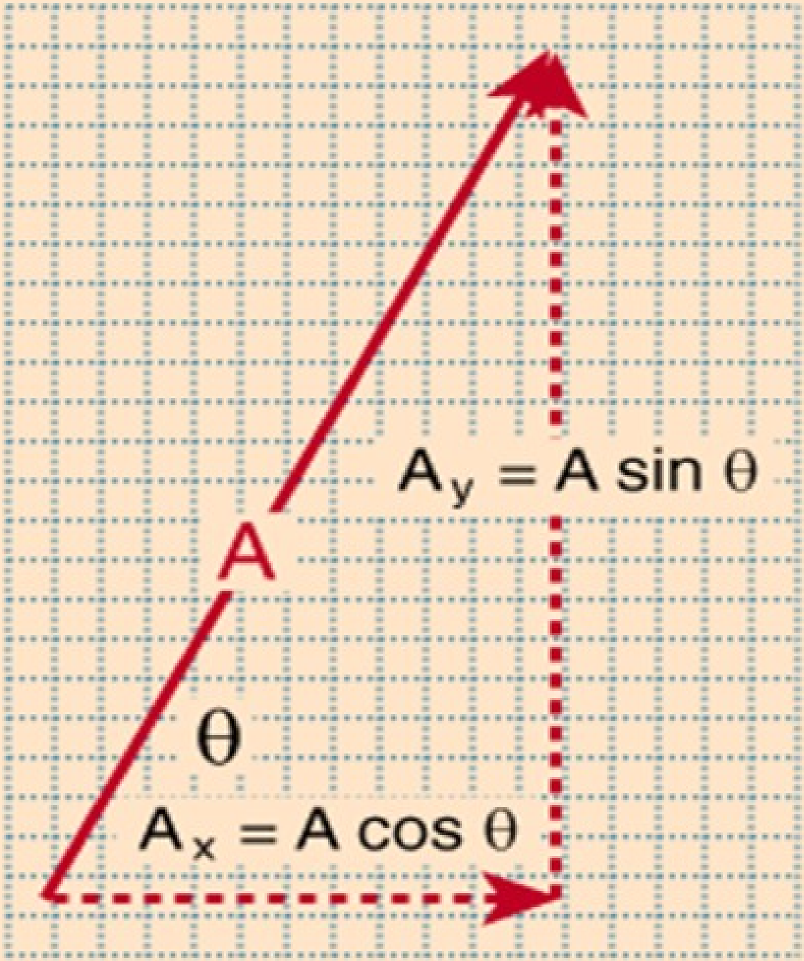
$$0.1\frac{g}{cm^3}\times\frac{1kg}{1000\,g}\times\frac{1000000cm^3}{1m^3}=100\frac{kg}{m^3}$$

Basic Vector Operations

Both a magnitude and a direction must be specified for a vector quantity, in contrast to a scalar quantity which can be quantified with just a number. Any number of vector quantities of the same type (i.e., same units) can be combined by basic vector operations.

Content


Vector Operations
include
Resolution into components
and the reverse
Calculation of polar form
which are used in
Addition of vectors
Graphical Addition
can be multiplied with a
Scalar product
which is used in calculation of
Work
can be multiplied with a
Vector Product
which is used for
Torque
and
Magnetic Force

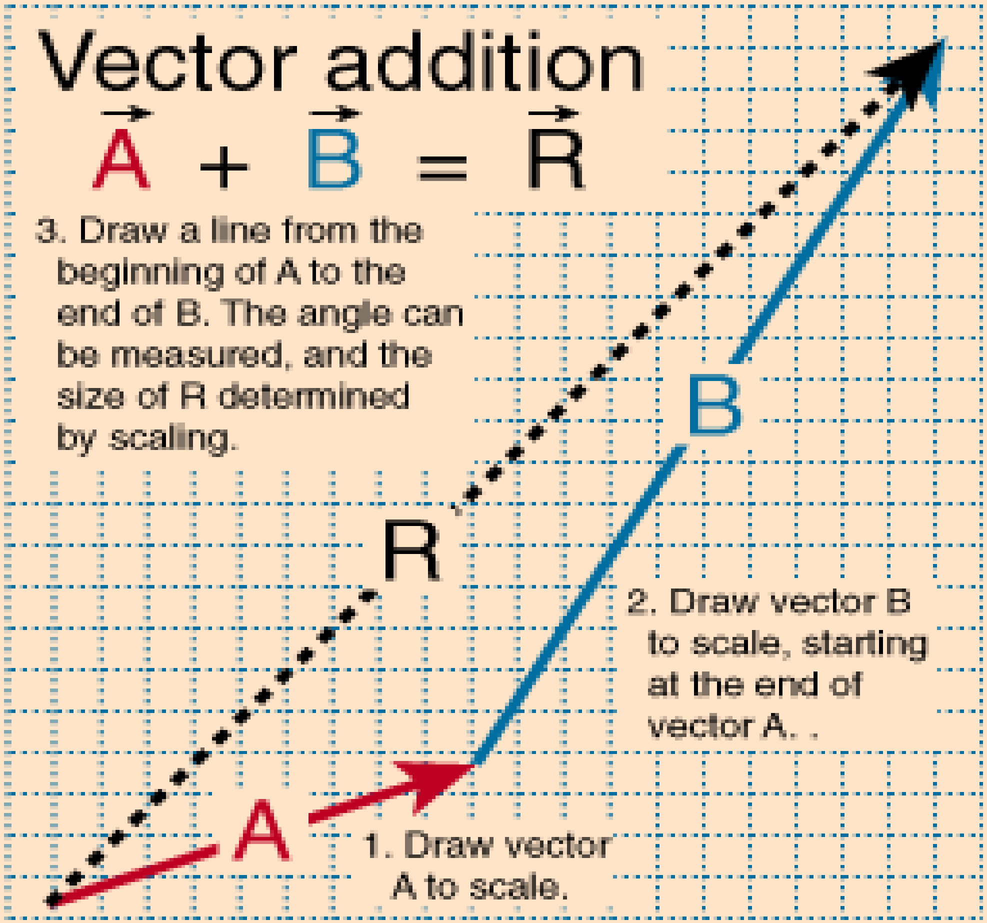


Vectors are resolved into components by use of the triangle trig relationships. You may change the length or angle of the polar form of the vector, and the components will be calculated below.

For vector A= 10 N
at angle 30 degrees,

the horizontal component is
= 8.66025403 N
and the vertical component is
= 4.99999999 N
The input to the boxes for units is arbitrary; they serve to emphasize that the process of vector addition is independent of the units of the vector.

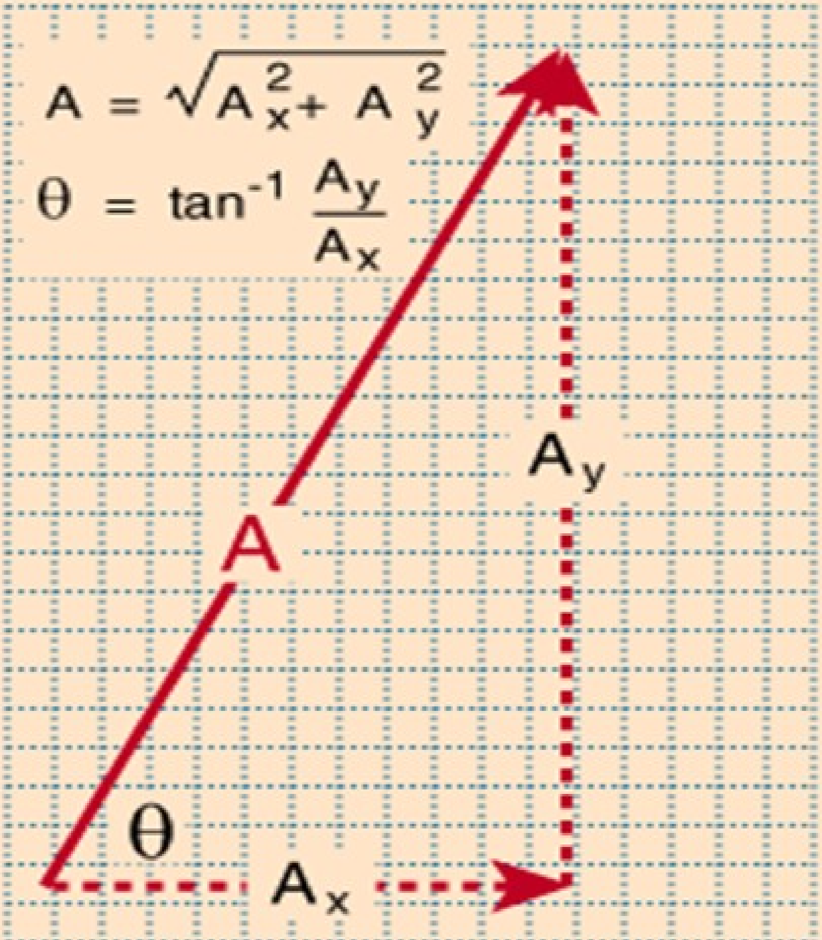
Graphical Vector Addition



Adding two vectors A and B graphically can be visualized like two successive walks, with the vector sum being the vector distance from the beginning to the end point. Representing the vectors by arrows drawn to scale, the beginning of vector B is placed at the end of vector A. The vector sum R can be drawn as the vector from the beginning to the end point.

The process can be done mathematically by finding the components of A and B, combining to form the components of R, and then converting to polar form.

Magnitude and Direction from Components

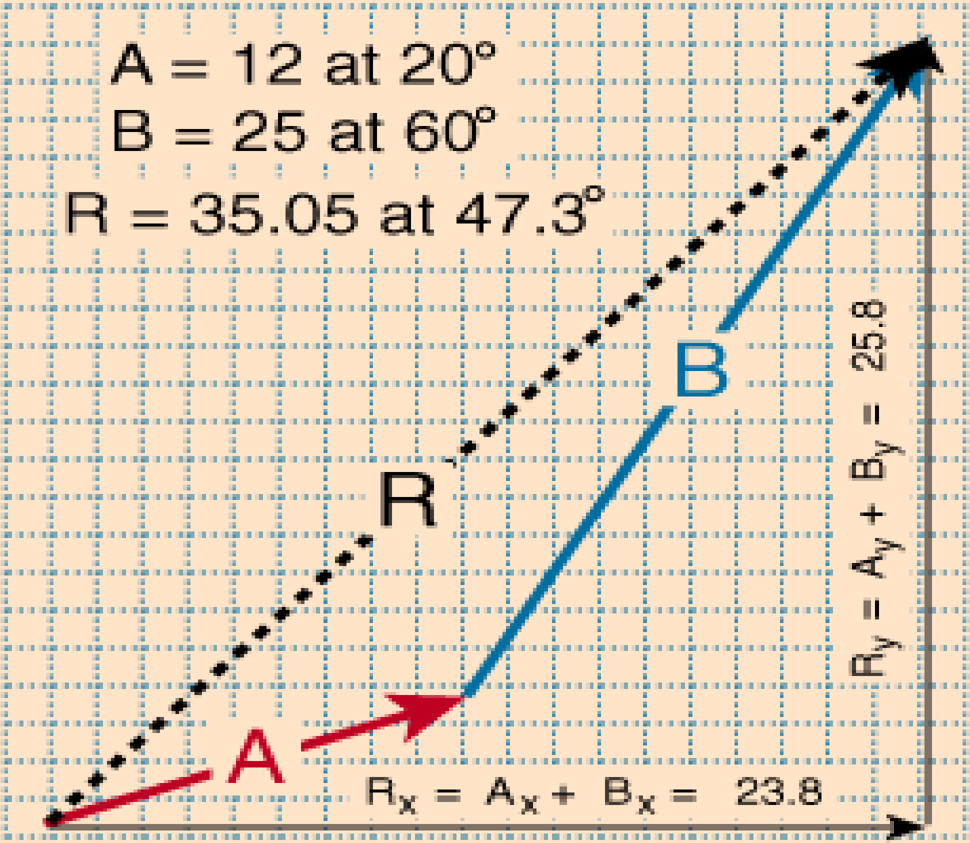


If the components of a vector are known, then its magnitude and direction can be calculated with the use of the Pythagorean relationship and triangle trig. This is called the polar form of the vector.

If the horizontal component is
= 5
and the vertical component is
= 4 ,

then the magnitude is
= 6.40312423
and the angle is
= 38.6598082 degrees.

Polar Form Example



After finding the components for the vectors A and B, and combining them to find the components of the resultant vector R, the result can be put in polar form by

$$R_x = A_x + B_x = 11.3 + 12.5 = 23.8$$

$$R_y = A_y + B_y = 4.1 + 21.7 = 25.8$$

$$R = \sqrt{R_x^2 + R_y^2}$$

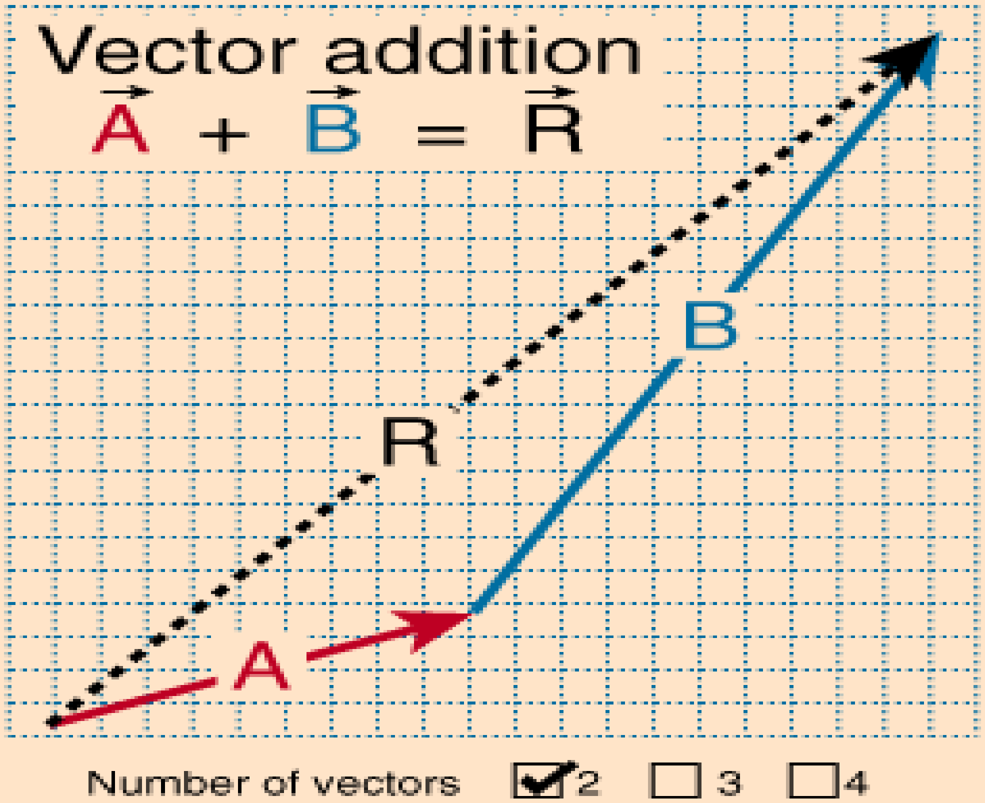
$$R = \sqrt{23.8^2 + 25.8^2} = 35.05$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1} 1.084$$

$$\theta = 47.3^\circ$$

Some caution should be exercised in evaluating the angle with a calculator because of ambiguities in the arctangent on calculators.

Vector Addition, Two Vectors



Vector addition involves finding vector components, adding them and finding the polar form of the resultant.

The addition of vector

A= 12 at 45 degrees,

and vector

B= 15 at 30 degrees,

yields components:

A_x	+	B_x	=	R_x
8.485281	+	12.99038	=	21.47566
A_y	+	B_y	=	R_y
8.485281	+	7.499999	=	15.98528

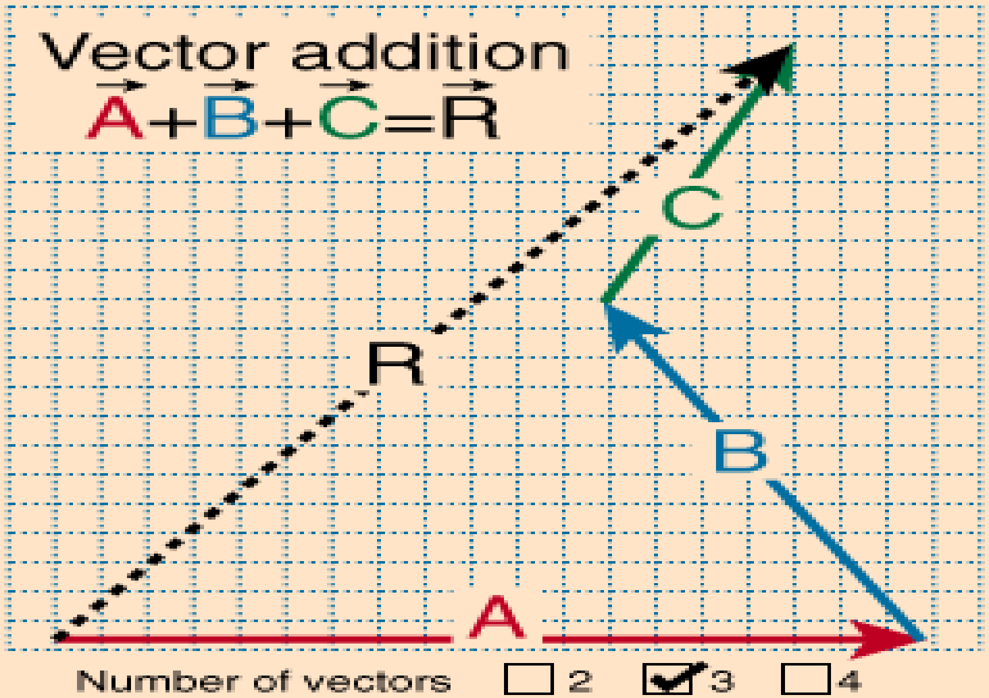
The resultant has magnitude

R = 26.7718751[illegible]

and angle

= 36.6619339[illegible] degrees.

Vector Addition, Three Vectors



Vector addition involves finding vector components, adding them and finding the polar form of the resultant.

The addition of vectors
A= 15 at 30 degrees,
B= 12 at 45 degrees,and
C= 10 at 60 degrees

yields components:

$A_x + B_x + C_x = R_x$

12.99038 + 8.485281 + 5.000000 = 26.47566

$A_y + B_y + C_y = R_y$

7.499999 + 8.485281 + 8.660254 = 24.64553

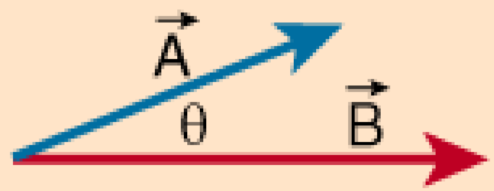
The resultant has magnitude
R = 36.1713023‹

and angle
= 42.9496999‹ degrees.

Scalar Product of Vectors

The scalar product and the vector product are the two ways of multiplying vectors which see the most application in physics and astronomy. The scalar product of two vectors can be constructed by taking the component of one vector in the direction of the other and multiplying it times the magnitude of the other vector. This can be expressed in the form:

$$\vec{A}\cdot\vec{B} = A\,B\cos\theta$$



$\vec{A}$ denotes vector

A denotes the magnitude of the vector.

If the vectors are expressed in terms of unit vectors i, j, and k along the x, y, and z directions, the scalar product can also be expressed in the form:

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

where

$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

$$\vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$$

The scalar product is also called the "inner product" or the "dot product" in some mathematics texts.

Scalar Product Calculation

You may enter values in any of the boxes below. Then click on the symbol for either the scalar product or the angle. The vectors A and B cannot be unambiguously calculated from the scalar product and the angle. If the angle is changed, then B will be placed along the x-axis and A in the xy plane.

Active formula: please click on the scalar product or the angle to update calculation.

$$\vec{A}\cdot\vec{B} = A\,B\cos\theta$$

The scalar product [A.B] = ([6])([12])(cos [45]) degrees.

B will be placed on the x axis and both A and B in the xy plane unless otherwise specified below.

$\vec{A}=$ [4.2426406] $\vec{i}$ + [4.2426406] $\vec{j}$ + [0] $\vec{k}$

$\vec{B}=$ [12] $\vec{i}$ + [0] $\vec{j}$ + [0] $\vec{k}$

$\vec{A}\cdot\vec{B} =$ ([4.242640])([12])+([4.242640])([0])+([0])([0]) = [A.B]

Scalar Product Applications

Geometrically, the scalar product is useful for finding the direction between arbitrary vectors in space. Since the two expressions for the product:

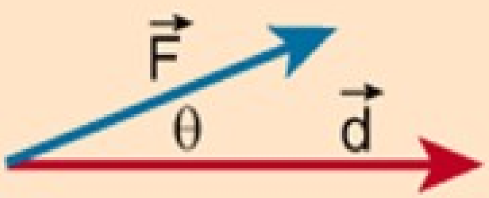
$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z = A\,B\cos\theta$$

involve the components of the two vectors and since the magnitudes A and B can be calculated from the components using:

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

then the cosine of the angle can be calculated and the angle determined.

One important physical application of the scalar product is the calculation of work:



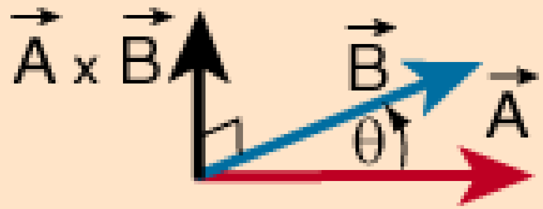
$$W = \vec{F}\cdot\vec{d} = F\cos\theta\, d$$

Work done by constant force, straight line motion.

Vector Product of Vectors

The vector product and the scalar product are the two ways of multiplying vectors which see the most application in physics and astronomy. The magnitude of the vector product of two vectors can be constructed by taking the product of the magnitudes of the vectors times the sine of the angle (<180 degrees) between them. The magnitude of the vector product can be expressed in the form:

$$\vec{A} \times \vec{B}_{\text{magnitude}} = A\,B \sin\theta$$



$\vec{A} \times \vec{B}$ is perpendicular to both A and B

and the direction is given by the right-hand rule. If the vectors are expressed in terms of unit vectors i, j, and k in the x, y, and z directions, then the vector product can be expressed in the rather cumbersome form:

$$\vec{A} \times \vec{B} = \vec{i}(A_y B_z - A_z B_y) - \vec{j}(A_x B_z - A_z B_x) + \vec{k}(A_x B_y - A_y B_x)$$

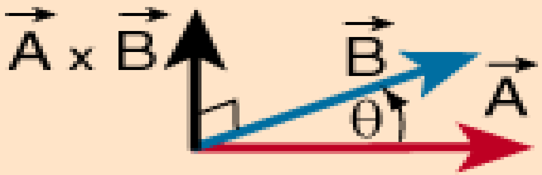
which may be stated somewhat more compactly in the form of a determinant.

Vector Product Calculation

You may enter values in any of the boxes below. Then click on the symbol for either the vector product or the angle.

$$\vec{A} \times \vec{B} = A\,B \sin\theta$$

The vector product [AxB] = ([15])([16])(sin [45]) degrees.



B will be placed on the x axis and both A and B in the xy plane unless otherwise specified below.

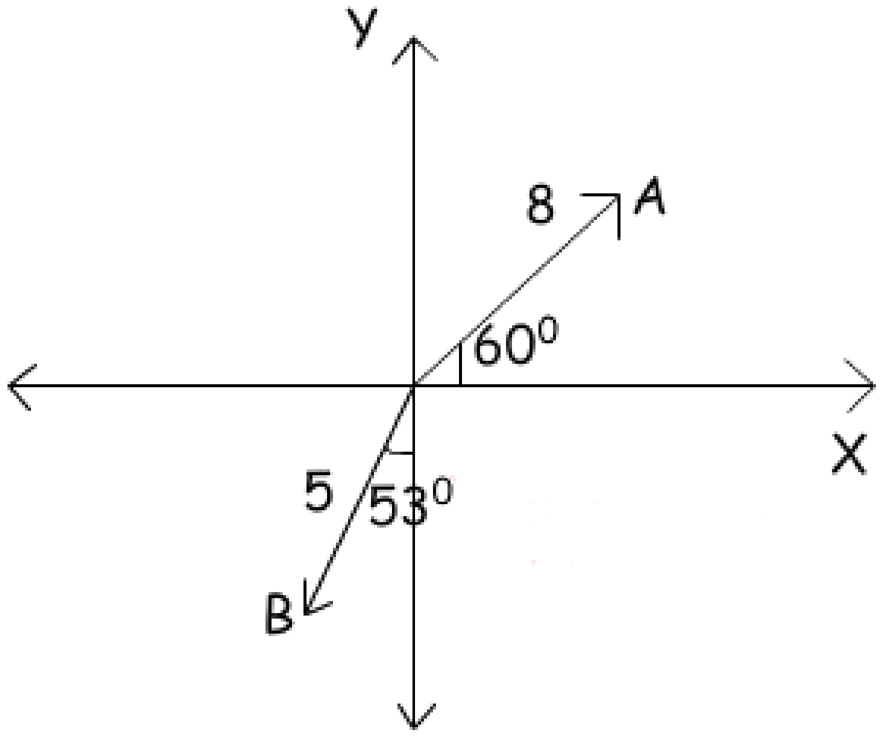
$\vec{A}=$ [10.606601] $\vec{i}$ + [10.606601] $\vec{j}$ + [0] $\vec{k}$

$\vec{B}=$ [16] $\vec{i}$ + [0] $\vec{j}$ + [0] $\vec{k}$

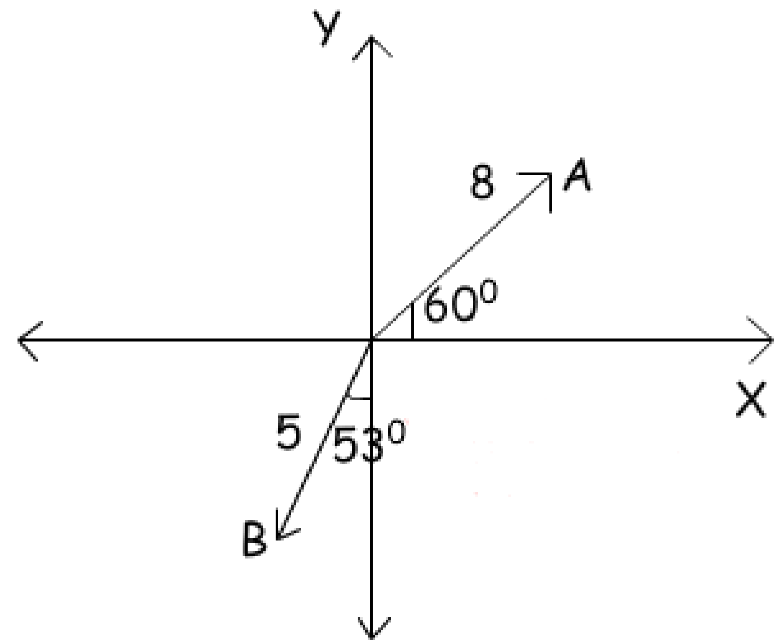
$\vec{A} \times \vec{B} =$ [0] $\vec{i}$ + [0] $\vec{j}$ + [-169.7056] $\vec{k}$

$\vec{A} \times \vec{B} =$ [AxB] magnitude.

Example Find the resultant vector of A and B given in the graph below. ($\sin 30° = 1/2$, $\sin 60° = \sqrt{3}/2$, $\sin 53° = 4/5$, $\cos 53° = 3/5$)



We use trigonometric equations first and find the components of the vectors then, make addition and subtraction between the vectors sharing same direction.



Components of A;

$A_x = A.\cos 60^0$

$A_x = 8.1/2 = 4$

$A_y = A.\sin 60^0$

$A_y = 8.\frac{\sqrt{3}}{2} = 4\sqrt{3}$

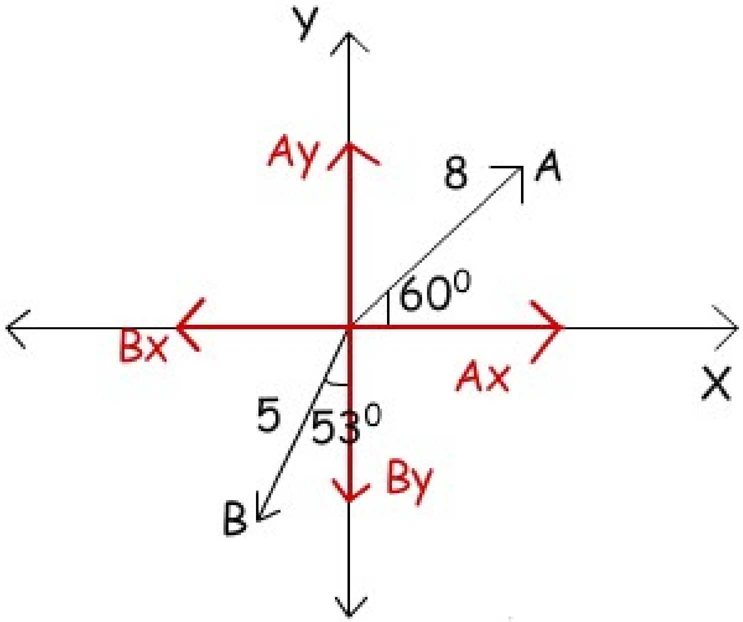
Components of B;

$B_x = B.\sin 53^0$

$B_x = 5.4/5 = 4$

$B_y = B.\cos 53^0$

$B_y = 5.3/5 = 3$

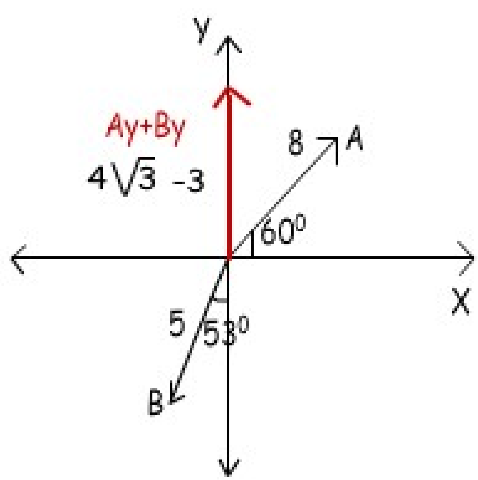


We sum the vectors having same direction:

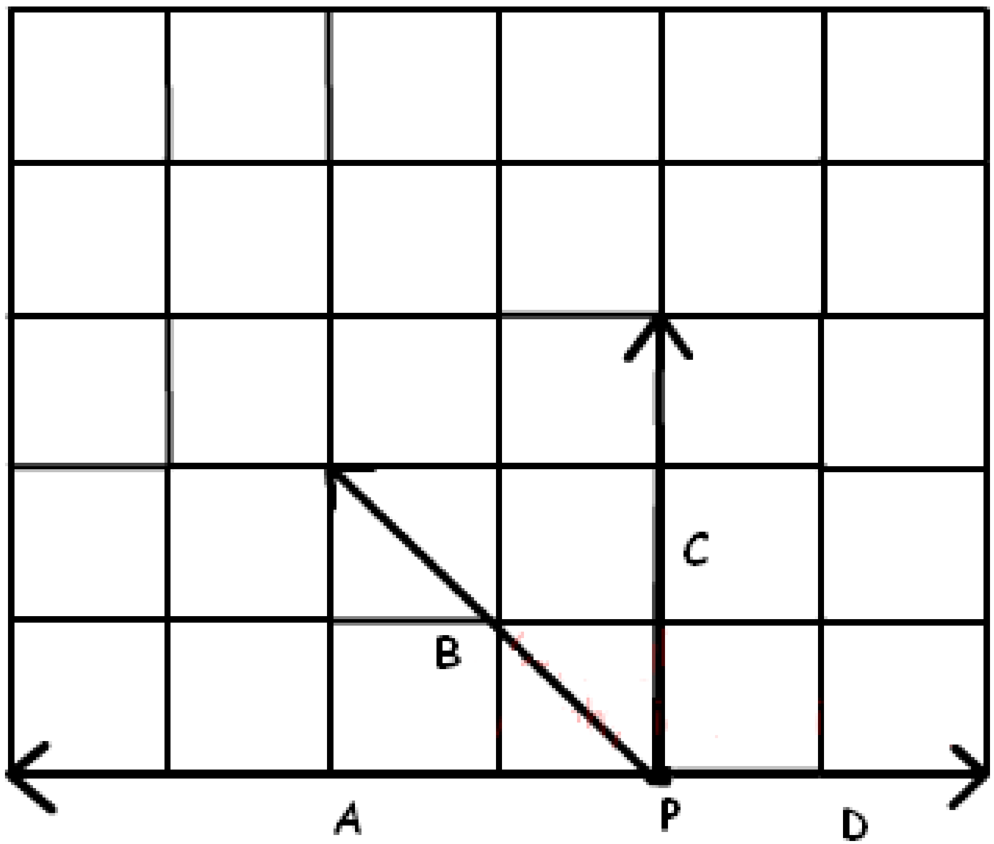
$Ax+Bx=4-4=0$

$Ay+By=4\sqrt{3}-3$

We put "-" in front of Bx, and By because we take right side and upward direction as positive



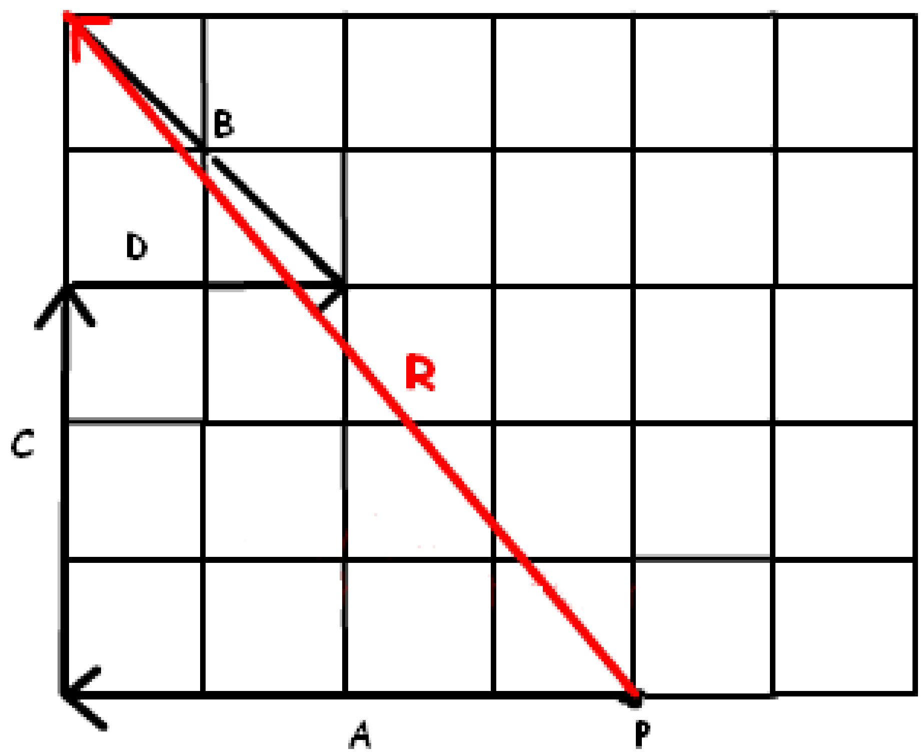
Find resultant of the following forces acting on an object at point P in figure given below.



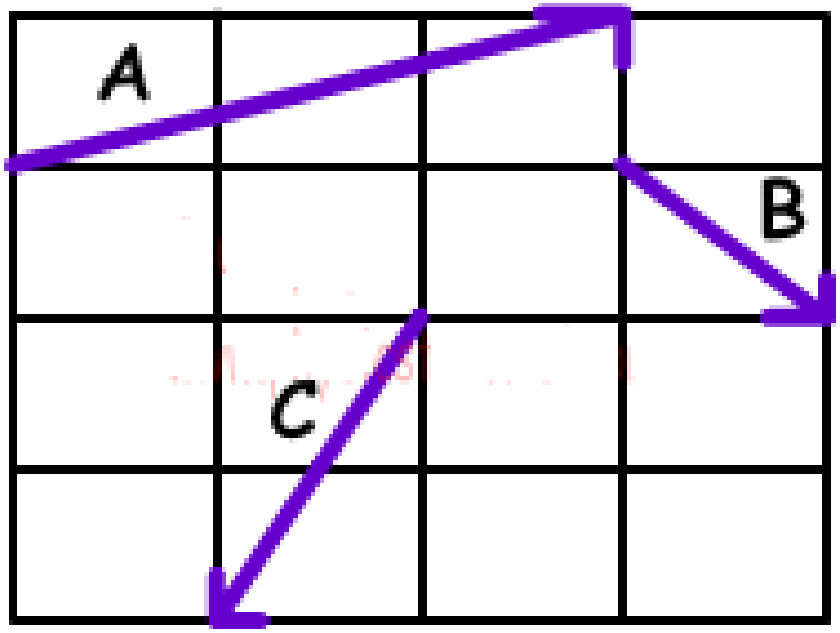
We add all vectros to find resultant force. Start with vector A and add vector C to it.

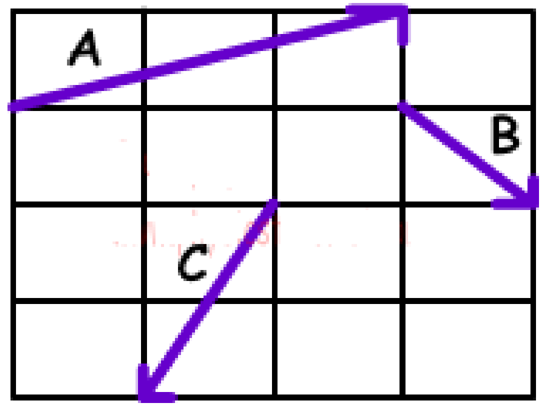
After that, add vector D and C and draw resultant vector by the starting point to the end.

Examine given solution below, resultant force is given in red color.

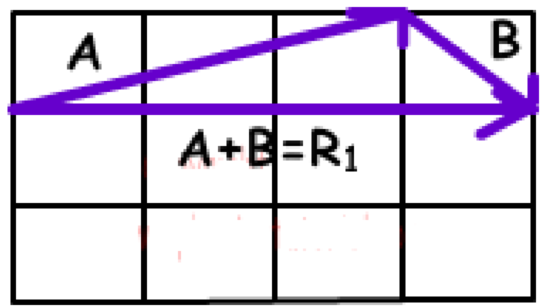


Find A+B+C.

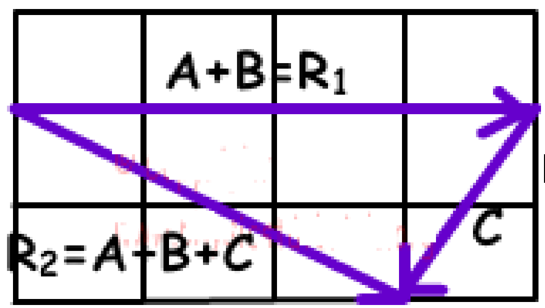




First, we find A+B then add it to vector C.

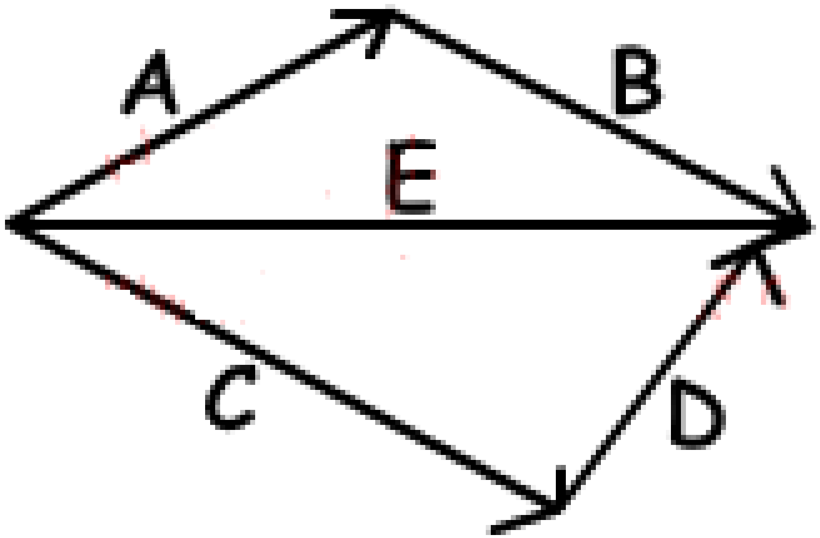


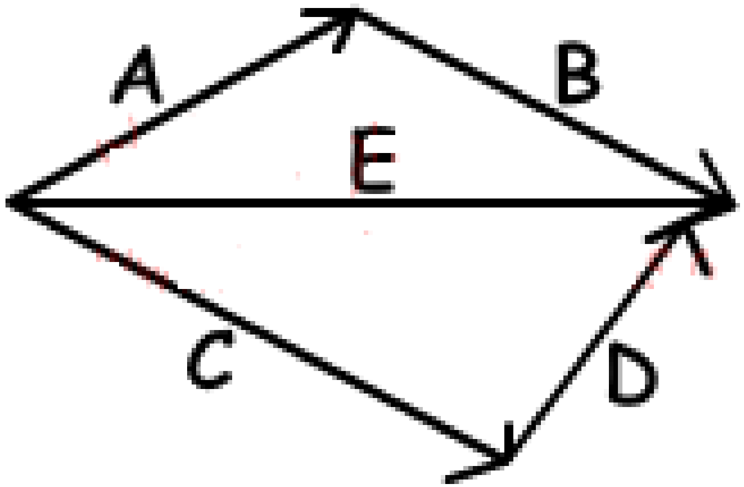
We find R_1, now we add C to R_1 to find resultant vector.



R_2=A+B+C

Find resultant vector.



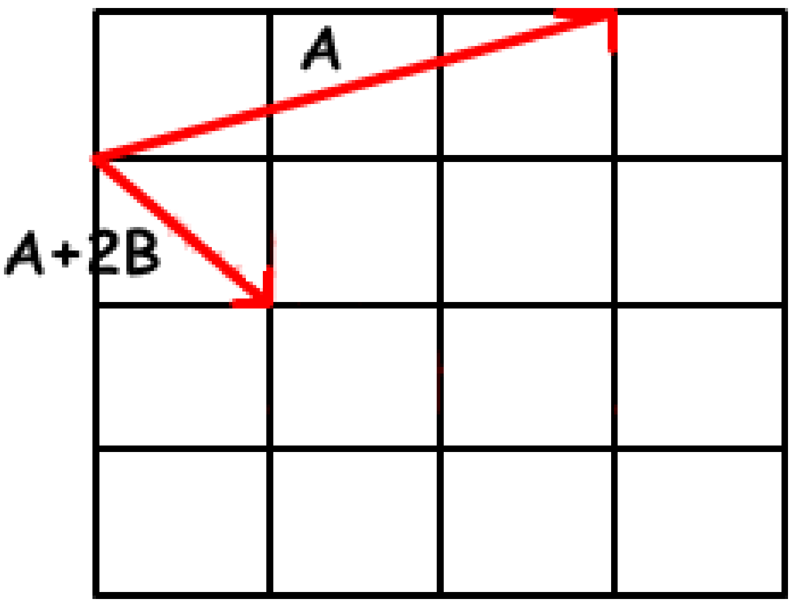


Since; A+B=E and C+D=E

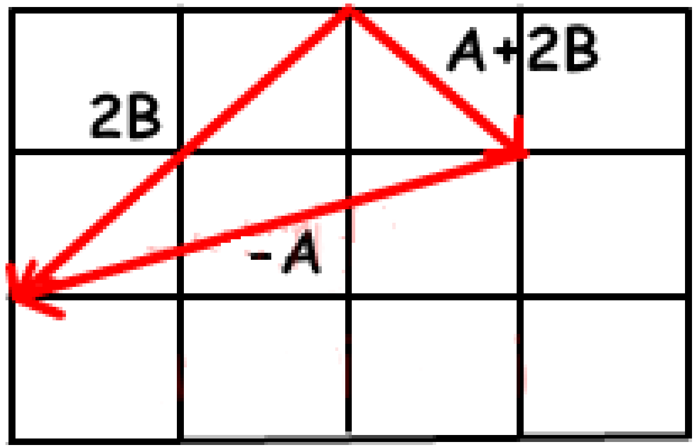
R=A+B+C+D+E

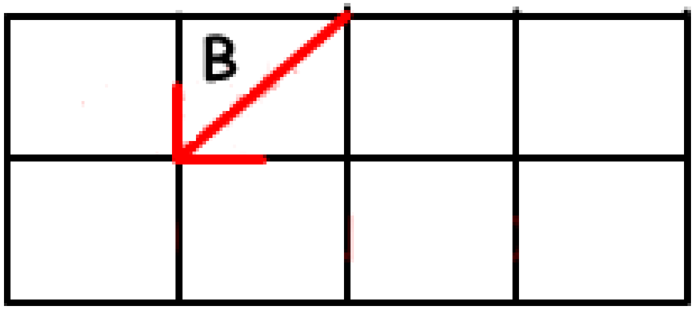
R=E+E+E=3E

A and A+2B vectors are given below. Find vector B.



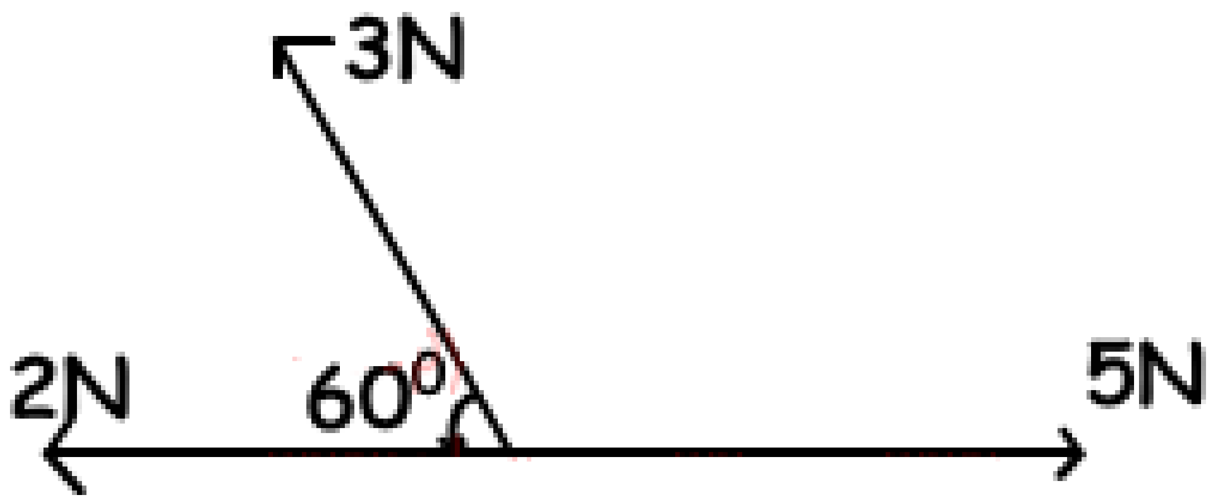
We use vector addition properties.
A+2B-A=2B

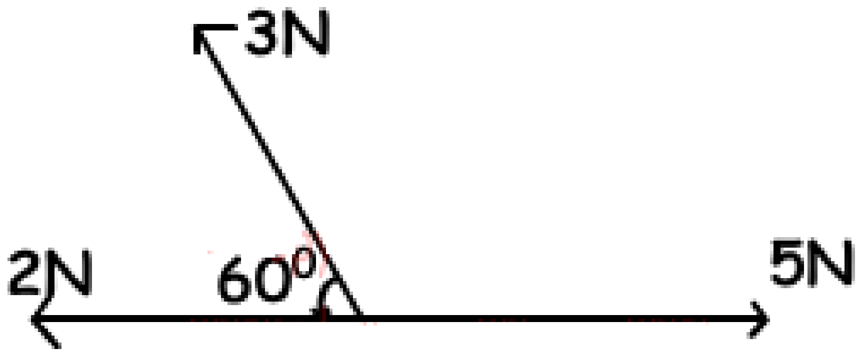




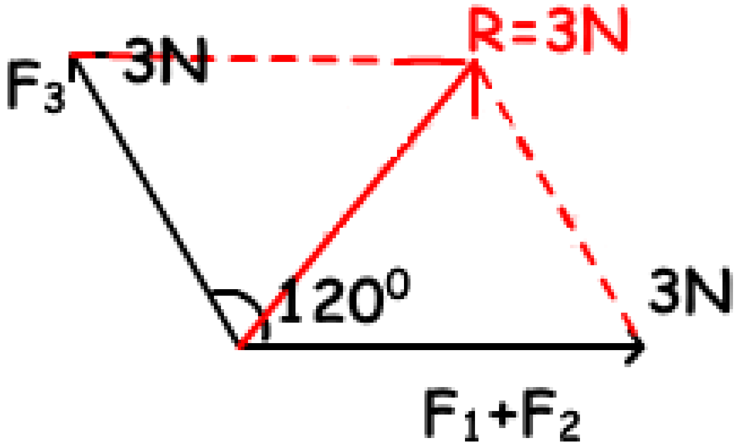
To get vector B, we multiply 2B with 1/2.

Find resultant vector.



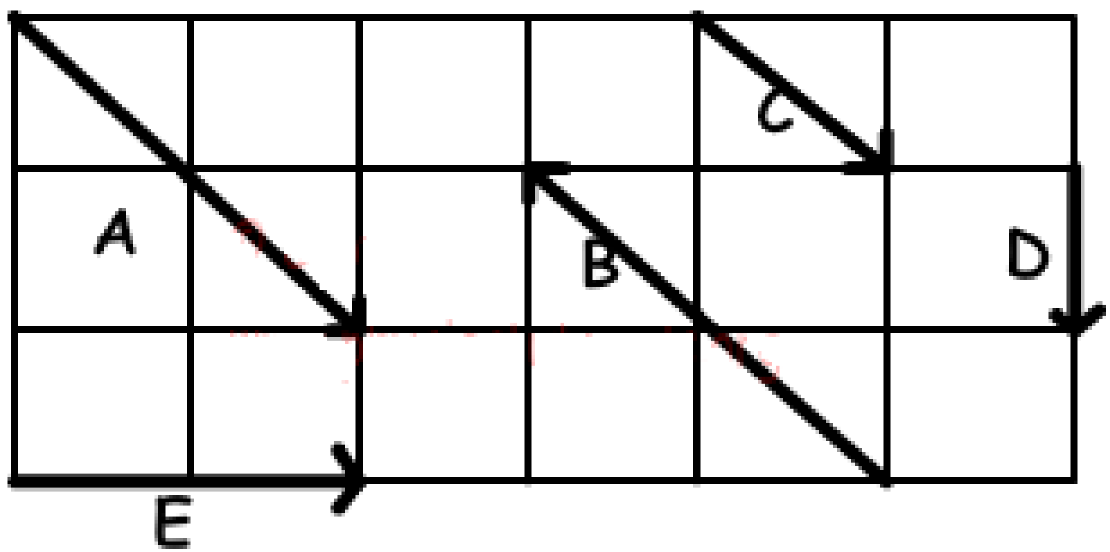


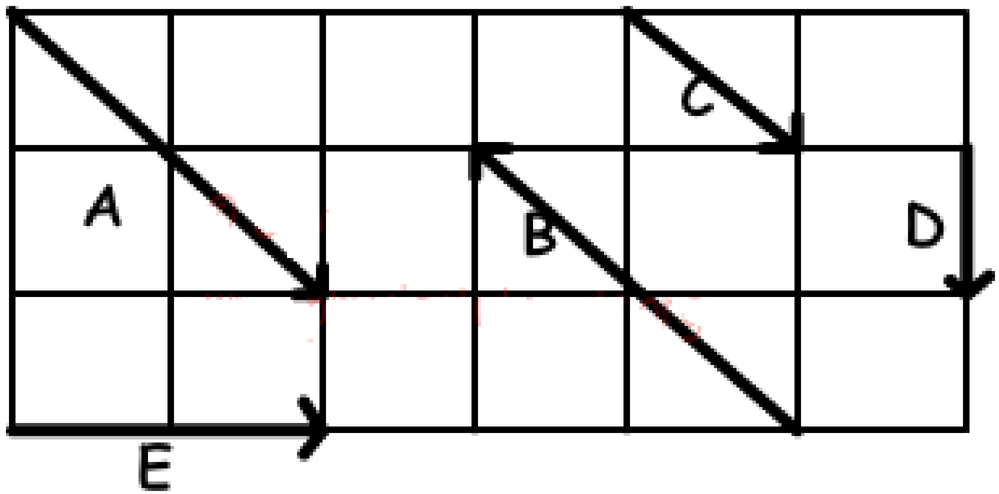
$F_1+F_2=5-2=3N$



$F_1+F_2+F_3=R=3N$

Which one of the following statements is true?





I. A=B in magnitude
II. A=2C
III. E=2D
IV. A=B

As you can see in the figure given above, A and B are equal in magnitude, so I. is true.
If you multiply C with 2, you get A, this means that II. is also true
E =2D in magnitude but not in direction. Thus; III. is false.

SOME EXAMPLES
on
UNIT CONVERSIONS and VECTORS

Unit Conversions

10	meters x 3.28 =	32.8	feet
75	centimeters x .3937 =	29.5275	inches
45	miles x 5280 =	237600	feet
15	miles x 1609 =	24135	meters
33	Newtons x .2248 =	7.4184	pounds
15	Newtons x 100000 =	1500000	dynes
75	kilograms x .06852 =	5.139	slugs
30	meters/sec x 2.24 =	67.2	miles/hour
50	km/hour x 0.278 =	13.9000000(	meters/sec
20	atmosphere x 101.3 =	2026	kilopascals

50	atmosphere x 14.69 =	734.5 lb/in^2
20	lb/in^2 x 51.7 =	1034 mmHg
5000	mmHg x 1333 =	6665000 $dyne/cm^2$
400	$dynes/cm^2$ x 0.1 =	40 pascals
55	mmHg x 133.3 =	7331.50000(pascals
10000	cm^3 x .000001 =	0.01 m^3
40000	in^3 x .000163 =	0.652 m^3
45	in^3 x 16.39 =	737.550000(cm^3
950	ft^3 x .02832 =	26.904 m^3
56	liters x .001 =	0.056 m^3

10 cm/h^2 = ….. m/s^2

100 kg/m^3 = …. g/cm^3

$$10\,\frac{cm}{hr^2}\times\frac{1\,hr}{3600\,s}\times\frac{1\,hr}{3600\,s}\times\frac{1\,m}{100\,cm}=772\times10^{-11}\,\frac{cm}{s^2}$$

$$100\,\frac{kg}{m^3}\times\frac{1000\,g}{1kg}\times\frac{1m^3}{1000000cm^3}=0.1\,\frac{g}{cm^3}$$

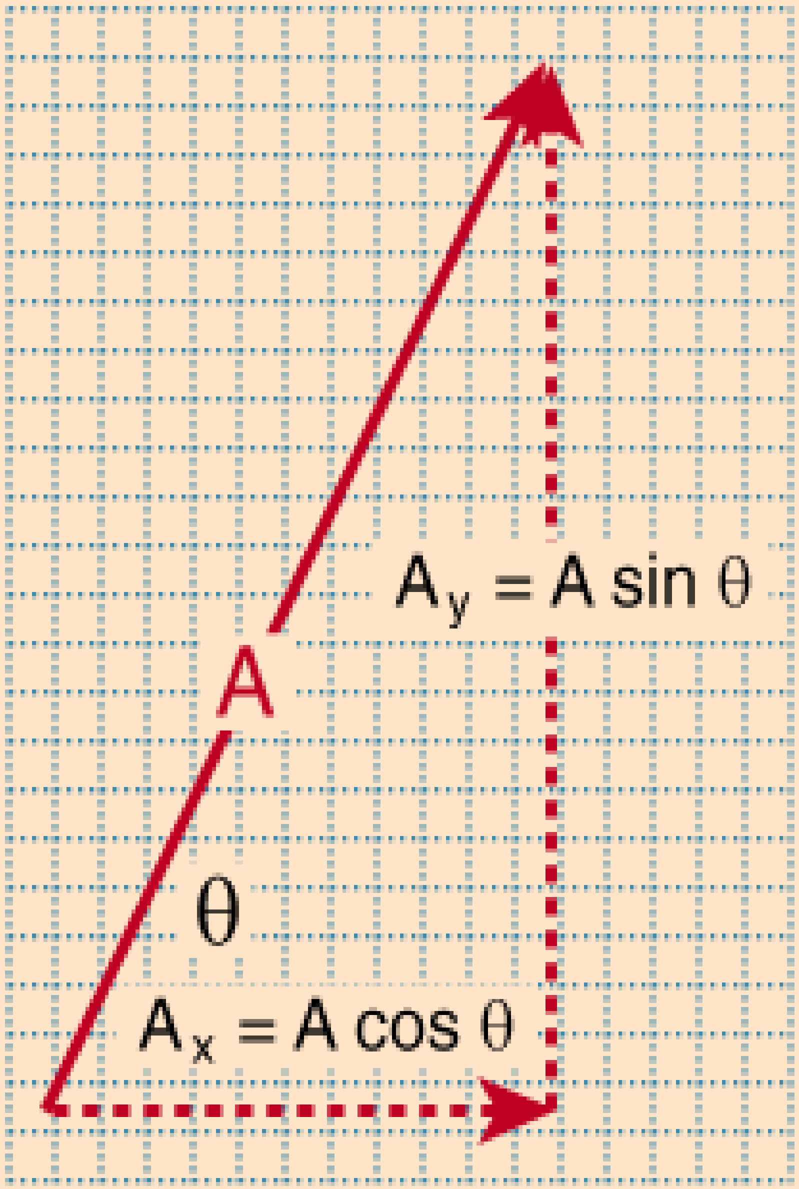
0.05 mega = …….. nano
6 mili = …….. mikro

0.05 mega = 0.05×10^{15} nano
6 mili = 6×10^{3} mikro

7 nano = …….. giga
50 nano = ……… tera

7 nano = 7×10^{-18} giga
50 nano = 50×10^{-21} tera

Resolving a Vector Into Components

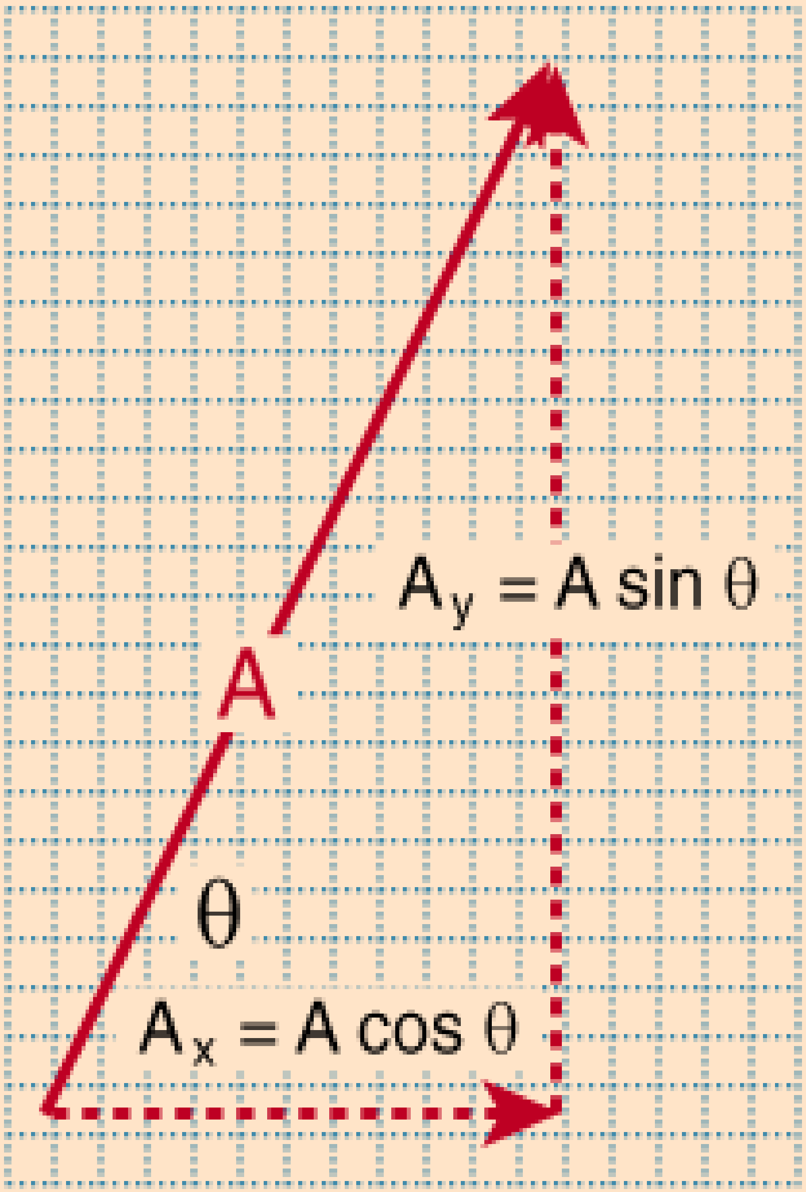


For vector A= 50 N at angle 30 degrees,

the horizontal component is = 43.30127018 N

and the vertical component is = 24.99999999 N

Resolving a Vector Into Components



For vector A= 50 N

at angle 45 degrees,

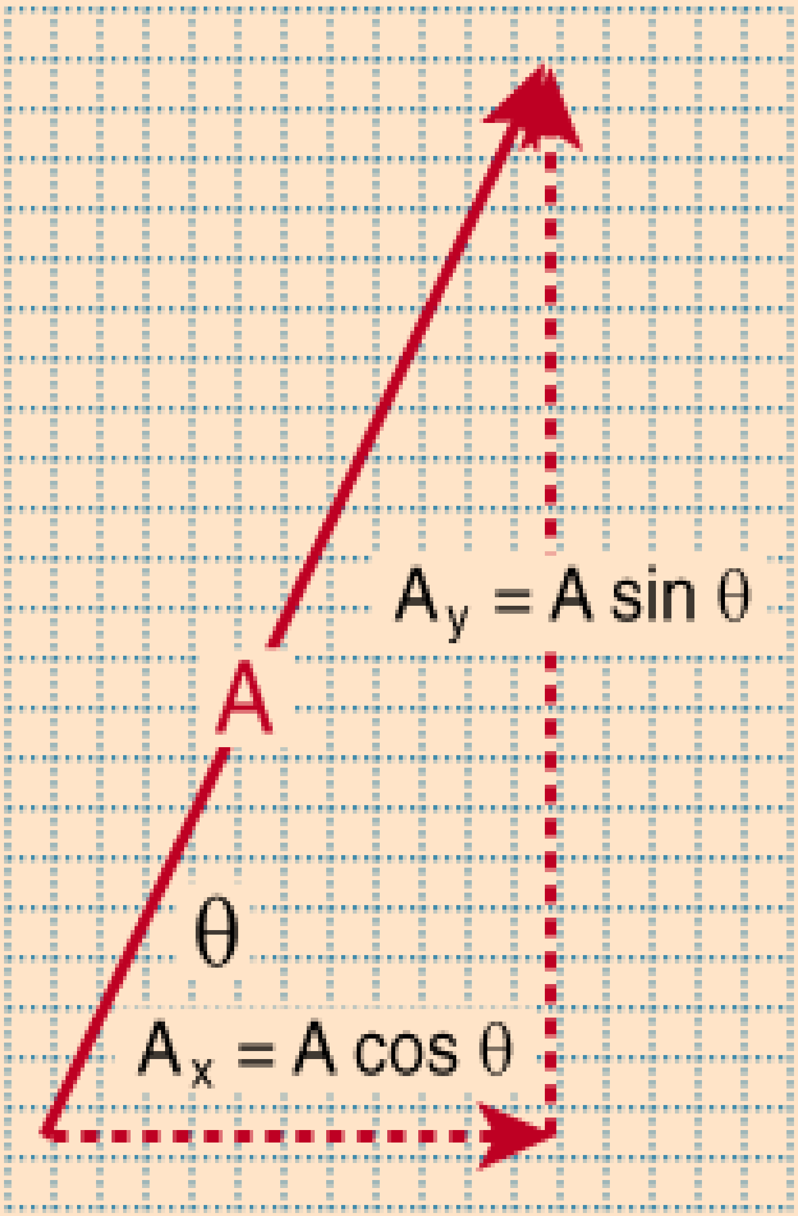
the horizontal component is

= 35.3553390! N

and the vertical component is

= 35.3553390! N

Resolving a Vector Into Components



For vector A= 50

N

at angle 60 degrees,

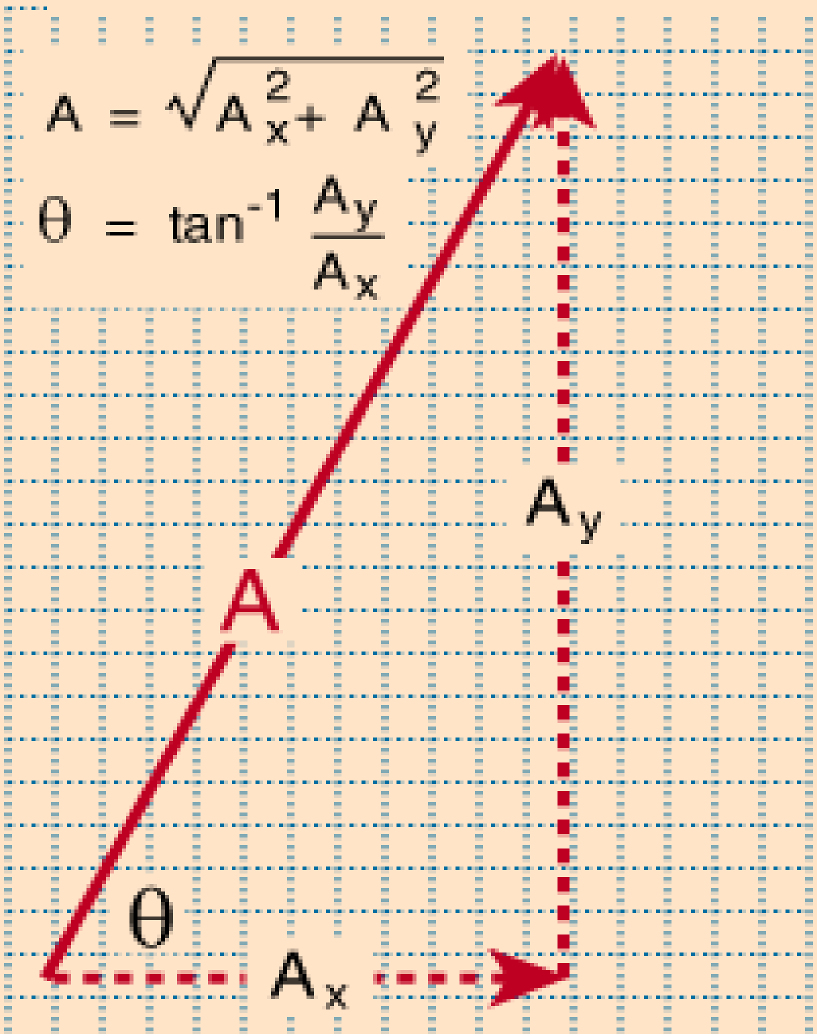
the horizontal component is

= 25.0000000(N

and the vertical component is

= 43.3012701(N

Magnitude and Direction from Components



If the horizontal component is

= 3 N

and the vertical component is

= 4 N ,

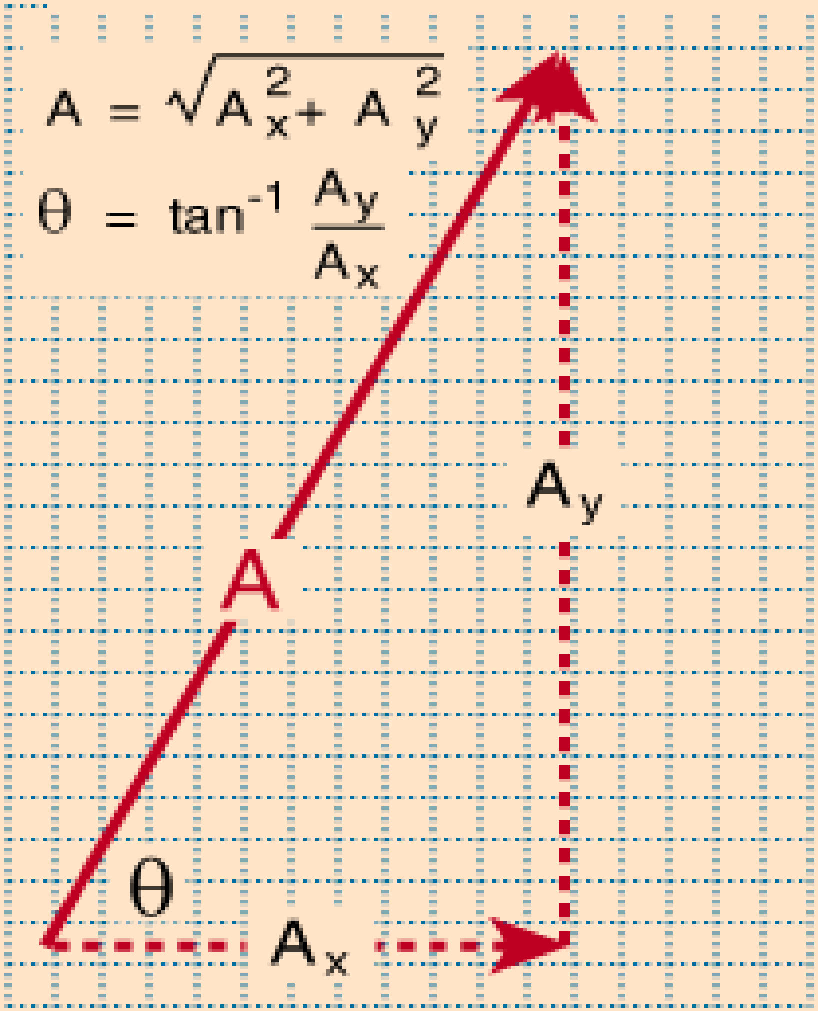
then the magnitude is

= 5 N

and the angle is

= 53.1301023$ degrees.

Magnitude and Direction from Components



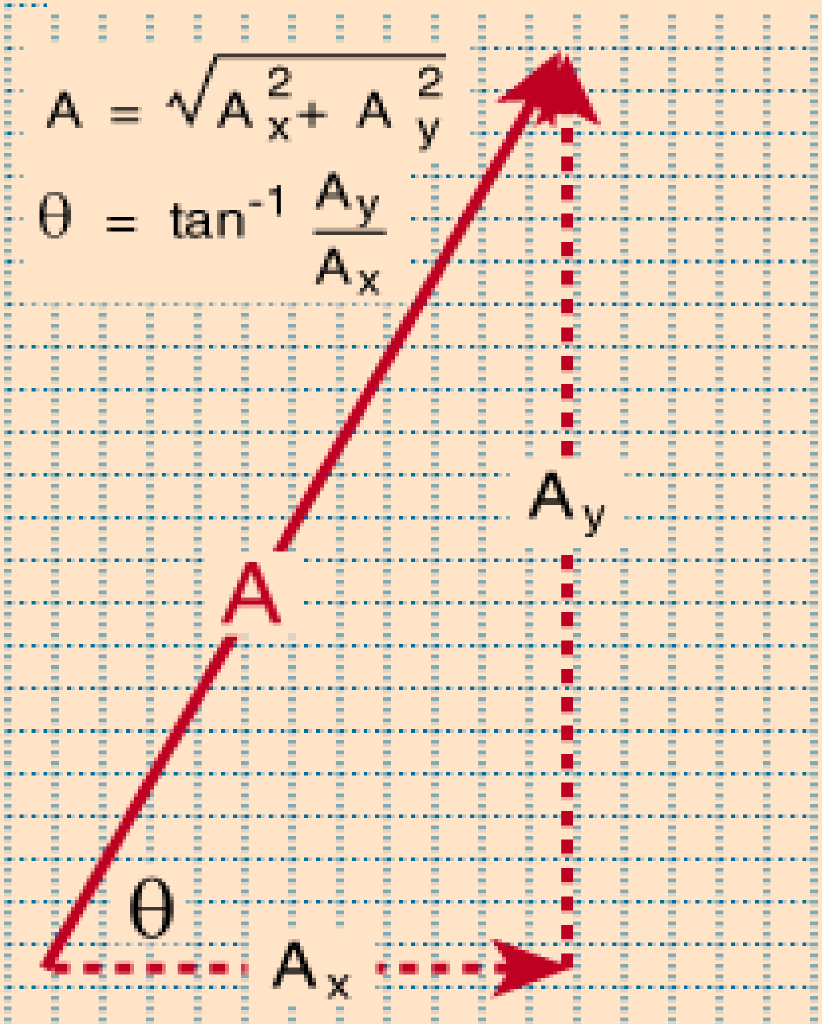
If the horizontal component is = 4 N

and the vertical component is = 4 N ,

then the magnitude is = 5.65685424$ N

and the angle is = 45 degrees.

Magnitude and Direction from Components



If the horizontal component is

= 4 N

and the vertical component is

= 3 N ,

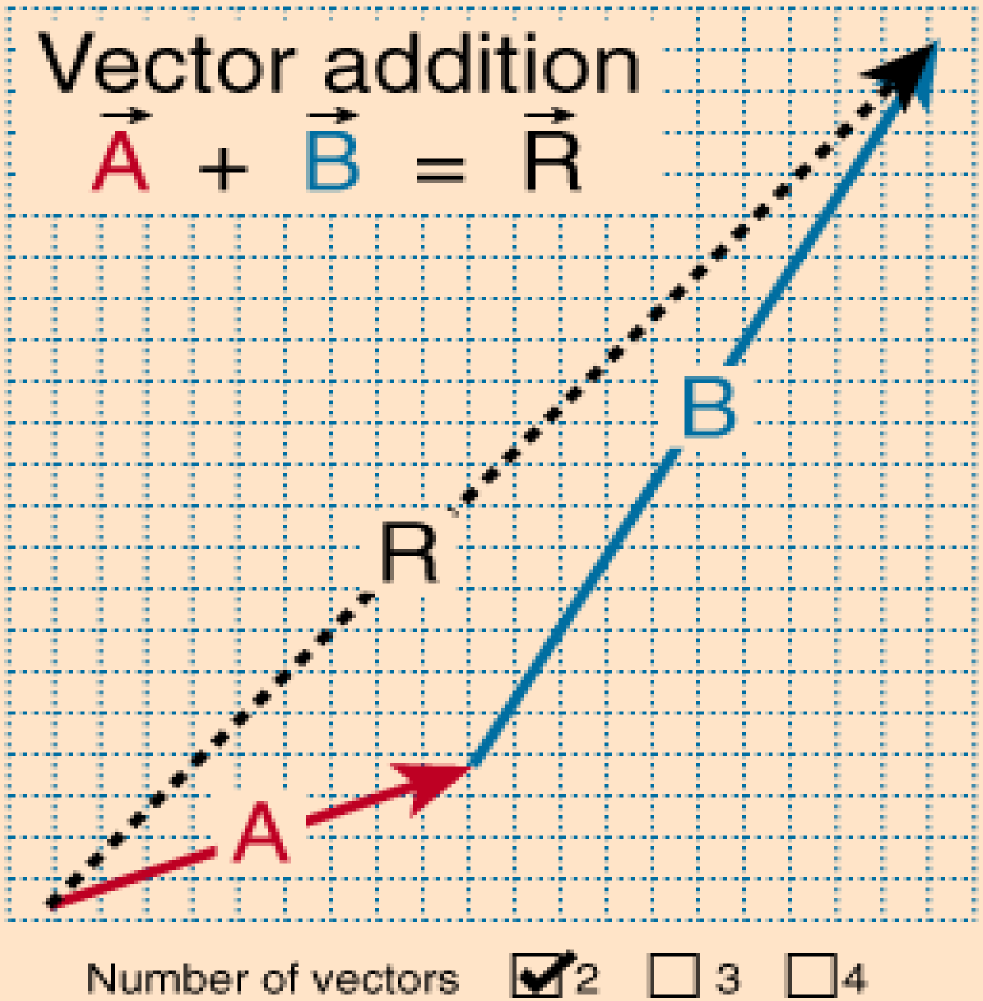
then the magnitude is

= 5 N

and the angle is

= 36.8698976 degrees.

Vector Addition, Two Vectors



The addition of vector

A= 5 at 30 degrees,

and vector

B= 4 at 135 degrees,

yields components:

$A_x + B_x = R_x$

4.330127 + -2.82842 = 1.501699

$A_y + B_y = R_y$

2.499999 + 2.828427 = 5.328427

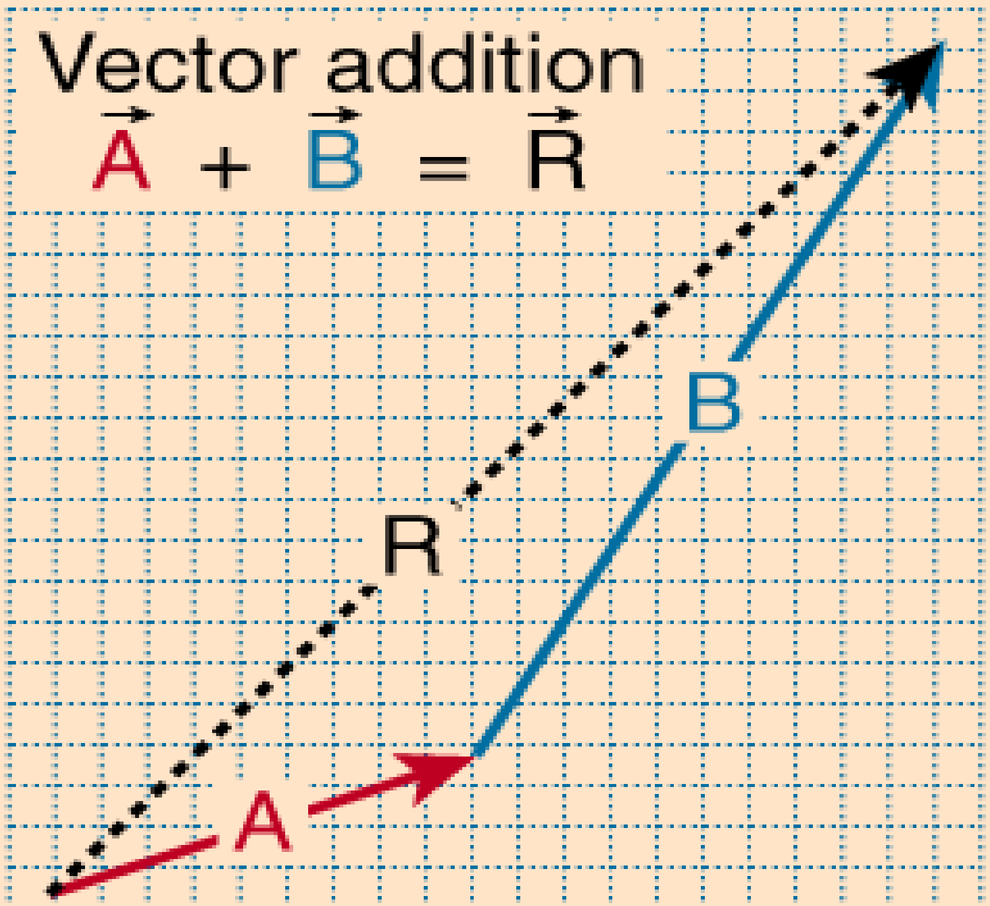
The resultant has magnitude

R = 5.53599477

and angle

= 74.2606780 degrees.

Vector Addition, Two Vectors



Number of vectors ☑2 ☐3 ☐4

The addition of vector

A= 5 at -30 degrees,

and vector

B= 4 at 60 degrees,

yields components:

$A_x + B_x = R_x$

4.330127 + 2.000000 = 6.330127

$A_y + B_y = R_y$

-2.49999 + 3.464101 = 0.964101

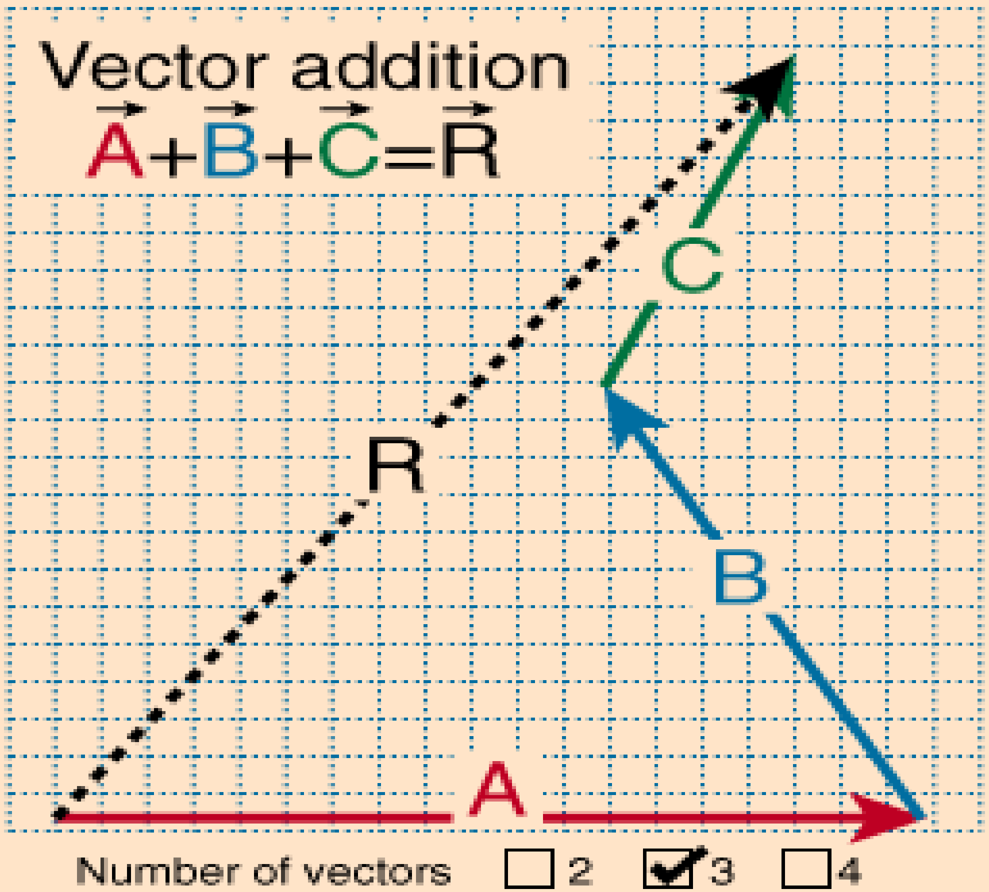
The resultant has magnitude

R = 6.40312423

and angle

= 8.65980825 degrees.

Vector Addition, Three Vectors



The addition of vectors

A= 10 at 30 degrees,

B= 25 at 120 degrees, and

C= 30 at -45 degrees

yields components:

$A_x + B_x + C_x = R_x$

8.660254 + -12.4999 + 21.21320 = 17.37345

$A_y + B_y + C_y = R_y$

4.999999 + 21.65063 + -21.2132 = 5.437431

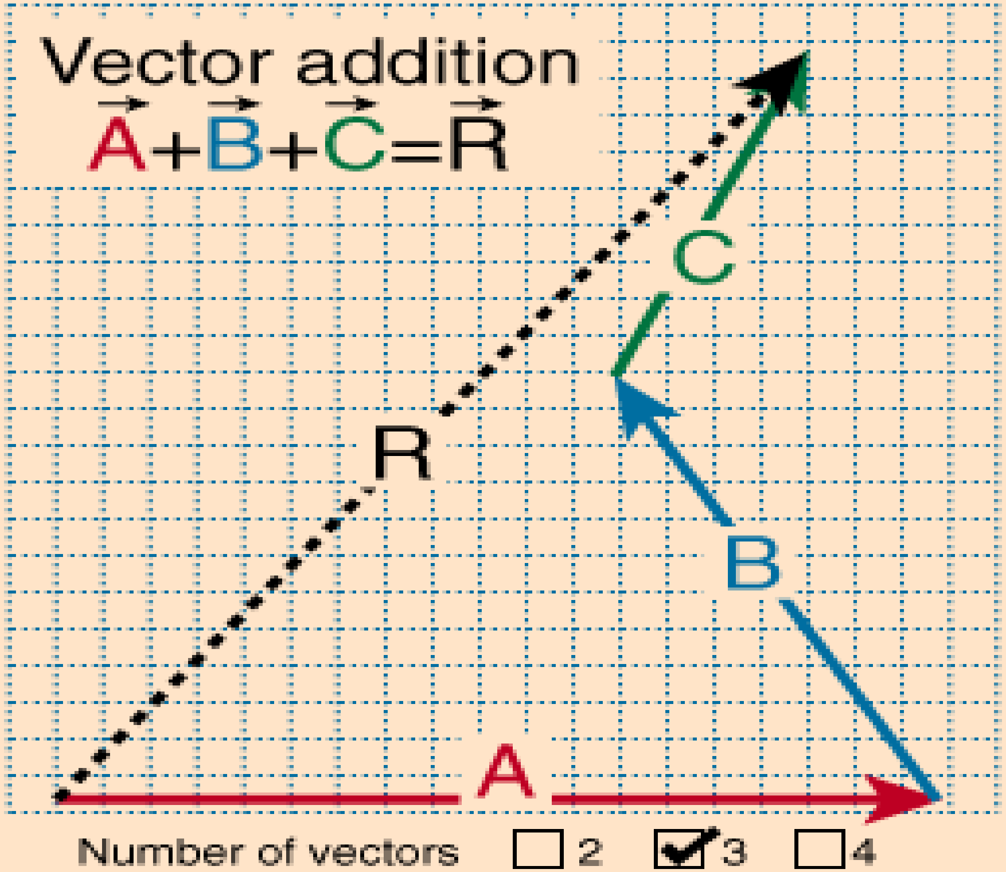
The resultant has magnitude

R = 18.2044688

and angle

= 17.3787367 degrees.

Vector Addition, Three Vectors



The addition of vectors

A= 10 at -60 degrees,

B= 25 at 30 degrees, and

C= 30 at 135 degrees

yields components:

$A_x + B_x + C_x = R_x$

5.000000 + 21.65063 + -21.2132(

= 5.437431

$A_y + B_y + C_y = R_y$

-8.66025(+ 12.49999 + 21.21320

= 25.05294

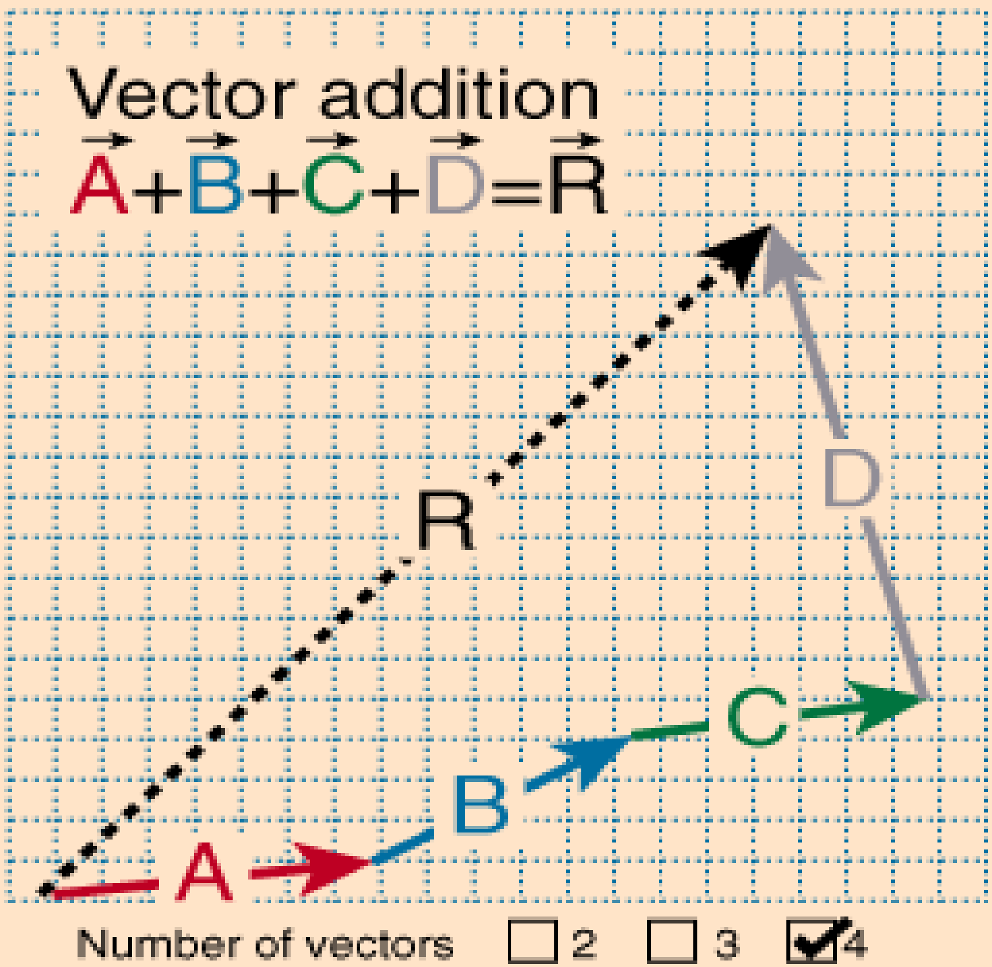
The resultant has magnitude

R = 25.6362231(

and angle

= 77.7545797(degrees.

Vector Addition, Four Vectors



The addition of vectors

A= 10 at 30 degrees,

B= 15 at -60 degrees,

C= 12 at 135 degrees,

D= 20 at 45 degrees,

yields components:

$A_x + B_x + C_x + D_x = R_x$

8.660254 + 7.500000 + -8.48528 +

14.14213 = 21.81710

$A_y + B_y + C_y + D_y = R_y$

4.999999 + -12.9903 + 8.485281 +

14.14213 = 14.63703

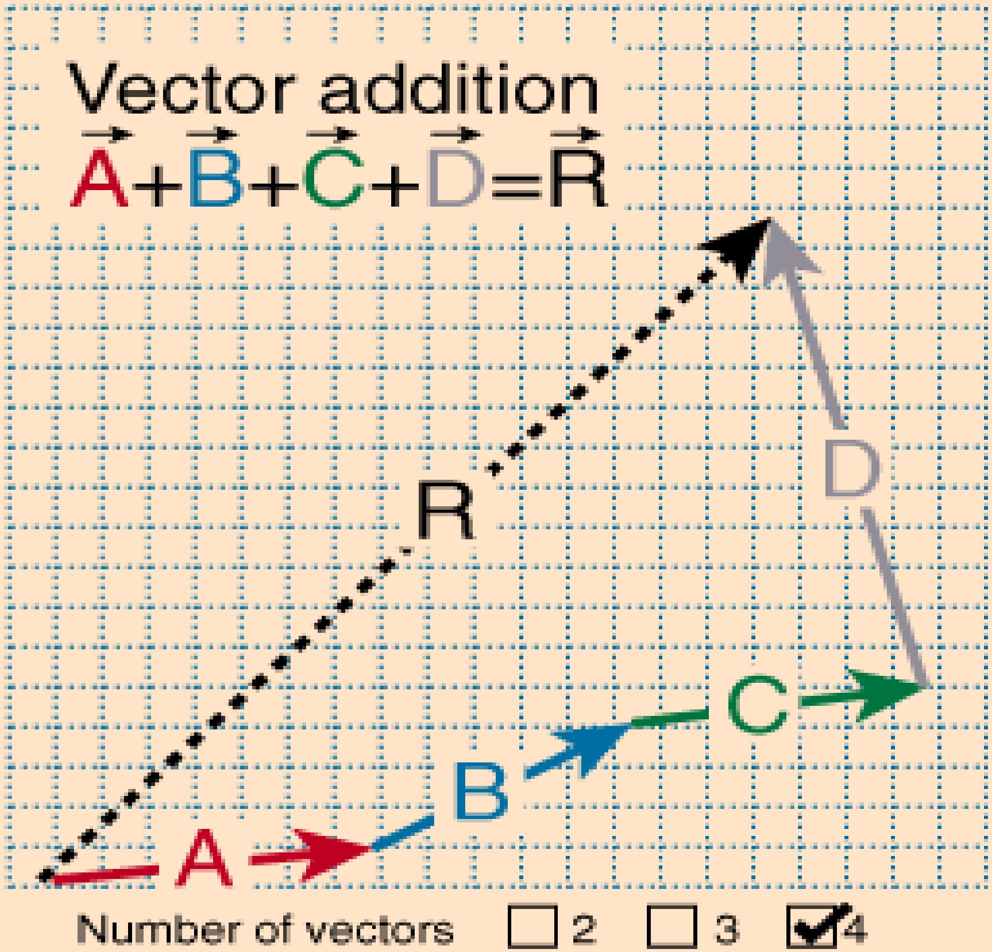
The resultant has magnitude

R = 26.2722103

and angle

= 33.8575480 degrees.

Vector Addition, Four Vectors



The addition of vectors

A= 20 at -45 degrees,
B= 15 at 120 degrees,
C= 18 at 60 degrees,
D= 30 at -60 degrees,

yields components:

$A_x + B_x + C_x + D_x = R_x$

14.14213 + -7.49999 + 9.000000 + 15.00000 = 30.64213

$A_y + B_y + C_y + D_y = R_y$

-14.1421 + 12.99038 + 15.58845 + -25.9807 = -11.5440

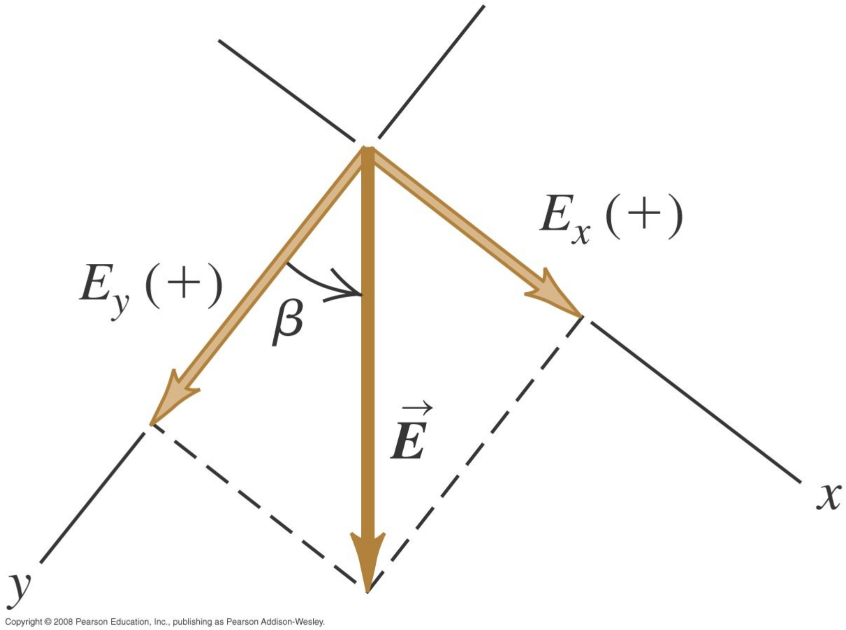
The resultant has magnitude

R = 32.7445534

and angle

= 339.356697 degrees.

Q1.1



What are the x– and y–components of the vector $\vec{E}$?

A. $E_x = E \cos \beta$, $E_y = E \sin \beta$

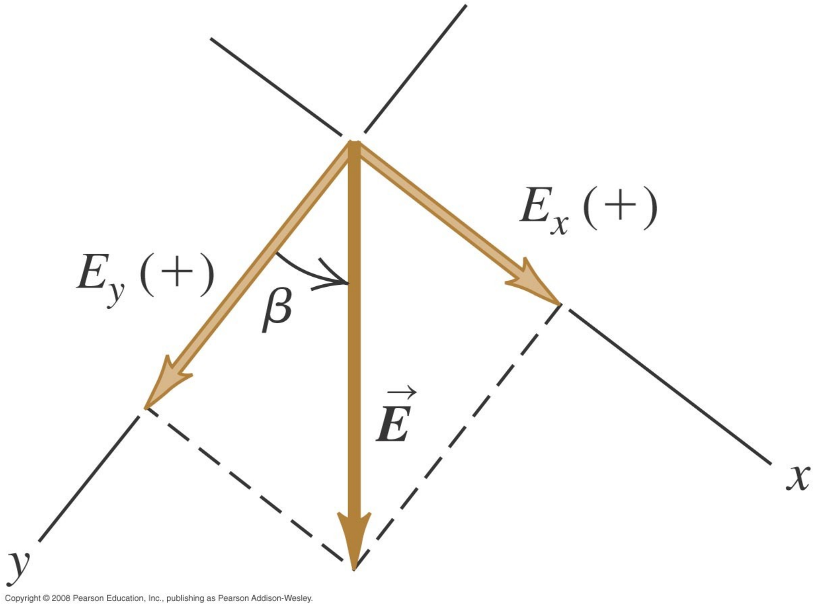
B. $E_x = E \sin \beta$, $E_y = E \cos \beta$

C. $E_x = -E \cos \beta$, $E_y = -E \sin \beta$

D. $E_x = -E \sin \beta$, $E_y = -E \cos \beta$

E. $E_x = -E \cos \beta$, $E_y = E \sin \beta$

A1.1



What are the x– and y–components of the vector $\vec{E}$?

A. $E_x = E \cos \beta$, $E_y = E \sin \beta$

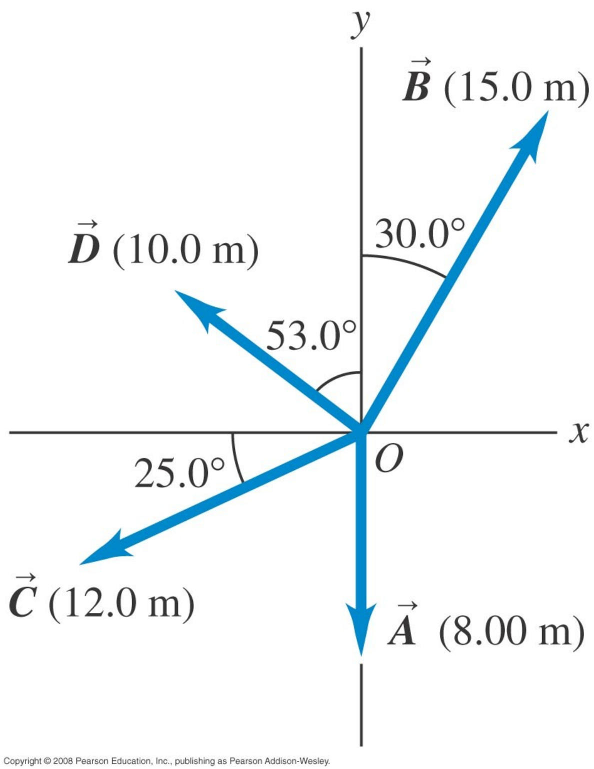
B. $E_x = E \sin \beta$, $E_y = E \cos \beta$ ✔

C. $E_x = -E \cos \beta$, $E_y = -E \sin \beta$

D. $E_x = -E \sin \beta$, $E_y = -E \cos \beta$

E. $E_x = -E \cos \beta$, $E_y = E \sin \beta$

Q1.2



Consider the vectors shown. Which is a correct statement about $\vec{A}+\vec{B}$?

A. x–component > 0, y–component > 0

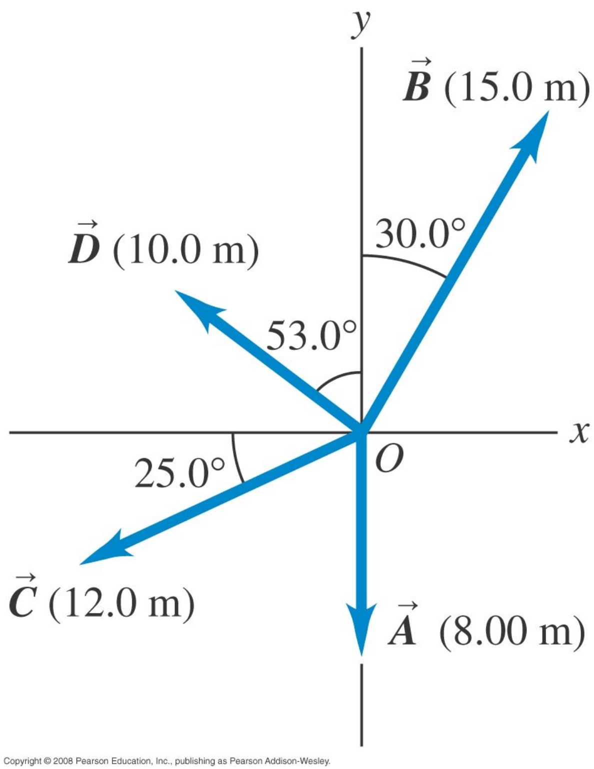
B. x–component > 0, y–component < 0

C. x–component < 0, y–component > 0

D. x–component < 0, y–component < 0

E. x–component = 0, y–component > 0

A1.2



Consider the vectors shown. Which is a correct statement about $\vec{A} + \vec{B}$?

A. x–component > 0, y–component > 0

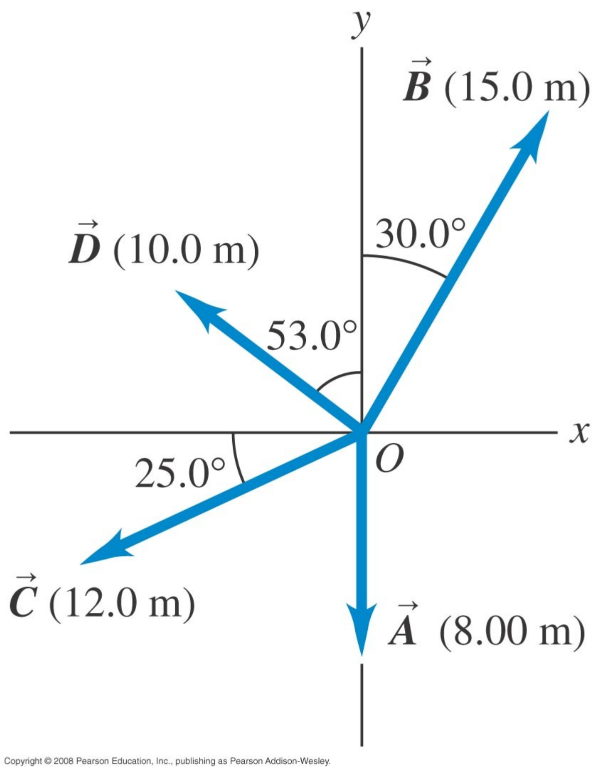
B. x–component > 0, y–component < 0

C. x–component < 0, y–component > 0

D. x–component < 0, y–component < 0

E. x–component = 0, y–component > 0

Q1.3



Consider the vectors shown. Which is a correct statement about $\vec{A} - \vec{B}$?

A. x–component > 0, y–component > 0

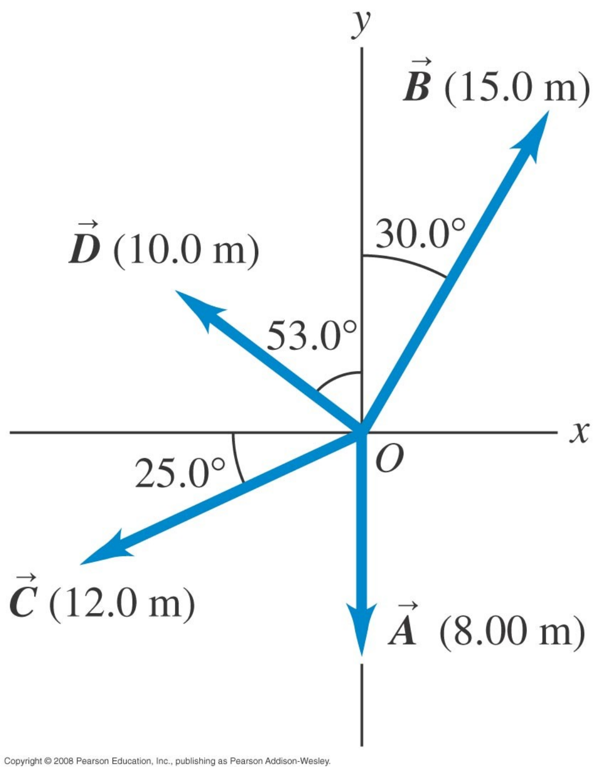
B. x–component > 0, y–component < 0

C. x–component < 0, y–component > 0

D. x–component < 0, y–component < 0

E. x–component = 0, y–component > 0

A1.3



Consider the vectors shown. Which is a correct statement about $\vec{A} - \vec{B}$?

A. x–component > 0, y–component > 0

B. x–component > 0, y–component < 0

C. x–component < 0, y–component > 0

✓ D. x–component < 0, y–component < 0

E. x–component = 0, y–component > 0

Q1.4

Which of the following statements is correct for *any* two vectors $\vec{A}$ and $\vec{B}$?

A. the magnitude of $\vec{A}+\vec{B}$ is $A+B$

B. the magnitude of $\vec{A}+\vec{B}$ is $A-B$

C. the magnitude of $\vec{A}+\vec{B}$ is greater than or equal to $|A-B|$

D. the magnitude of $\vec{A}+\vec{B}$ is greater than the magnitude of $\vec{A}-\vec{B}$

E. the magnitude of $\vec{A}+\vec{B}$ is $\sqrt{A^2+B^2}$

A1.4

Which of the following statements is correct for *any* two vectors $\vec{A}$ and $\vec{B}$?

A. the magnitude of $\vec{A}+\vec{B}$ is $A + B$

B. the magnitude of $\vec{A}+\vec{B}$ is $A - B$

✓ C. the magnitude of $\vec{A}+\vec{B}$ is greater than or equal to $|A - B|$

D. the magnitude of $\vec{A}+\vec{B}$ is greater than the magnitude of $\vec{A}-\vec{B}$

E. the magnitude of $\vec{A}+\vec{B}$ is $\sqrt{A^2 + B^2}$

Q1.5

Which of the following statements is correct for *any* two vectors $\vec{A}$ and $\vec{B}$?

A. the magnitude of $\vec{A} - \vec{B}$ is $A - B$

B. the magnitude of $\vec{A} - \vec{B}$ is $A + B$

C. the magnitude of $\vec{A} - \vec{B}$ is greater than or equal to $|A - B|$

D. the magnitude of $\vec{A} - \vec{B}$ is less than the magnitude of $\vec{A} + \vec{B}$

E. the magnitude of $\vec{A} - \vec{B}$ is $\sqrt{A^2 + B^2}$

A1.5

Which of the following statements is correct for *any* two vectors $\vec{A}$ and $\vec{B}$?

A. the magnitude of $\vec{A} - \vec{B}$ is $A - B$

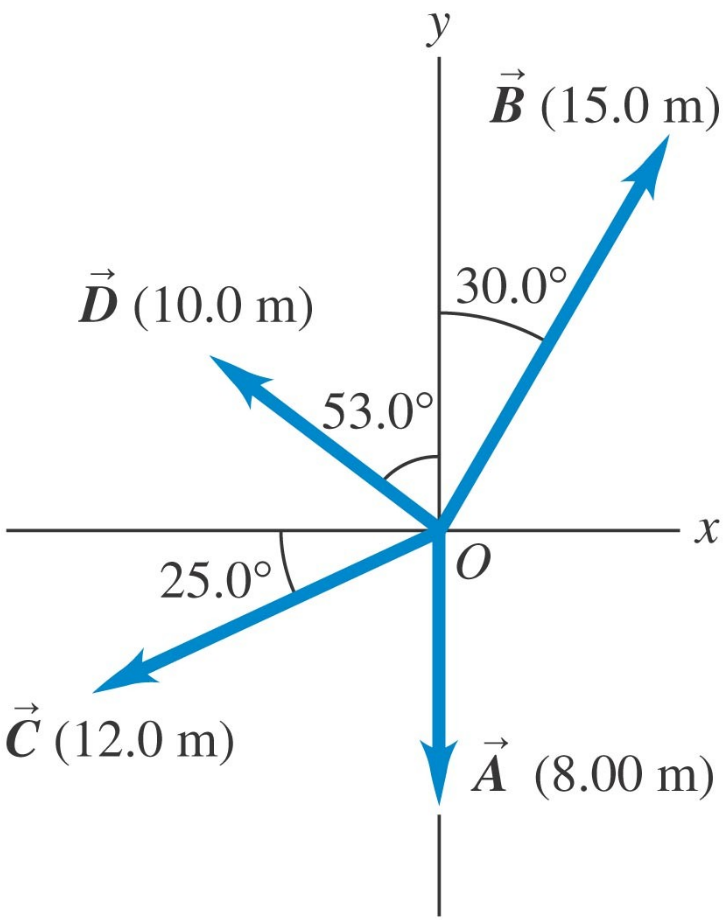
B. the magnitude of $\vec{A} - \vec{B}$ is $A + B$

C. the magnitude of $\vec{A} - \vec{B}$ is greater than or equal to $|A - B|$

D. the magnitude of $\vec{A} - \vec{B}$ is less than the magnitude of $\vec{A} + \vec{B}$

E. the magnitude of $\vec{A} - \vec{B}$ is $\sqrt{A^2 + B^2}$

Q1.6



Consider the vectors shown.

What are the components of the vector $\vec{E} = \vec{A} + \vec{D}$?

A. $E_x = -8.00$ m, $E_y = -2.00$ m

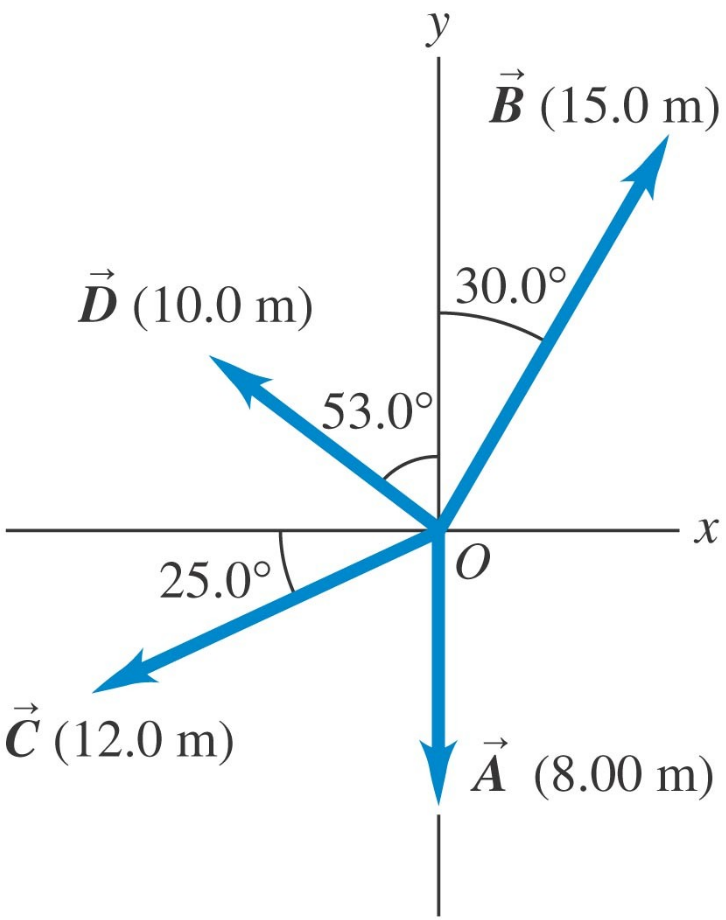
B. $E_x = -8.00$ m, $E_y = +2.00$ m

C. $E_x = -6.00$ m, $E_y = 0$

D. $E_x = -6.00$ m, $E_y = +2.00$ m

E. $E_x = -10.0$ m, $E_y = 0$

A1.6



Consider the vectors shown.

What are the components of the vector $\vec{E} = \vec{A} + \vec{D}$?

✔ A. $E_x = -8.00$ m, $E_y = -2.00$ m

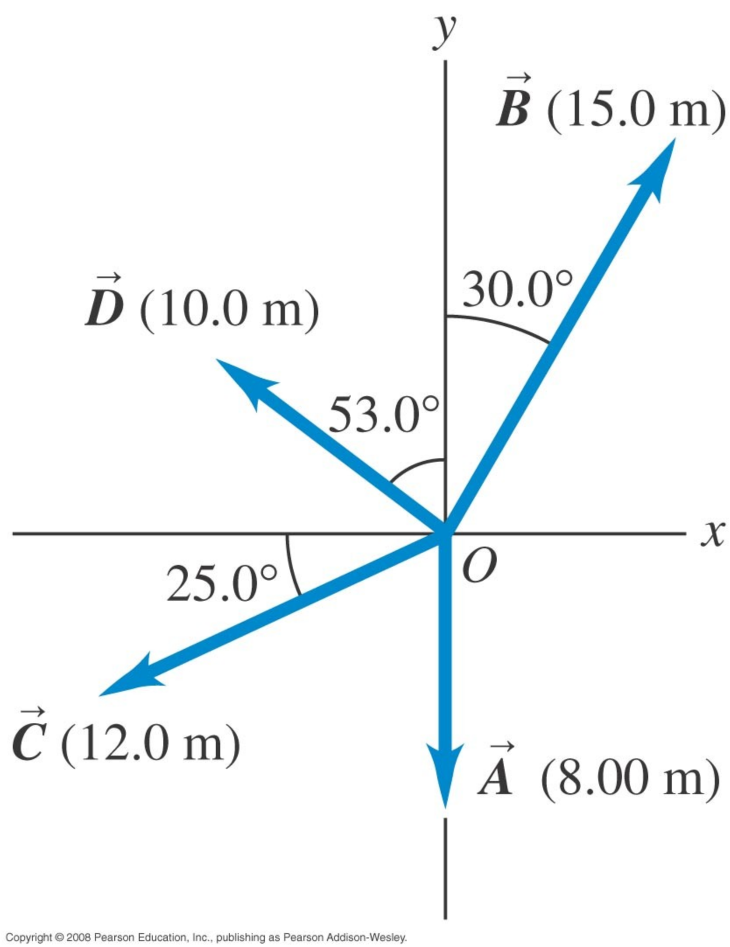
B. $E_x = -8.00$ m, $E_y = +2.00$ m

C. $E_x = -6.00$ m, $E_y = 0$

D. $E_x = -6.00$ m, $E_y = +2.00$ m

E. $E_x = -10.0$ m, $E_y = 0$

Q1.7



Consider the vectors shown.

What is the dot product $\vec{C} \bullet \vec{D}$?

A. $(120\ \text{m}^2)\cos 78.0°$

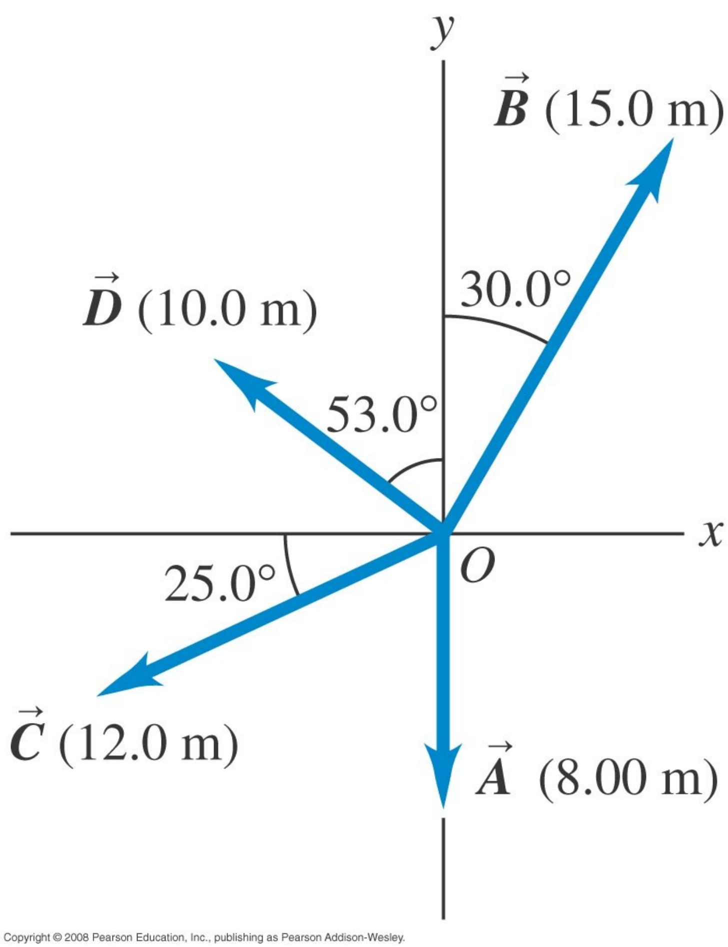
B. $(120\ \text{m}^2)\sin 78.0°$

C. $(120\ \text{m}^2)\cos 62.0°$

D. $(120\ \text{m}^2)\sin 62.0°$

E. none of these

A1.7



Consider the vectors shown.

What is the dot product $\vec{C} \bullet \vec{D}$?

A. (120 m^2) cos 78.0°

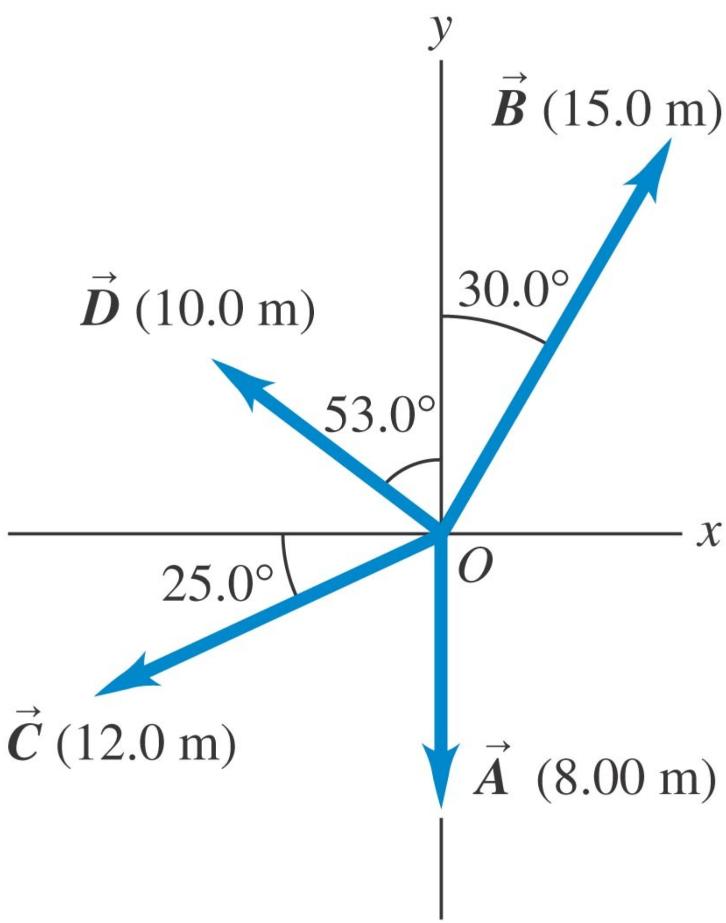
B. (120 m^2) sin 78.0°

✔ C. (120 m^2) cos 62.0°

D. (120 m^2) sin 62.0°

E. none of these

Q1.8



Consider the vectors shown.

What is the cross product $\vec{A} \times \vec{C}$?

A. $(96.0\ \text{m}^2)\sin 25.0°\ \hat{k}$

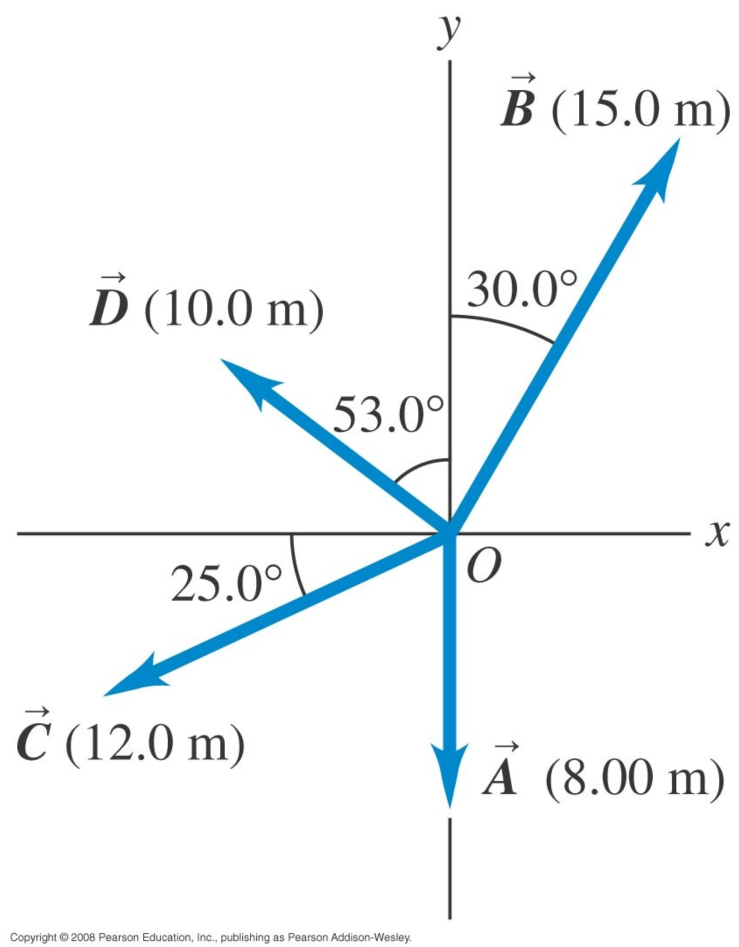
B. $(96.0\ \text{m}^2)\cos 25.0°\ \hat{k}$

C. $-(96.0\ \text{m}^2)\sin 25.0°\ \hat{k}$

D. $-(96.0\ \text{m}^2)\cos 25.0°\ \hat{k}$

E. none of these

A1.8



Consider the vectors shown.

What is the cross product $\vec{A} \times \vec{C}$?

A. $(96.0\ \text{m}^2)\sin 25.0°\ \hat{k}$

B. $(96.0\ \text{m}^2)\cos 25.0°\ \hat{k}$

C. $-(96.0\ \text{m}^2)\sin 25.0°\ \hat{k}$

✓ D. $-(96.0\ \text{m}^2)\cos 25.0°\ \hat{k}$

E. none of these

Q1.9

Consider the two vectors

$$\vec{A} = 3\hat{i} + 4\hat{j}$$

$$\vec{B} = -8\hat{i} + 6\hat{j}$$

What is the dot product $\vec{A} \bullet \vec{B}$?

A. zero

B. 14

C. 48

D. 50

E. none of these

A1.9

Consider the two vectors

$$\vec{A} = 3\hat{\boldsymbol{i}} + 4\hat{\boldsymbol{j}}$$

$$\vec{B} = -8\hat{\boldsymbol{i}} + 6\hat{\boldsymbol{j}}$$

What is the dot product $\vec{A} \bullet \vec{B}$?

✔ A. zero

B. 14

C. 48

D. 50

E. none of these

Q1.10

Consider the two vectors

$$\vec{A} = 3\hat{i} + 4\hat{j}$$

$$\vec{B} = -8\hat{i} + 6\hat{j}$$

What is the cross product $\vec{A} \times \vec{B}$?

A. $6\hat{k}$

B. $-6\hat{k}$

C. $50\hat{k}$

D. $-50\hat{k}$

E. none of these

A1.10

Consider the two vectors

$$\vec{A} = 3\hat{\boldsymbol{i}} + 4\hat{\boldsymbol{j}}$$

$$\vec{B} = -8\hat{\boldsymbol{i}} + 6\hat{\boldsymbol{j}}$$

What is the cross product $\vec{A} \times \vec{B}$?

A. $6\hat{\boldsymbol{k}}$

B. $-6\hat{\boldsymbol{k}}$

✓ C. $50\hat{\boldsymbol{k}}$

D. $-50\hat{\boldsymbol{k}}$

E. none of these

Q1.11

Consider the two vectors

$$\vec{A} = 3\hat{i} - 4\hat{j}$$

$$\vec{B} = 6\hat{k}$$

What is the dot product $\vec{A} \bullet \vec{B}$?

A. zero

B. –6

C. +6

D. 42

E. –42

A1.11

Consider the two vectors

$$\vec{A} = 3\hat{i} - 4\hat{j}$$

$$\vec{B} = 6\hat{k}$$

What is the dot product $\vec{A} \bullet \vec{B}$?

✓ A. zero

B. –6

C. +6

D. 42

E. –42

Q1.12

Consider the two vectors

$$\vec{A} = 3\hat{i} - 4\hat{j}$$

$$\vec{B} = 6\hat{k}$$

What is the cross product $\vec{A} \times \vec{B}$?

A. zero

B. $24\hat{i} + 18\hat{j}$

C. $-24\hat{i} - 18\hat{j}$

D. $-18\hat{i} + 24\hat{j}$

E. $-18\hat{i} - 24\hat{j}$

A1.12

Consider the two vectors

$$\vec{A} = 3\hat{i} - 4\hat{j}$$

$$\vec{B} = 6\hat{k}$$

What is the cross product $\vec{A} \times \vec{B}$?

A. zero

B. $24\hat{i} + 18\hat{j}$

C. $-24\hat{i} - 18\hat{j}$

D. $-18\hat{i} + 24\hat{j}$

E. $-18\hat{i} - 24\hat{j}$

MOTION

Definition of Motion: Change in position of an object with respect to time.

More About Motion:

A moving object changes its position as the time passes/changes.

In general motion is described as the movement of object.

Objects which are moving will not be at a same position after certain interval of time.

When the object remains at same position after certain interval of time then the state of the object is known as “rest”.

In physics, motion is a change in position of an object with respect to time and its reference point. Motion is typically described in terms of displacement, direction, velocity, acceleration, and time

Change in position of an object with time is measured with the units of distance. It describes the speed of an object. (How fast or slow an object is moving.)

General types of motion: Circular motion, Periodic motion, Translatory motion.

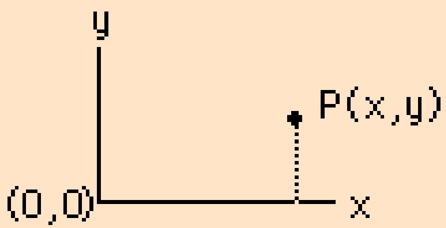
Examples of different types of motion:
1. **Translatory Motion**: March-past of soldiers in a parade
Motion of a stone dropped from a building.
2. **Circular Motion:** Blades of an Electric fan (Switched on).
3. **Periodic Motion:** “Child on a swing”
“Motion of needle in a sewing machine”.

Position

Specifying the position of an object is essential in describing motion. In one dimension some typical ways are

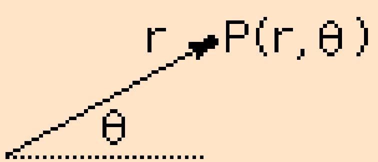
$x(t)$ is used to represent position as a function of time

In two dimensions, either cartesian or polar coordinates may be used, and the use of unit vectors is common. A position vector r may be expressed in terms of the unit vectors.

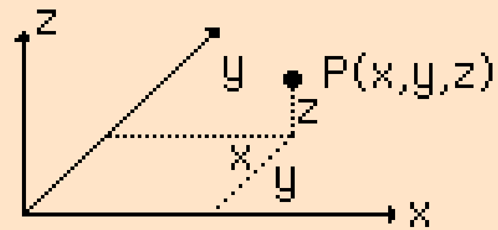


$$\vec{r} = x\hat{i} + y\hat{j} = r\hat{1}_r$$

$\vec{r}(t)$ describes position as a function of time

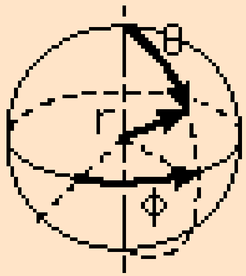


In three dimensions, cartesian or spherical polar coordinates are used, as well as other coordinate systems for specific geometries.



In cartesian coordinates

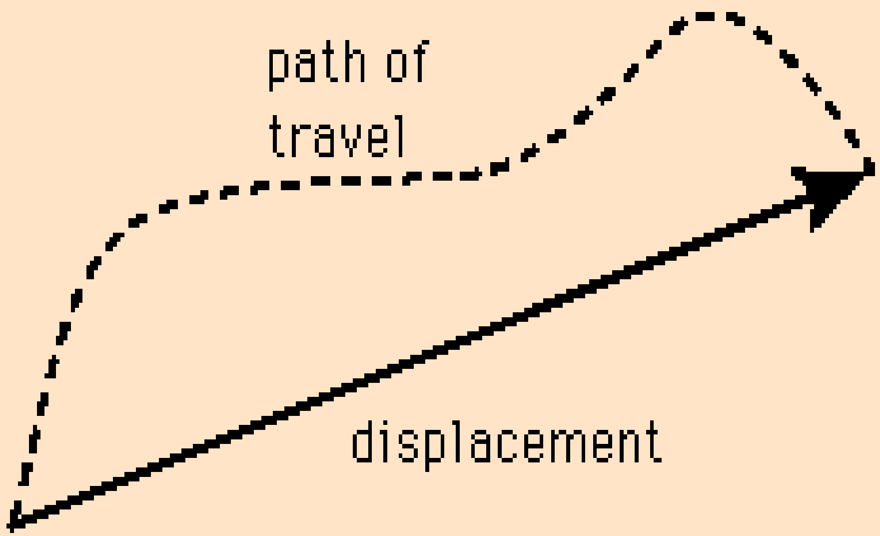
$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$



The vector change in position associated with a motion is called the displacement.

Displacement

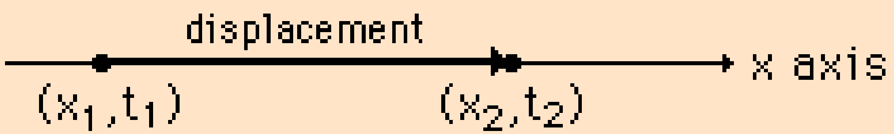
The displacement of an object is defined as the vector distance from some initial point to a final point. It is therefore distinctly different from the distance traveled except in the case of straight line motion. The distance traveled divided by the time is called the speed, while the displacement divided by the time defines the average velocity.



If the positions of the initial and final points are known, then the distance relationship can be used to find the displacement.

Velocity

The average speed of an object is defined as the distance traveled divided by the time elapsed. Velocity is a vector quantity, and average velocity can be defined as the displacement divided by the time. For the special case of straight line motion in the x direction, the average velocity takes the form:



$$v_{average} = \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

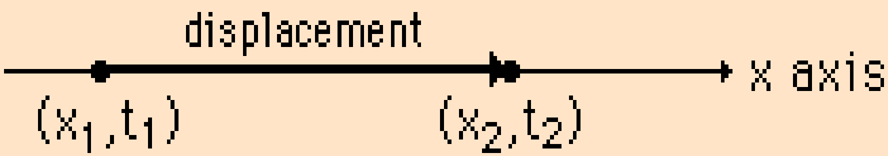
The units for velocity can be implied from the definition to be meters/second or in general any distance unit over any time unit.

You can approach an expression for the instantaneous velocity at any point on the path by taking the limit as the time interval gets smaller and smaller. Such a limiting process is called a derivative and the instantaneous velocity can be defined as

$$v_{instantaneous} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Average Velocity, Straight Line

The average speed of an object is defined as the distance traveled divided by the time elapsed. Velocity is a vector quantity, and average velocity can be defined as the displacement divided by the time. For the special case of straight line motion in the x direction, the average velocity takes the form:



$$v_{average} = \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

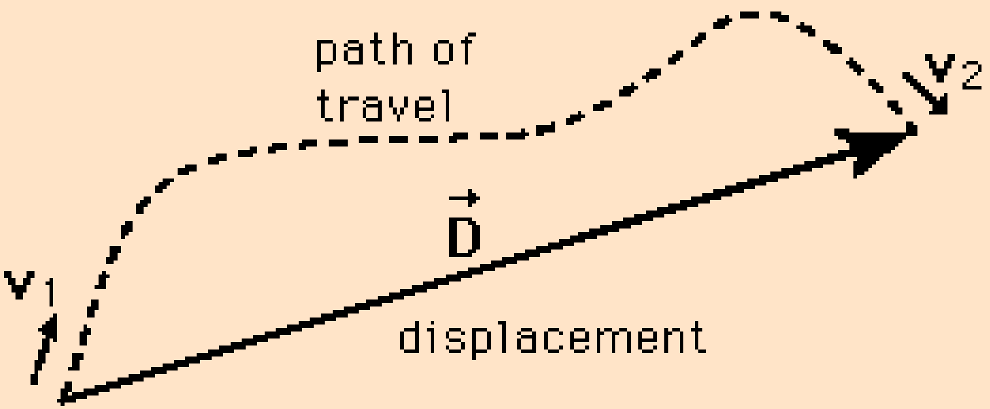
If the beginning and ending velocities for this motion are known, and the acceleration is constant, the average velocity can also be expressed as

$$v_{average} = \bar{v} = \frac{v_1 + v_2}{2}$$

For this special case, these expressions give the same result.

Average Velocity, General

The average speed of an object is defined as the distance traveled divided by the time elapsed. Velocity is a vector quantity, and average velocity can be defined as the displacement divided by the time. For general cases involving non-constant acceleration, this definition must be applied directly because the straight line average velocity expressions do not work.



$$\vec{v}_{average} = \frac{\vec{D}}{t}$$

Warning! The average velocity is not given by

$$\frac{\vec{v}_1 + \vec{v}_2}{2}$$

since the velocities are vectors in different directions and the acceleration is not constant.

If the positions of the initial and final points are known, then the distance relationship can be used to find the displacement.

Acceleration

Acceleration is defined as the rate of change of velocity. Acceleration is inherently a vector quantity, and an object will have non-zero acceleration if its speed and/or direction is changing. The average acceleration is given by

$$\vec{a}_{average} = \bar{\vec{a}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

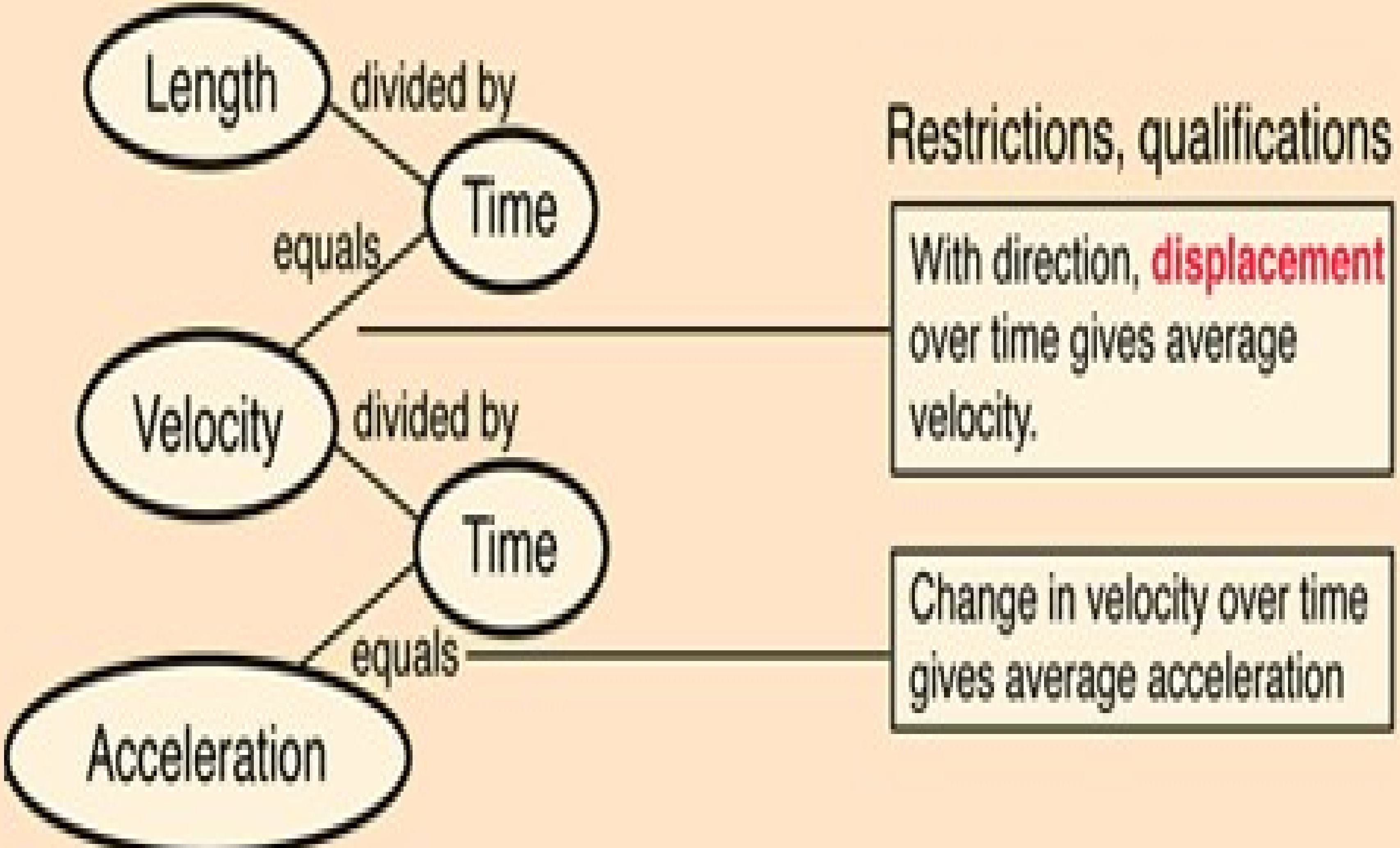
where the small arrows indicate the vector quantities. The operation of subtracting the initial from the final velocity must be done by vector addition since they are inherently vectors.

The units for acceleration can be implied from the definition to be meters/second divided by seconds, usually written m/s^2.

The instantaneous acceleration at any time may be obtained by taking the limit of the average acceleration as the time interval approaches zero. This is the derivative of the velocity with respect to time:

$$\vec{a}_{instantaneous} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

The Chain of Mechanical Quantities



Motion with constant velocity

The simplest type of motion (excluding the trivial case in which the body under investigation remains at rest) consists of motion with *constant velocity*. This type of motion occurs in everyday life whenever an object slides over a horizontal, low friction surface: *e.g.*, a puck sliding across a hockey rink. Figure shows the graph of displacement versus time for a body moving with constant velocity. It can be seen that the graph consists of a *straight-line*. This line can be represented algebraically as

$$x=x_0+vt$$

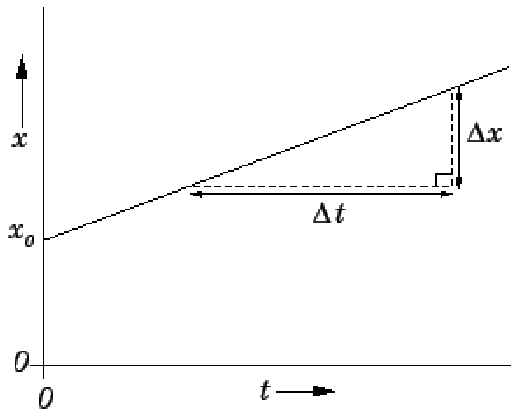


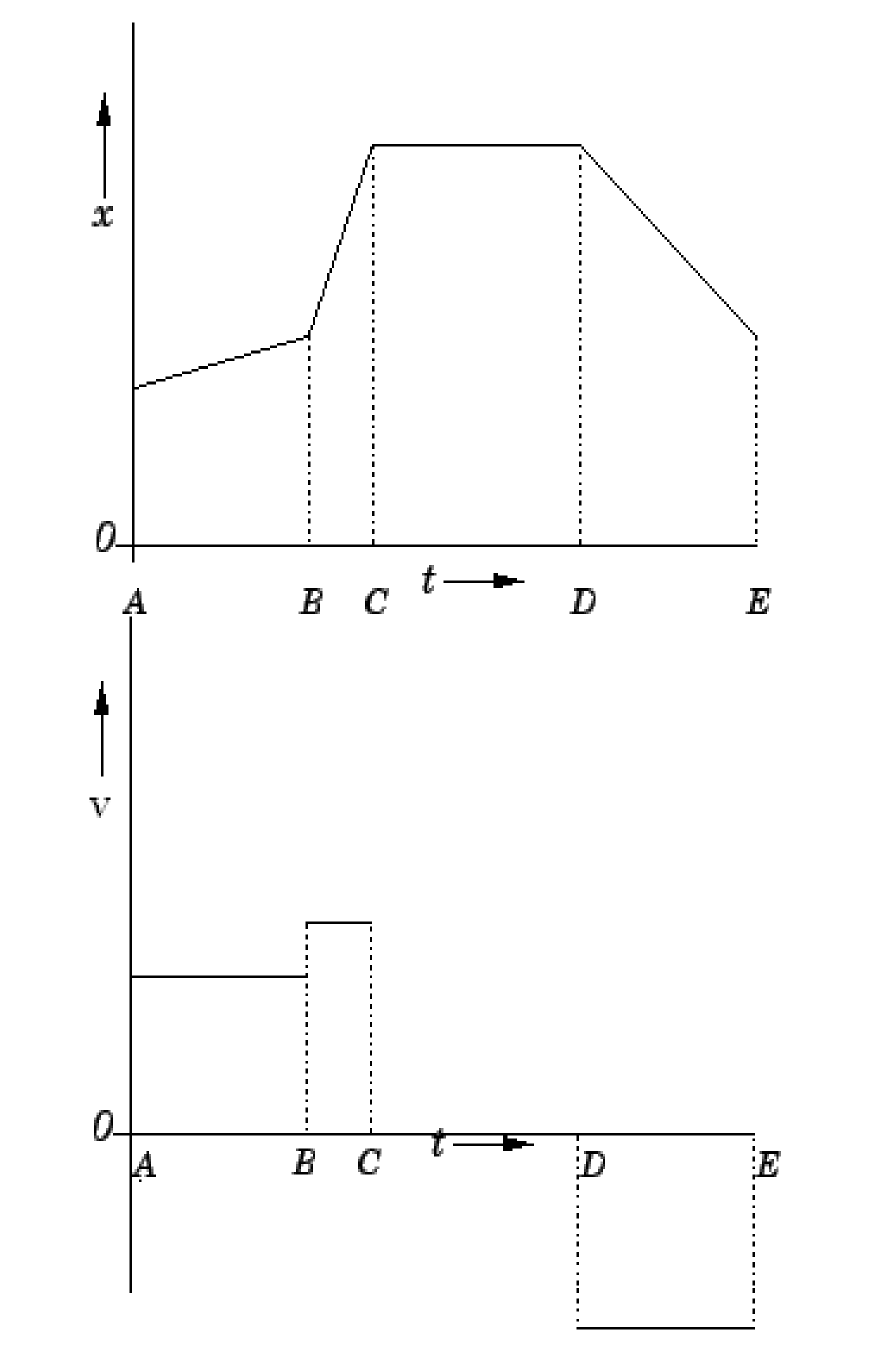
Figure : *Graph of displacement versus time for a body moving with constant velocity*

Here x_0, is the displacement at time t=0 : this quantity can be determined from the graph as the *intercept* of the straight-line with the -axis.

Likewise,$v=dx/dt$ is the constant velocity of the body: this quantity can be determined from the graph as the *gradient* of the straight-line (*i.e.*, the ratio $v=\Delta x/\Delta t$, as shown).

Note that $a=d^2x/dt^2$, as expected.

Figure shows a displacement versus time graph for a slightly more complicated case of motion with constant velocity. The body in question moves to the right (since x is clearly increasing with t) with a constant velocity (since the graph is a straight-line) between times A and B. The body then moves to the right (since x is still increasing in time) with a somewhat larger constant velocity (since the graph is again a straight line, but possesses a larger gradient than before) between times B and C. The body remains at rest (since the graph is horizontal) between times C and D. Finally, the body moves to the left (since x is decreasing with t) with a constant velocity (since the graph is a straight-line) between times D and E.



Constant Acceleration Motion

Constant acceleration motion can be characterized by formuli and by motion graphs.

Starting from rest at position zero

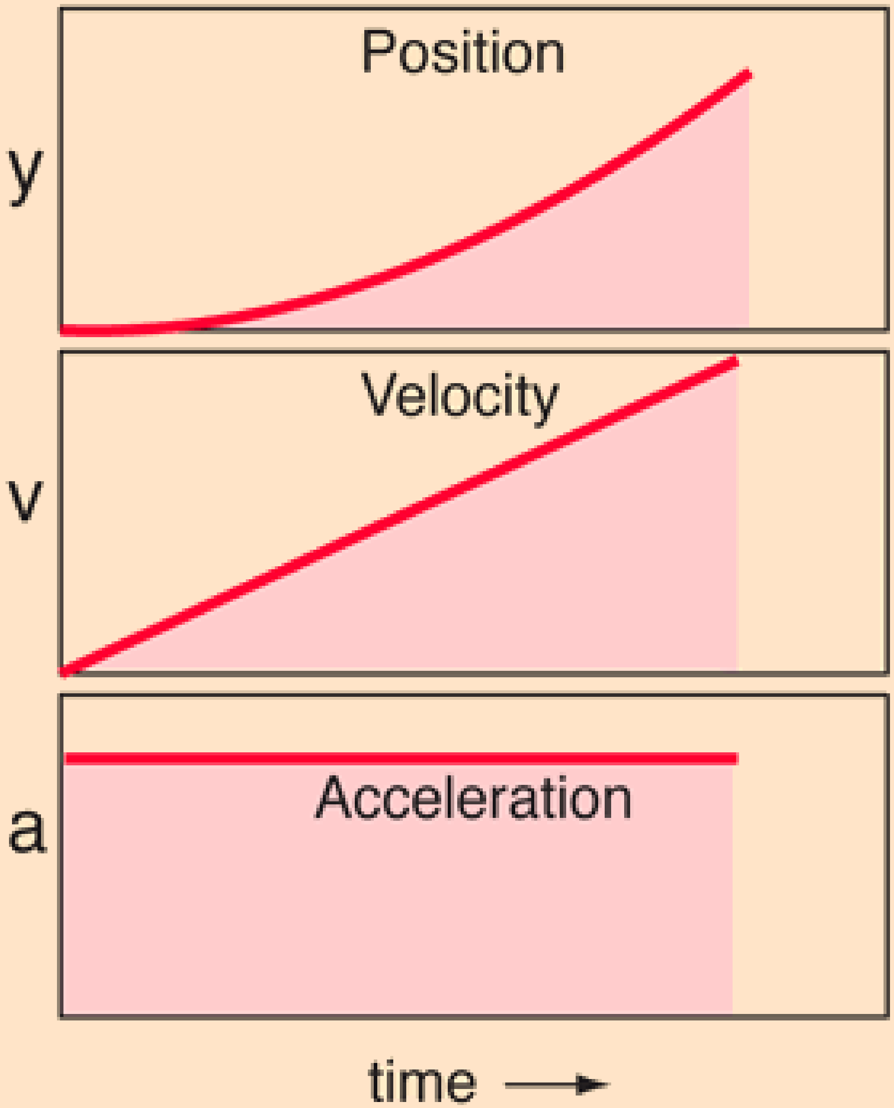
$$y = \frac{1}{2} a t^2$$

$$v = at$$

$$a = \text{constant}$$

accelerating at

$$9.8 \text{ m/s}^2$$



More generally

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + at$$

Velocity is equal to the slope of the position curve.

Acceleration is equal to the slope of the velocity curve.

Description of Motion in One Dimension

Motion is described in terms of displacement (x), time (t), velocity (v), and acceleration (a). Velocity is the rate of change of displacement and the acceleration is the rate of change of velocity. The average velocity and average acceleration are defined by the relationships:

Average velocity: $\bar{v} = \frac{\Delta x}{\Delta t}$ Average acceleration: $\bar{a} = \frac{\Delta v}{\Delta t}$

where the Greek letter Δ indicates the change in the quantity following it.

Constant acceleration equations.

1. $x = \bar{v}t$ $\bar{v} = \frac{v_0 + v}{2}$
2. $v = v_0 + at$
3. $x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2ax$

A bar above any quantity indicates that it is the average value of that quantity. If the acceleration is constant, then equations 1,2 and 3 represent a complete description of the motion. Equation 4 is obtained by a combination of the others. Click on any of the equations for an example.

Forms of Motion Equations

1. $x = \bar{v}\, t$ $\qquad \bar{v} = \dfrac{v_0 + v}{2}$

2. $v = v_0 + at$

Combining equations 1 and 2 leads to the useful form in equation 3 below.

From 1: $x = \bar{v}\, t = \left[\dfrac{v_0 + v}{2}\right] t$ and substituting for v from 2:

$x = \left[\dfrac{v_0 + v_0 + at}{2}\right] t$ gives 3. :

3. $x = v_0 t + \frac{1}{2} a t^2$

The motion equations for the case of constant acceleration can be developed by integration of the acceleration. The process can be reversed by taking successive derivatives.

$$y = \int v\,dt$$

$$= \int (v_0 + at)\,dt$$

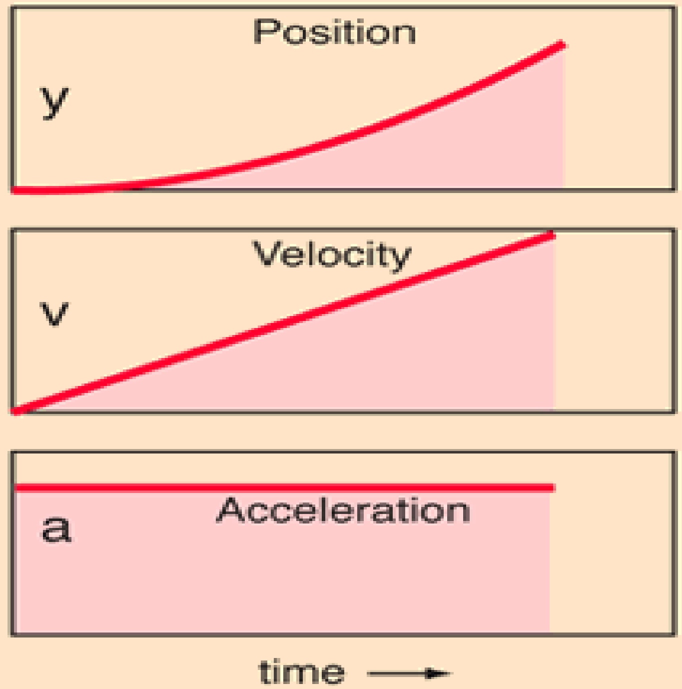
$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

Integrate velocity to get position

$$v = \int a\,dt = v_0 + at$$

Integrate acceleration to get velocity

$$a = \text{constant}$$



Motion relationships in one dimension.

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

Derivative of position is velocity

$$v = \frac{dy}{dt}$$

$$v = v_0 + at$$

Derivitive of velocity is acceleration

$$a = \frac{dv}{dt} = a$$

On the left hand side above, the constant acceleration is integrated to obtain the velocity. For this indefinite integral, there is a constant of integration. But in this physical case, the constant of integration has a very definite meaning and can be determined as an intial condition on the movement. Note that if you set t=0, then $v = v_0$, the initial value of the velocity. Likewise the further integration of the velocity to get an expression for the position gives a constant of integration. Checking the case where t=0 shows us that the constant of integration is the initial position x_0. It is true as a general property that when you integrate a second derivative of a quantity to get an expression for the quantity, you will have to provide the values of two constants of integration. In this case their specific meanings are the initial conditions on the distance and velocity.

Forms of Motion Equations

1. $x = \bar{v}\,t \qquad \bar{v} = \dfrac{v_0 + v}{2}$

2. $v = v_0 + at$

Combining equations 1 and 2 leads to the useful form in equation 4 below.

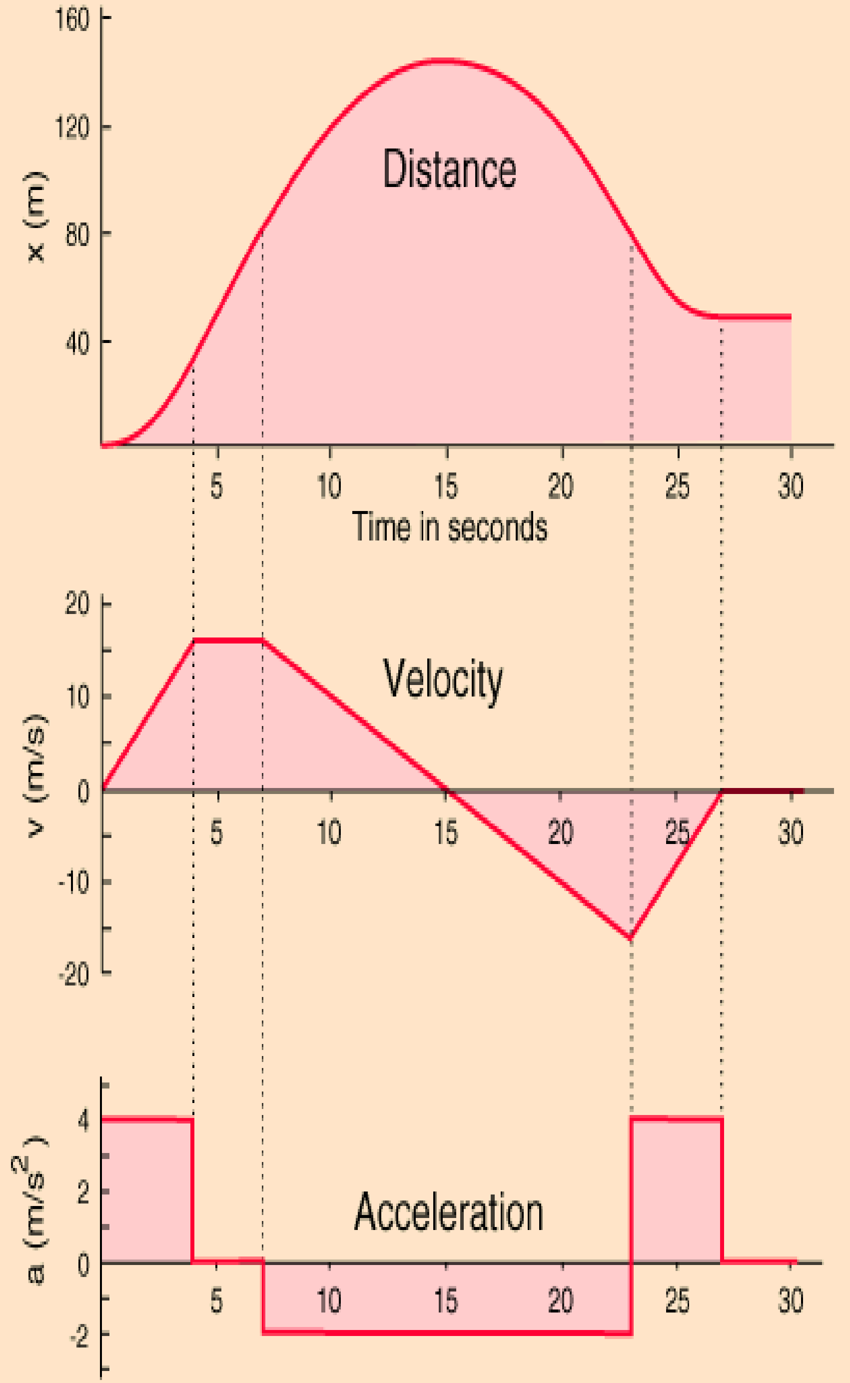
From 1: $x = \bar{v}\,t = \left[\dfrac{v_0 + v}{2}\right] t$ and substituting for t from 2:

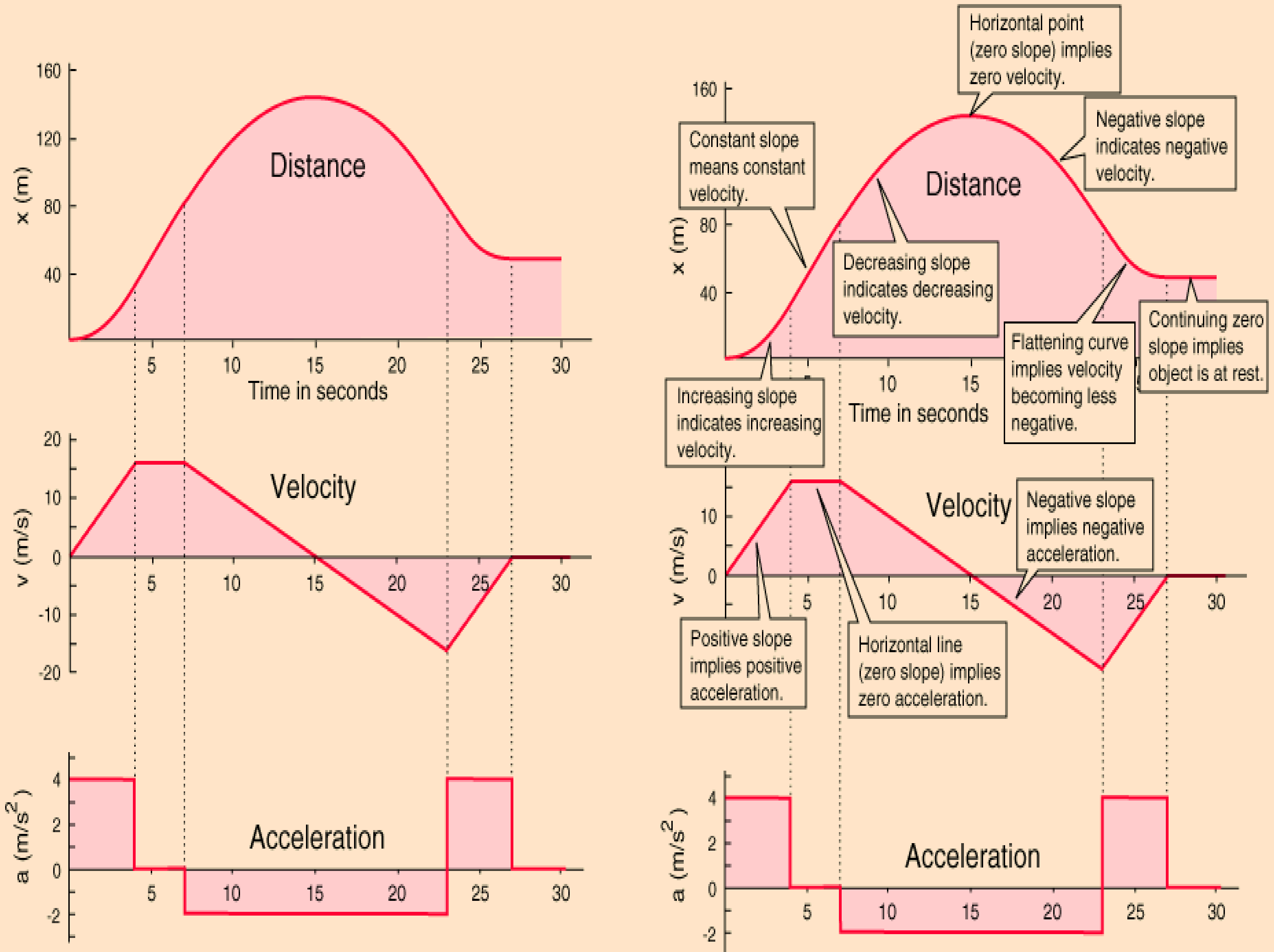
$x = \left[\dfrac{v_0 + v}{2}\right]\left[\dfrac{v - v_0}{a}\right] = \dfrac{v^2 - v_0^2}{2a}$ which is equation 4:

4. $v^2 = v_0^2 + 2ax$

Motion Graphs

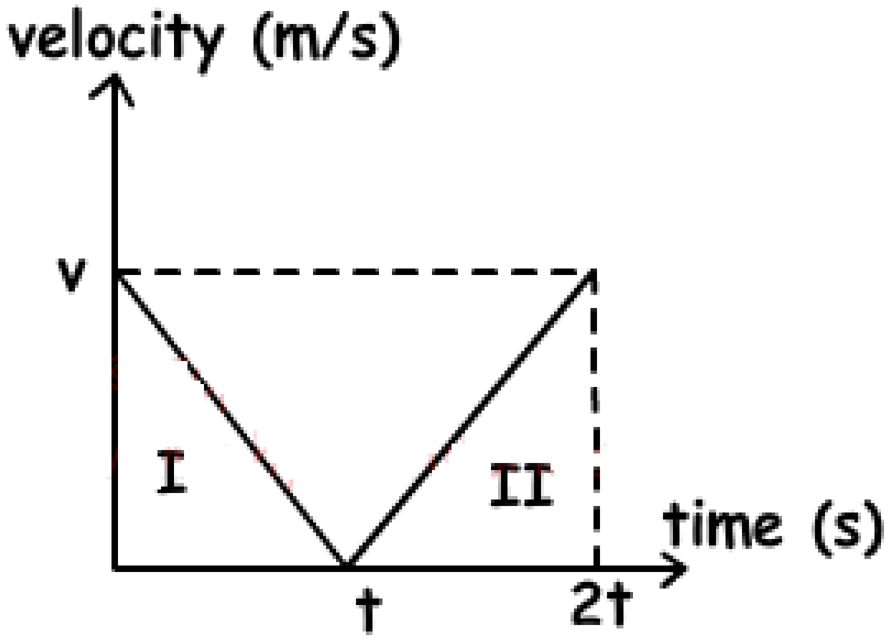
Constant acceleration motion can be characterized by motion equations and by motion graphs. The graphs of distance, velocity and acceleration as functions of time below were calculated for one-dimensional motion using the motion equations in a spreadsheet. The acceleration does change, but it is constant within a given time segment so that the constant acceleration equations can be used. For variable acceleration (i.e., continuously changing), then calculus methods must be used to calculate the motion graphs.





x (m)
Distance
Time in seconds
v (m/s)
Velocity
a (m/s^2)
Acceleration
Horizontal point (zero slope) implies zero velocity.
Constant slope means constant velocity.
Negative slope indicates negative velocity.
Decreasing slope indicates decreasing velocity.
Increasing slope indicates increasing velocity.
Flattening curve implies velocity becoming less negative.
Continuing zero slope implies object is at rest.
Negative slope implies negative acceleration.
Positive slope implies positive acceleration.
Horizontal line (zero slope) implies zero acceleration.

Velocity vs. time graph of an object traveling along a straight line given below.



a) Draw the acceleration vs. time graph,

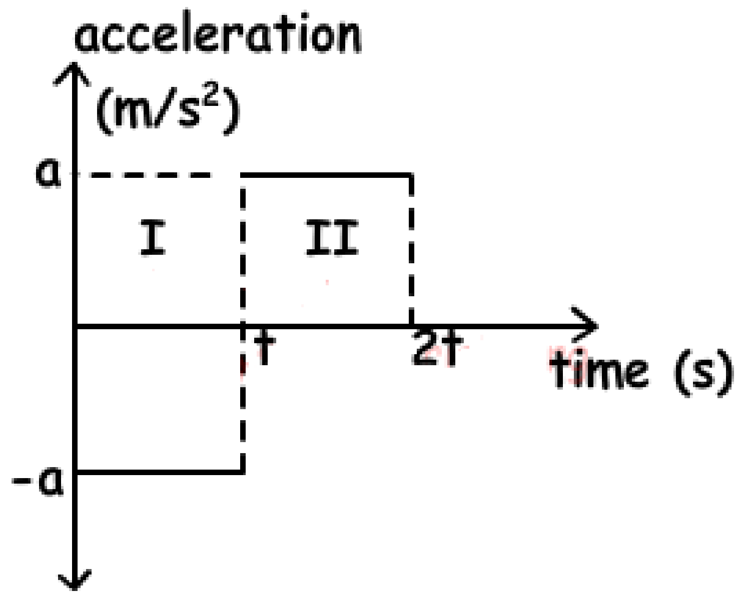
b) Draw the position vs. time graph of the object.

a) Slope of the velocity vs. time graph gives us acceleration. In first interval, slope of the line is constant and negative, thus, acceleration of the object is also constant and negative. In other words, object does slowing down motion in positive direction with negative acceleration.

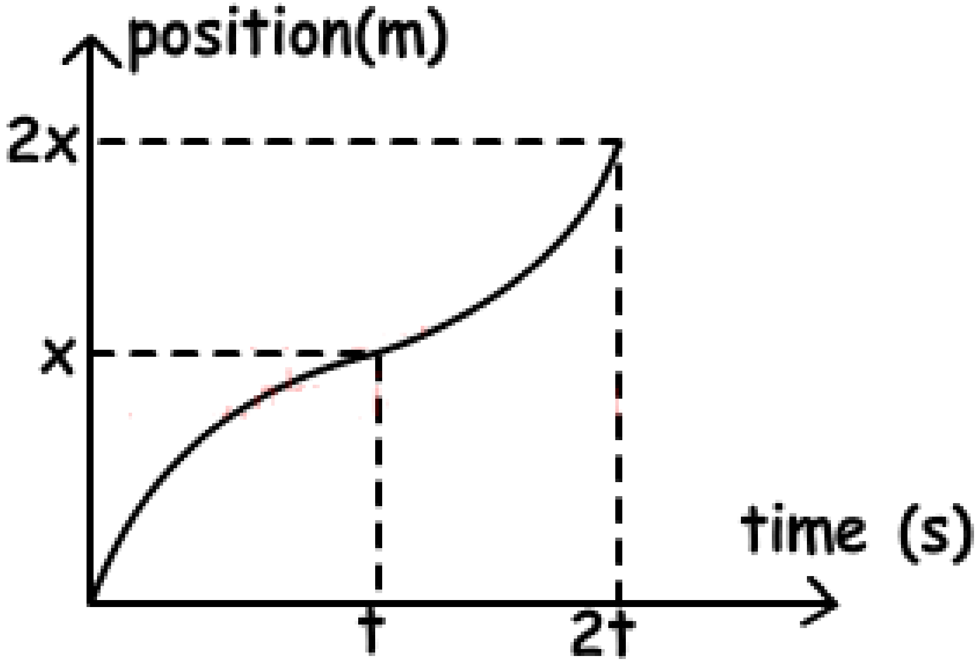
Slope=(0-v)/t=-a

In the second interval, slope is constant and positive, so acceleration is also constant and positive. Object does speeding up motion in positive direction.

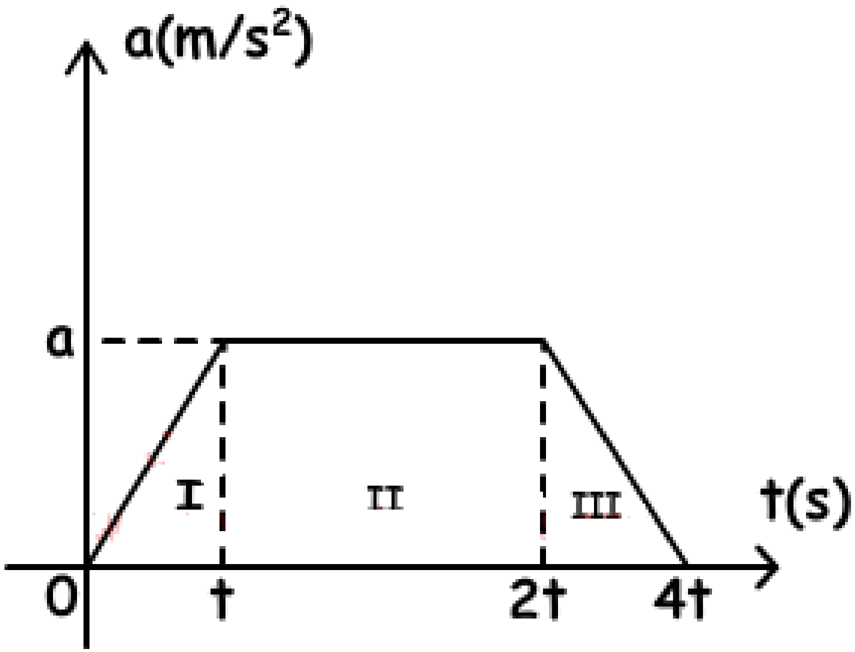
Slope=(v-0)/t=+a

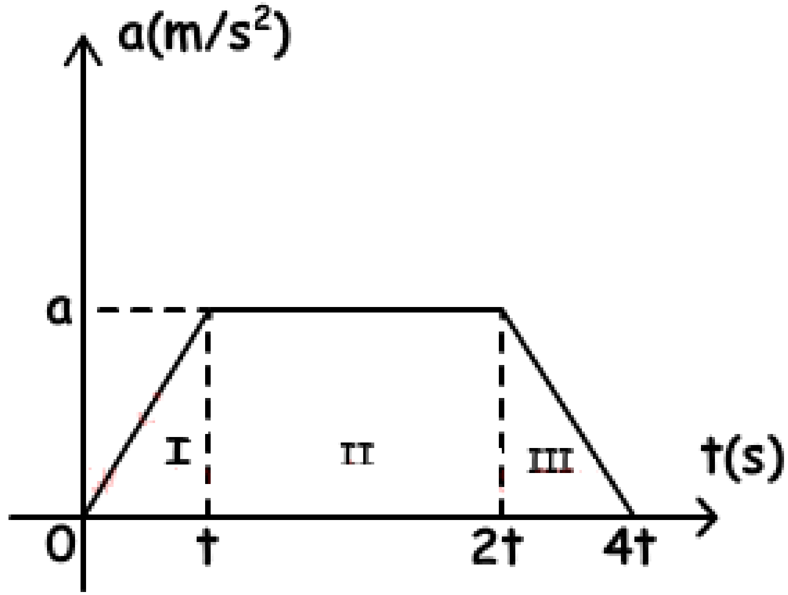


b) In the first and second interval velocity of the object changes constantly thus; position time graph becomes like in the picture given below.



An object is stationary at t=0. Picture given below shows the acceleration vs. time graph of this object. Find the intervals in which object speeds up in positive direction.





Area under the acceleration vs. time graph gives us the change in velocity.

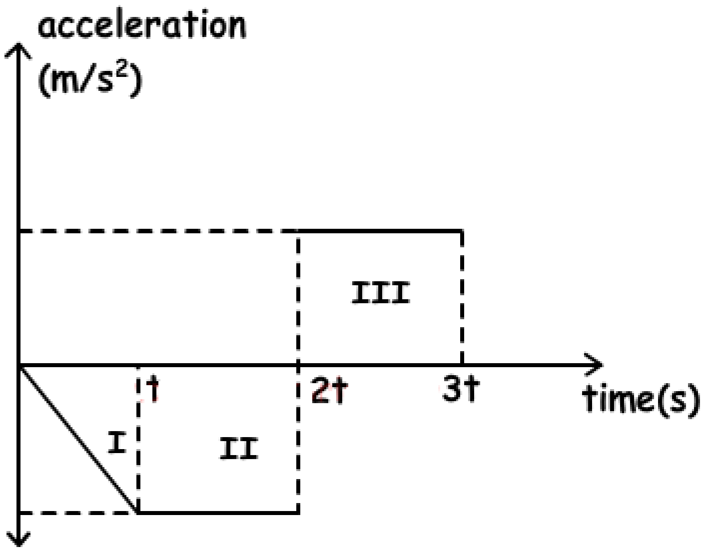
I. In this interval, acceleration is increasing with the time, thus object speeds up with increasing acceleration.

II. In this interval, acceleration of the object is constant, so object speeds up constantly.

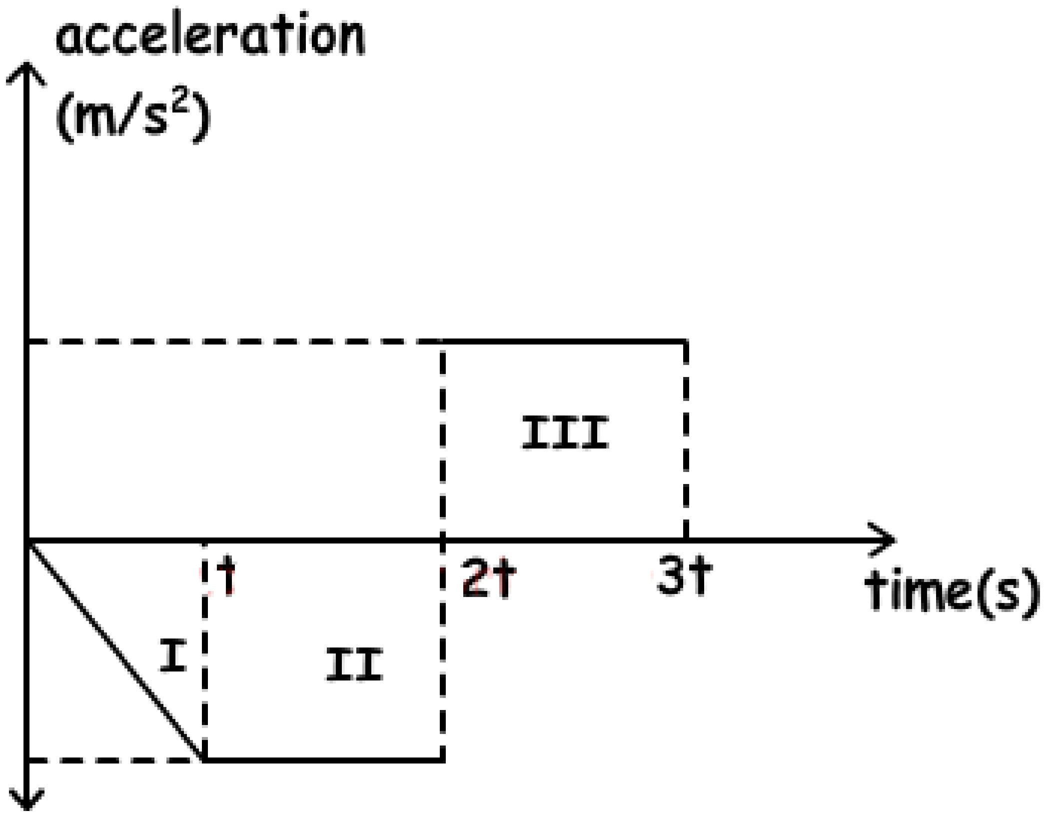
III. In this interval, acceleration of the object is decreasing with the time, so object speeds up with decreasing acceleration.

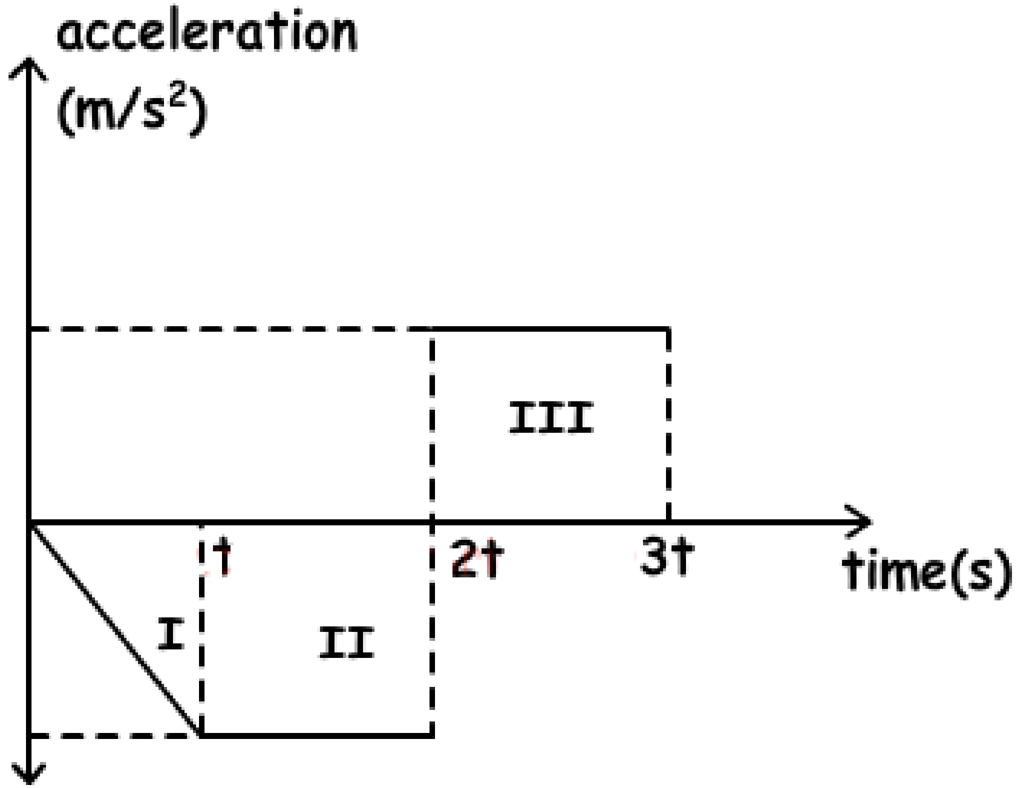
In all intervals, velocity of the object increases.

3. An object is stationary at t=0. Picture given below shows the acceleration vs. time graph of this object. Find the intervals in which object speeds up in positive direction.



An object is stationary at t=0. Picture given below shows the acceleration vs. time graph of this object. Find the intervals in which object speeds up in positive direction.



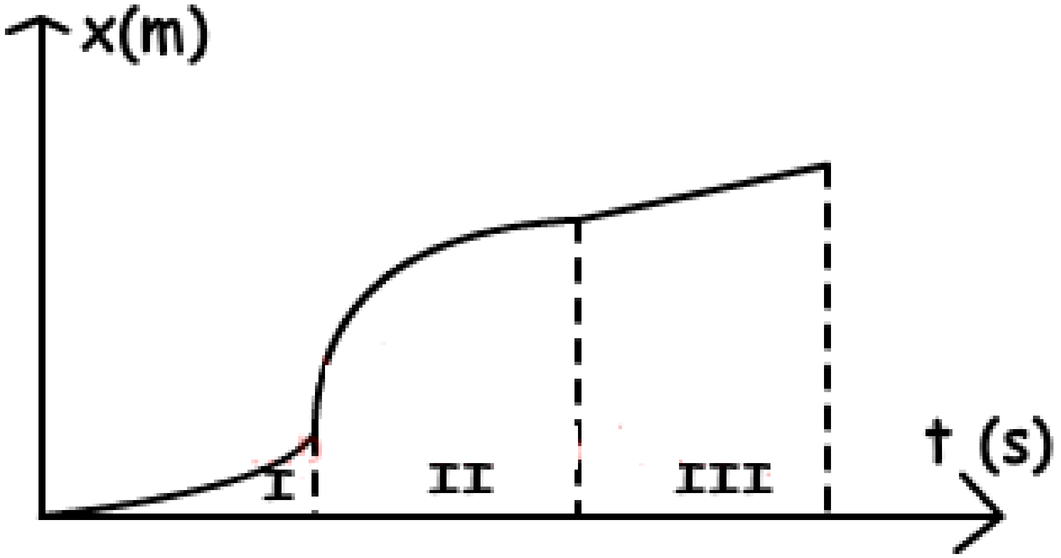


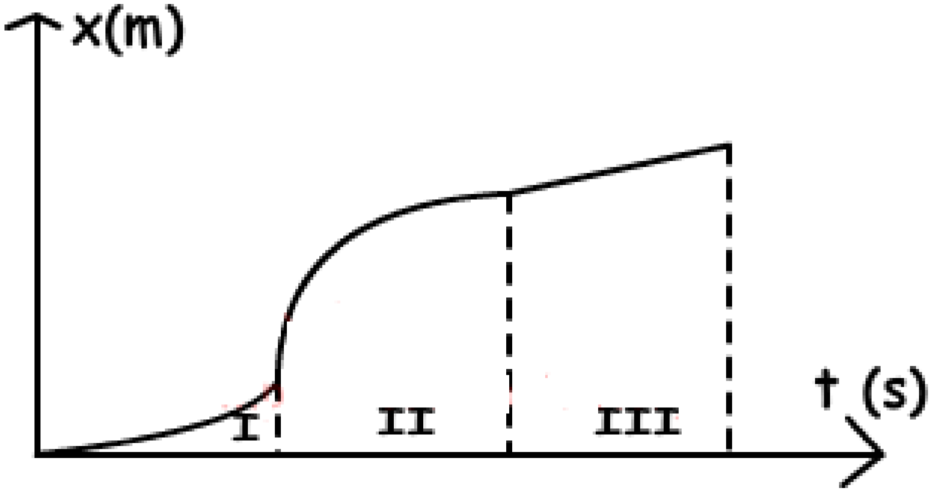
I. Object speeds up with increasing acceleration in negative direction.

II. Object speeds up with constant acceleration in negative direction.

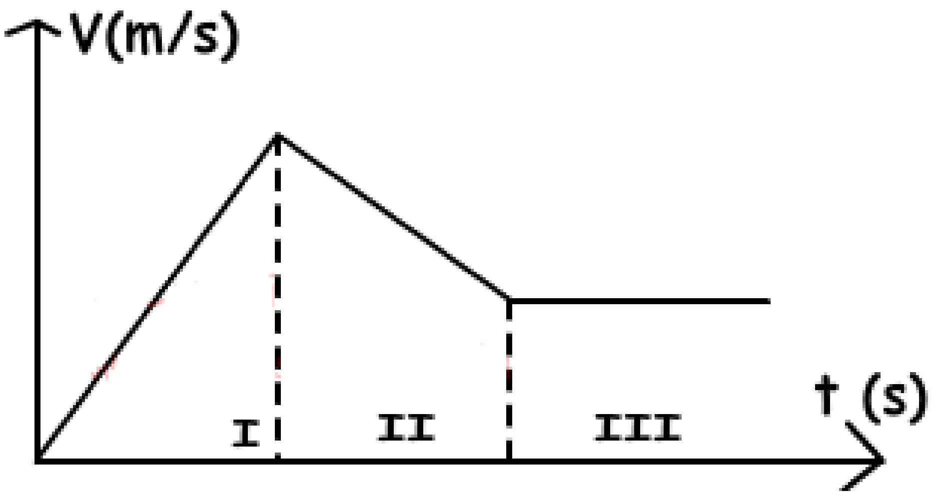
III. In this interval, object slows down with constant acceleration. (acceleration (+), velocity (-))

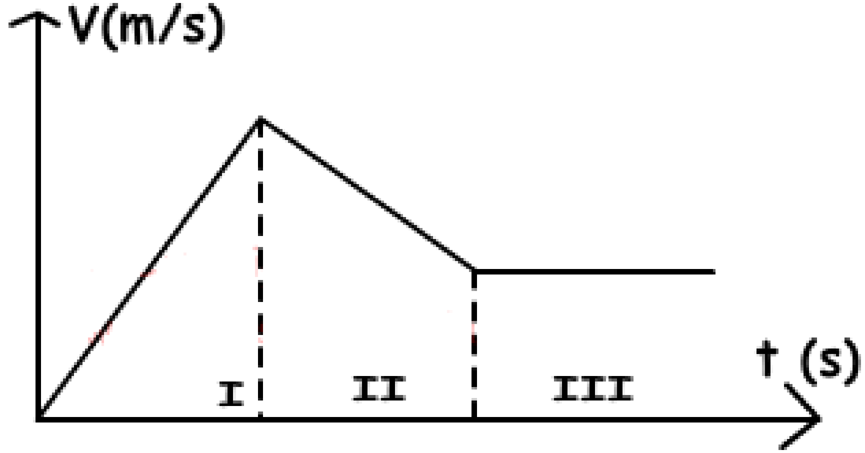
Position vs. time graph of a car is given below. In which intervals direction of velocity and direction of acceleration are same.



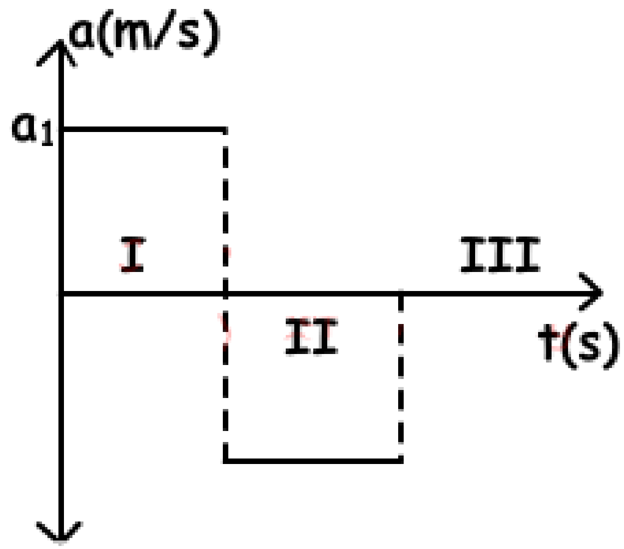


We draw velocity vs. time graph using position time graph.





Slope of the velocity vs. time graph gives us acceleration.



An object is dropped from 320 m high. Find the time of motion and velocity when it hits the ground.(**$g=10m/s^2$**)

$h=1/2.g.t^2$, $v=g.t$

$h=320m$

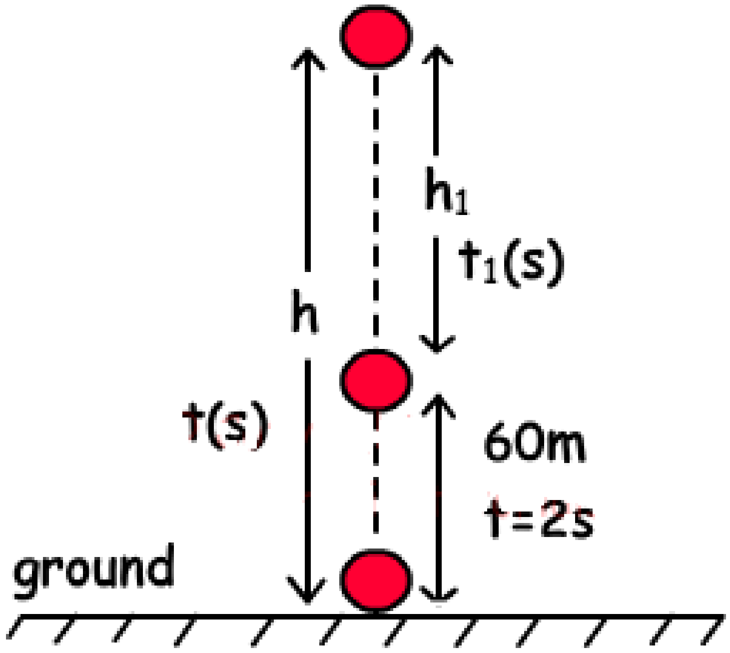
$g=10m/s^2$

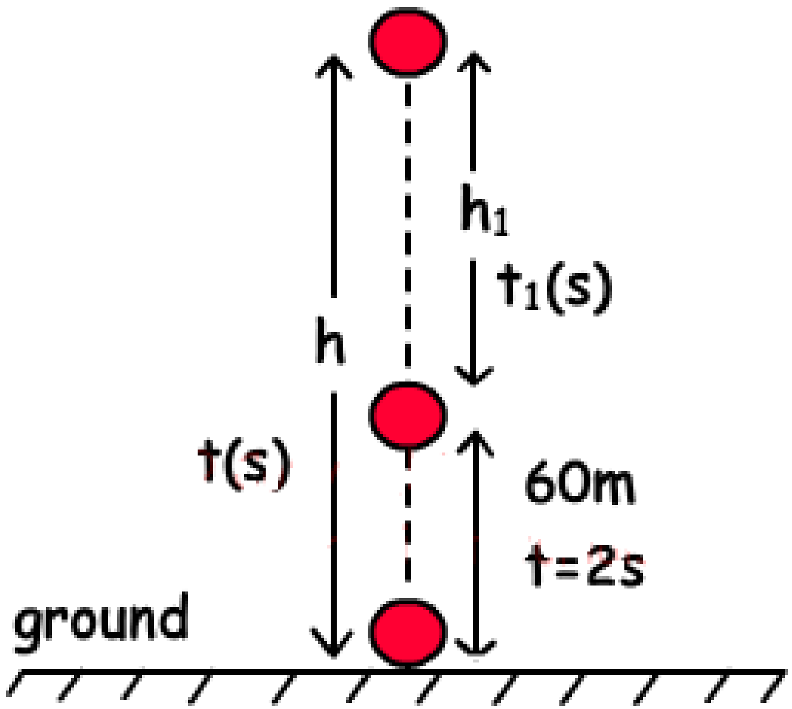
$320=1/2.10.t^2$

$t=8s.$

$v=g.t=10.8=80m/s$

An object does free fall and it takes 60m distance during last 2 seconds of its motion. Find the height it is dropped.(**g=10m/s²**)





t is the time of motion

$h=1/2.g.t^2$

$h_1=1/2.g.t_1{}^2$

put $t_1=t-2$ and $h-h_1=60$ in the equation,

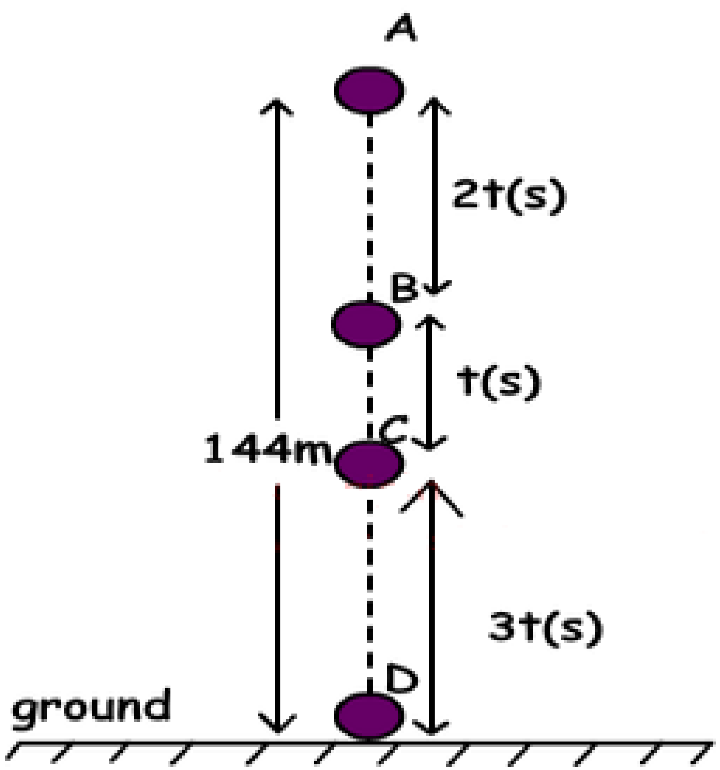
$1/2.g.t^2-1/2.g.t_1{}^2=60$

$5t^2-5(t^2-4t+4)=60$

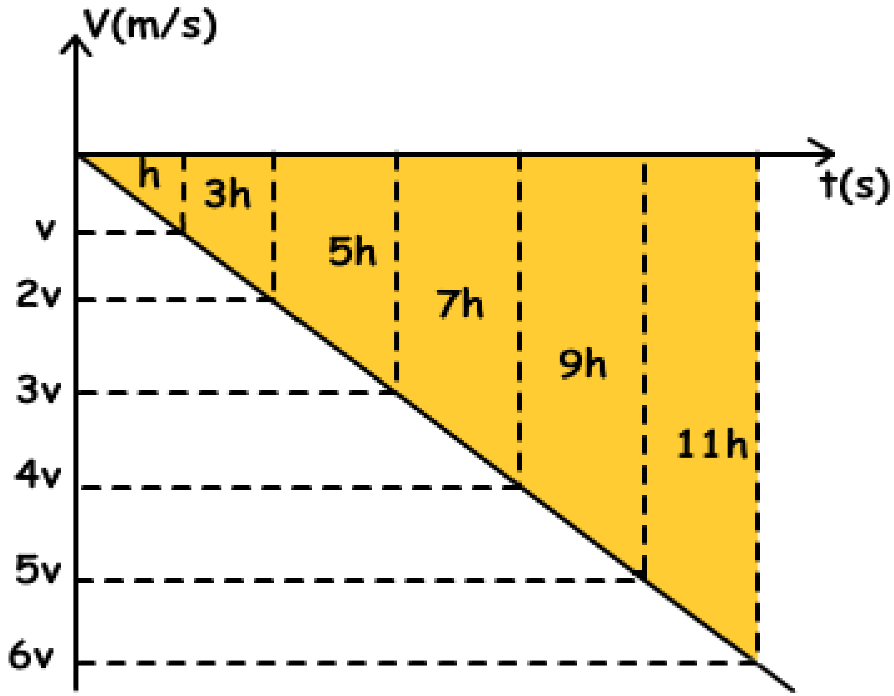
$t=4s$

$h=1/2.g.t^2=1/2.10.4^2=80m$

An object is dropped from 144m height and it does free fall motion. Distance it travels and time of motion are given in the picture below. Find the distance between points B-C.



We can draw velocity time graph of object and area under this graph gives us position of the object.



As you can see from the velocity time graph, object travels 5h distance during 2t-3t which is the distance between the points B and C.

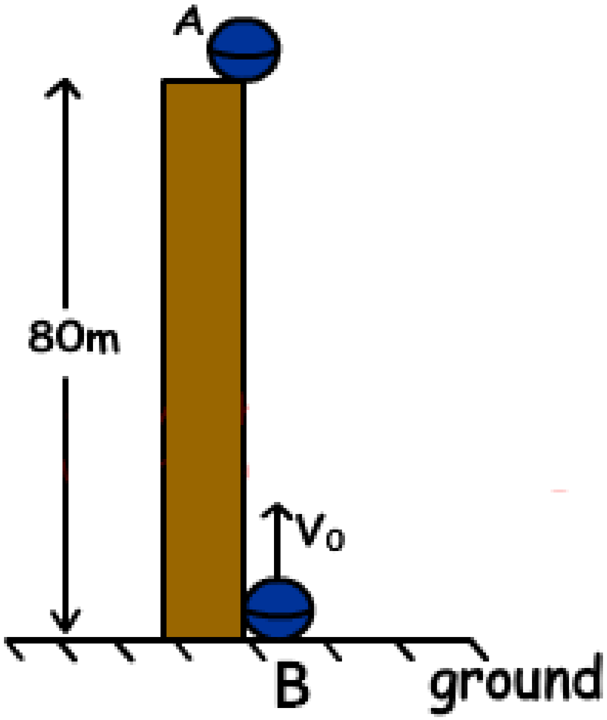
All distance traveled is 36h

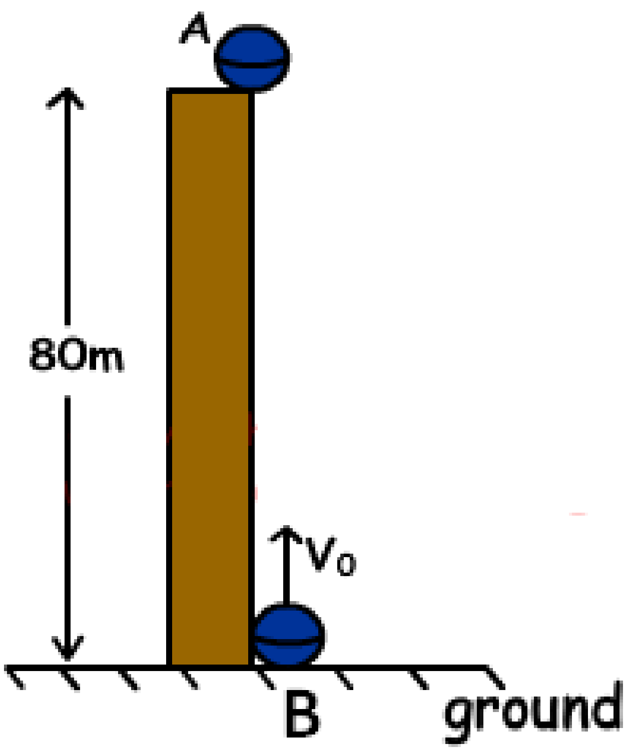
144m=36h

h=4m

Distance between B-C=5h=5.4m=20m

Look at the given picture below. Object K does free fall motion and object B thrown upward at the same time. They collide after 2s. Find the initial velocity of object B. ($g=10m/s^2$)





Object A does free fall motion

$h_A=1/2.10.2^2=20m$

$h_L=v_0.t-1/2.g.t^2$

$h_L=v_0.2-1/2.10.2^2$

$h_L=2v_0-20$

$h_K+h_L=80m$

$20m+h_L=80m$

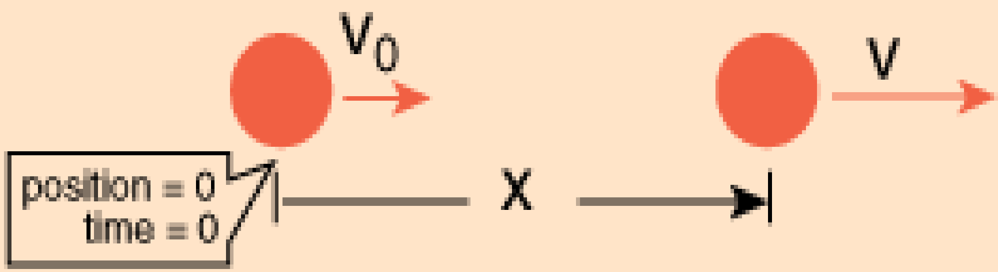
$2v_0-20=60m$

$v_0=40m/s$

ONE DIMENSIONAL MOTION EXAMPLES

Motion Example

Initial velocity = 0 m/s, Final velocity = 4 m/s



Equations of motion

1. $x = \bar{v}\,t \qquad \bar{v} = \dfrac{v_0 + v}{2}$
2. $v = v_0 + at$
3. $x = v_0 t + \frac{1}{2} a t^2$

Distance x = 3 m

Initial velocity v0 = 0 m/s

Final velocity v = 4 m/s

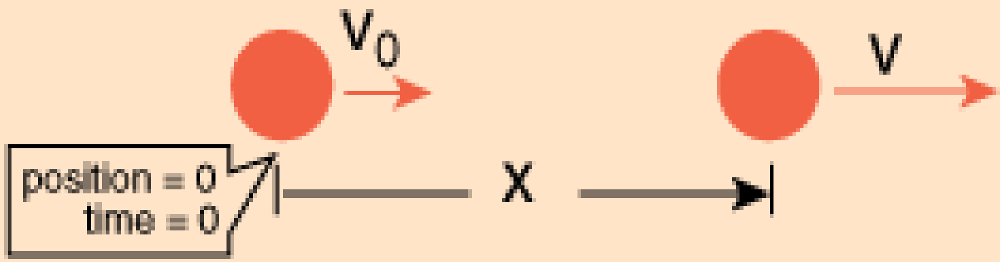
Average velocity = 2 m/s

Acceleration a = 2.66666666(m/s^2

Time t = 1.5 s

Motion Example

Initial velocity = 2 m/s, Final velocity = 8 m/s



Equations of motion

1. $x = \bar{v}\,t \qquad \bar{v} = \dfrac{v_0 + v}{2}$
2. $v = v_0 + at$
3. $x = v_0 t + \frac{1}{2}at^2$

Distance x = 10 m

Initial velocity v0 = 2 m/s

Final velocity v = 8 m/s

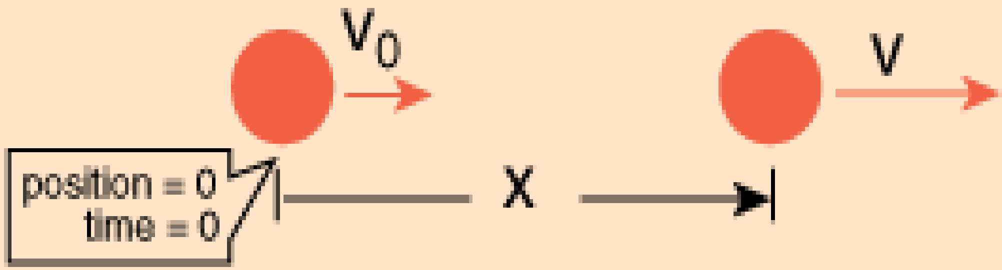
Average velocity = 5 m/s

Acceleration a = 3 m/s^2

Time t = 2 s

Motion Example

Initial velocity = 8 m/s, Final velocity = 0 m/s



Equations of motion

1. $x = \bar{v}\,t \qquad \bar{v} = \dfrac{v_0 + v}{2}$
2. $v = v_0 + at$
3. $x = v_0 t + \frac{1}{2}at^2$

Distance x = 15 m

Initial velocity v0 = 8 m/s

Final velocity v = 0 m/s

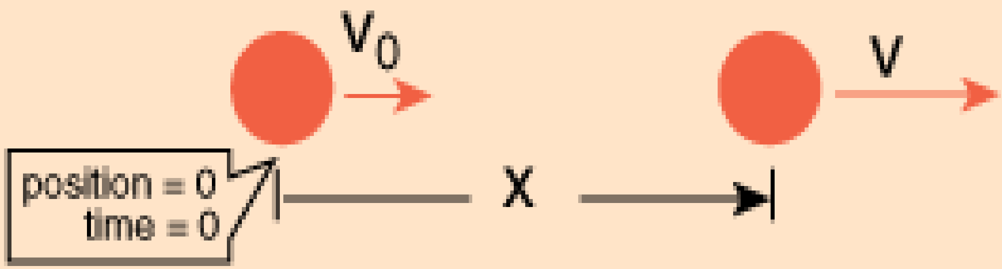
Average velocity = 4 m/s

Acceleration a = -2.13333333 m/s^2

Time t = 3.75 s

Motion Example

Initial velocity = 8 m/s, Final velocity = 2 m/s



Equations of motion

1. $x = \bar{v}t \quad \bar{v} = \dfrac{v_0 + v}{2}$
2. $v = v_0 + at$
3. $x = v_0 t + \frac{1}{2}at^2$

Distance x = 20 m

Initial velocity v0 = 8 m/s

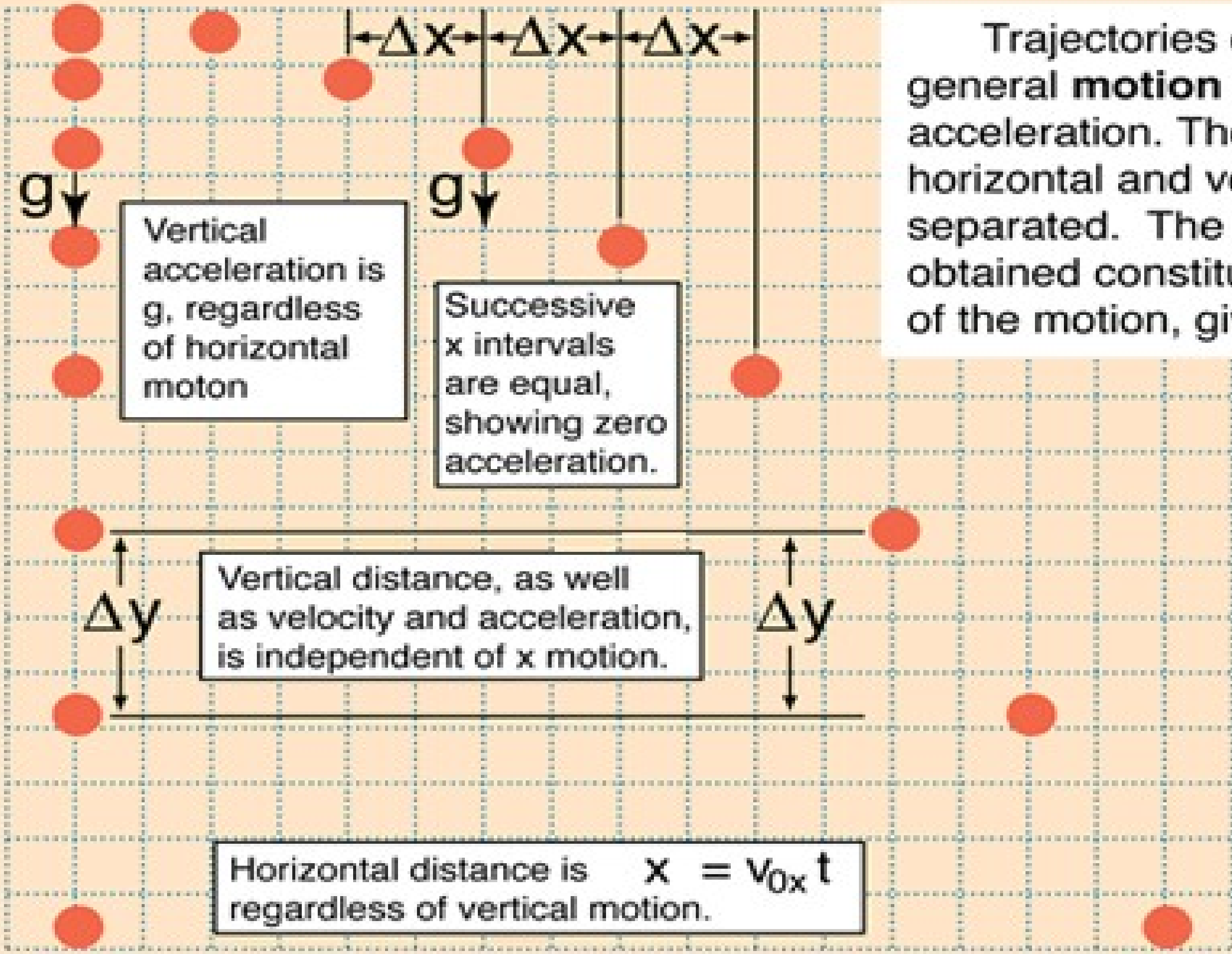
Final velocity v = 2 m/s

Average velocity = 5 m/s

Acceleration a = -1.5 m/s^2

Time t = 4 s

General Equations for projectile motions



Trajectories can be described by the general **motion equations** for constant acceleration. The key idea is that the horizontal and vertical motions can be separated. The motion equations obtained constitute a complete description of the motion, given the initial conditions.

Horizontal Motion →

$$a_x = 0$$
$$v_x = v_{0x}$$
$$x = v_{0x}\,t$$

Vertical Motion ↓

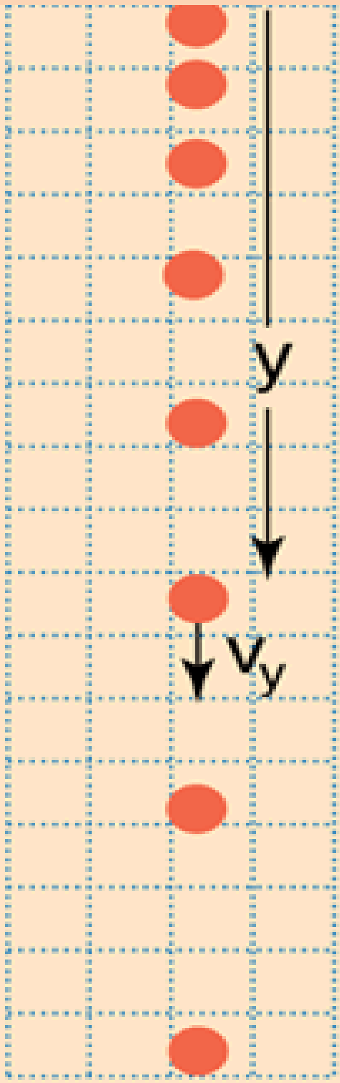
$$a_y = -g$$
$$v_y = v_{0y} - gt$$
$$y = v_{0y}\,t - \tfrac{1}{2}gt^2$$

+↑ Upward chosen as positive direction, so the y values will be negative.

SOME EXAMPLES
FOR
ONE DIMENSIONAL PROJECTILE MOTIONS

Freefall (a_x=0, v_x=v_{0x}=0,x=0, v_{0y}=0, y=h, hitting velocity v_y=20 m/s)

Images of an object in freefall at constant time intervals. Note that the distance traveled in each successive interval is larger.



↓g

$g = 9.8\ m/s^2$ so that the velocity increases 9.8 m/s each second.

In the absence of frictional drag, an object near the surface of the earth will fall with the constant acceleration of gravity g. Position and speed at any time can be calculated from the motion equations.

Illustrated here is the situation where an object is released from rest. It's position and speed can be predicted for any time after that. Since all the quantities are directed downward that direction is chosen as the positive direction in this case.

$$v_y = gt \qquad \text{Taking } g = 9.8\ m/s^2$$

$$y = \frac{1}{2} g t^2 \qquad = 32.15\ ft/s^2$$

At time t = 2.04081632€ s after being dropped,

the speed is v_y = 20 m/s

The distance from the starting point will be

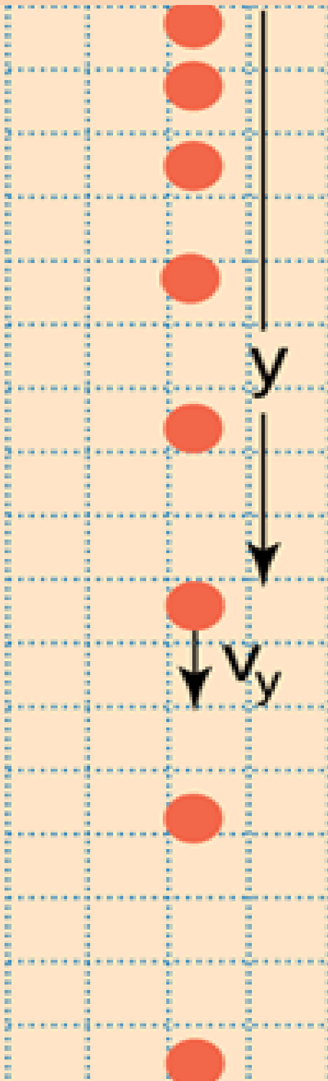
y = 20.40816326 m;

Freefall ($a_x=0$, $v_x=v_{0x}=0$, $x=0$, $v_{0y}=0$, $y=100m$)

Images of an object in freefall at constant time intervals. Note that the distance traveled in each successive interval is larger.

$\downarrow g$

$g = 9.8\ m/s^2$ so that the velocity increases 9.8 m/s each second.



In the absence of frictional drag, an object near the surface of the earth will fall with the constant acceleration of gravity g. Position and speed at any time can be calculated from the motion equations.

Illustrated here is the situation where an object is released from rest. It's position and speed can be predicted for any time after that. Since all the quantities are directed downward that direction is chosen as the positive direction in this case.

$$v_y = gt$$
$$y = \frac{1}{2} g t^2$$

Taking $g = 9.8\ m/s^2 = 32.15\ ft/s^2$

At time t = 4.51753951 s after being dropped,
the speed is v_y = 44.2718872 m/s

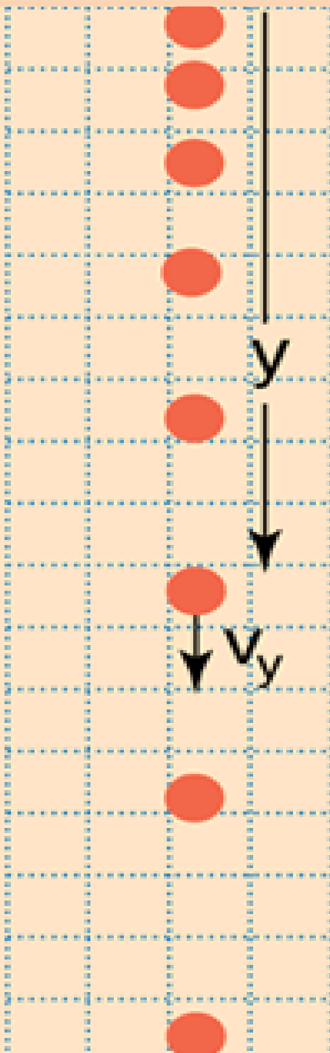
The distance from the starting point will be
y = 100 m

Freefall (a_x=0, v_x=v_{0x}=0, x=0, v_{0y}=0, y=h, t=10 s)

Images of an object in freefall at constant time intervals. Note that the distance traveled in each successive interval is larger.

$\downarrow g$

$g = 9.8\ m/s^2$ so that the velocity increases 9.8 m/s each second.



In the absence of frictional drag, an object near the surface o the earth will fall with the constant acceleration of gravity g. Position and speed at any time can be calculated from the motion equations.

Illustrated here is the situation where an object is released from rest. It's position and speed can be predicted for any time after that. Since all the quantities are directed downwarc that direction is chosen as the positive direction in this case.

$$v_y = gt$$
$$y = \frac{1}{2} g t^2$$

Taking $g = 9.8\ m/s^2 = 32.15\ ft/s^2$

At time t = 10 s after being dropped,

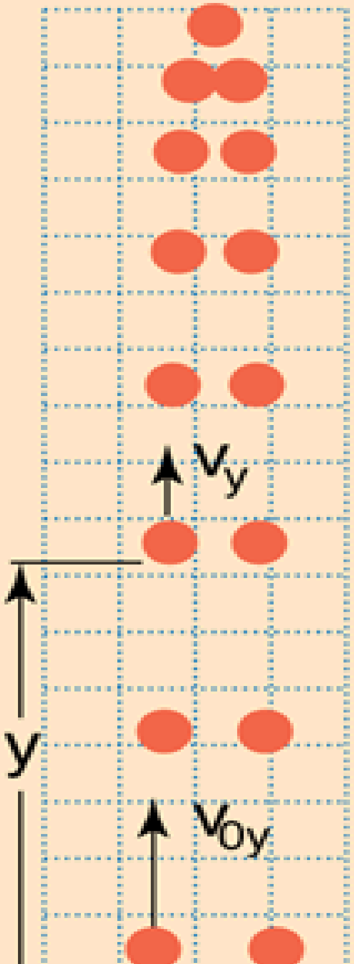
the speed is v_y = 98 m/s

The distance from the starting point will be

y = 490.000000(m

Vertical Motion ($a_x=0$, $v_x=v_{0x}=0, x=0$)

Vertical Trajectory



Vertical motion under the influence of gravity can be described by the basic motion equations. Given the constant acceleration of gravity g, the position and speed at any time can be calculated from the motion equations:

$$v_y = v_{0y} - gt \qquad \text{Taking } g = 9.8\ m/s^2$$

$$y = v_{0y}\, t - \frac{1}{2} g t^2$$

You may enter values for launch velocity and time in the boxes below and click outside the box to perform the calculation.

For launch speed v_{0y} = 12 m/s

and time t = 2.44 s ,

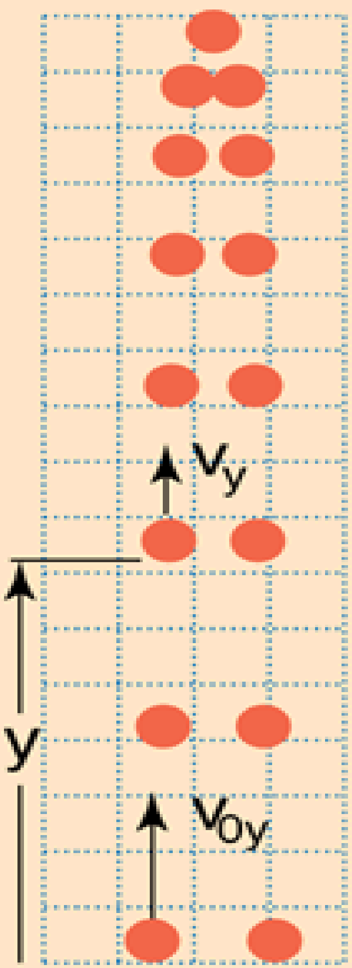
Peak at

7.346938775 m at

t= 1.224489795 s

Vertical Motion ($a_x=0$, $v_x=v_{0x}=0, x=0$)

Vertical Trajectory



Vertical motion under the influence of gravity can be described by the basic motion equations. Given the constant acceleration of gravity g, the position and speed at any time can be calculated from the motion equations:

$$v_y = v_{0y} - gt$$
$$y = v_{0y}t - \frac{1}{2}gt^2$$

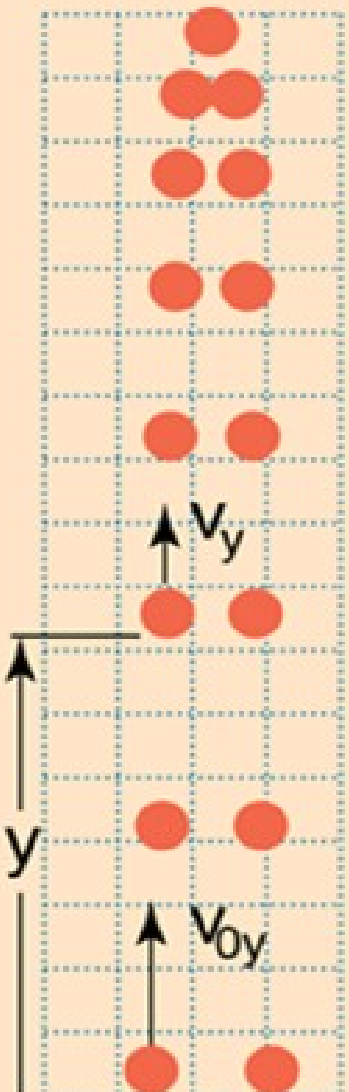
Taking $g = 9.8\ m/s^2$

You may enter values for launch velocity and time in the boxes below and click outside the box to perform the calculation.

For launch speed v_{0y} = 100 m/s
and time t = 10.20 s ,

Peak at
127.551020 m at
t= 5.102040816 s

Vertical Motion ($a_x=0$, $v_x=v_{0x}=0$, $x=0$)



Vertical motion under the influence of gravity can be described by the basic motion equations. Given the constant acceleration of gravity g, the position and speed at any time can be calculated from the motion equations:

$$v_y = v_{0y} - gt$$

$$y = v_{0y}t - \frac{1}{2}gt^2$$

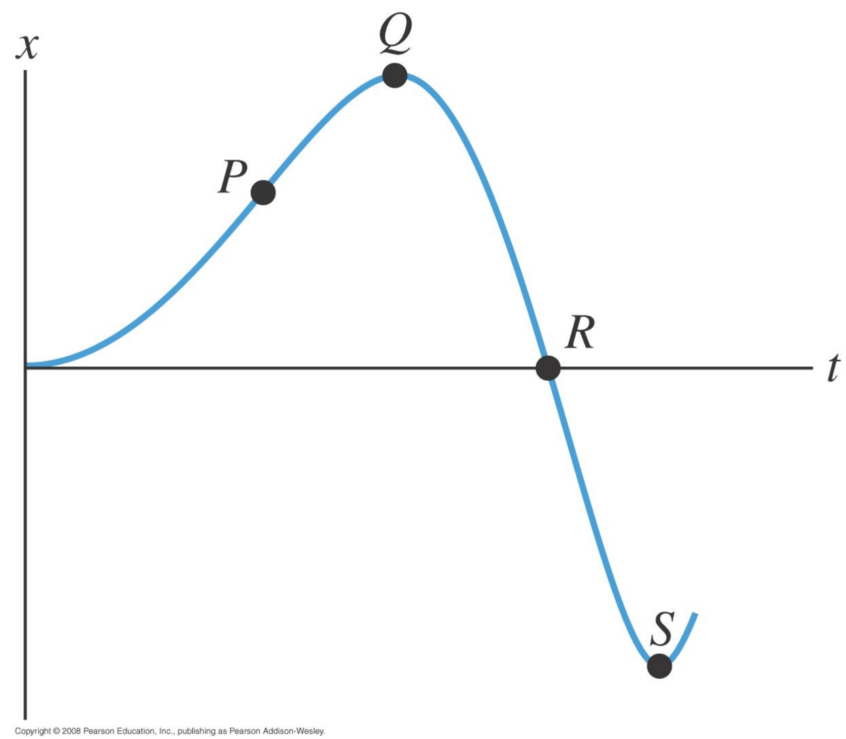
Taking $g = 9.8\ m/s^2$

You may enter values for launch velocity and time in the boxes below and click outside the box to perform the calculation.

For launch speed v_{0y} = 200 m/s
and time t = 40.80 s ,

Peak at
510.2040816 m at
t= 20.40 s

Q2.1

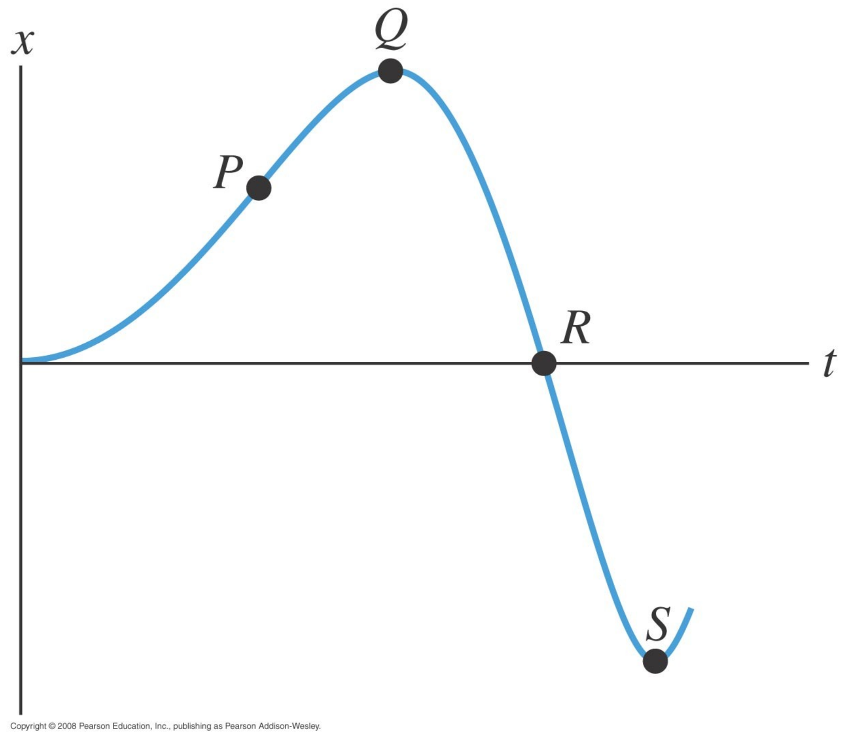


This is the *x*–*t* graph of the motion of a particle. Of the four points *P*, *Q*, *R*, and *S*, the velocity v_x is greatest (most positive) at

A. point *P*. B. point *Q*. C. point *R*. D. point *S*.

E. not enough information in the graph to decide

A2.1

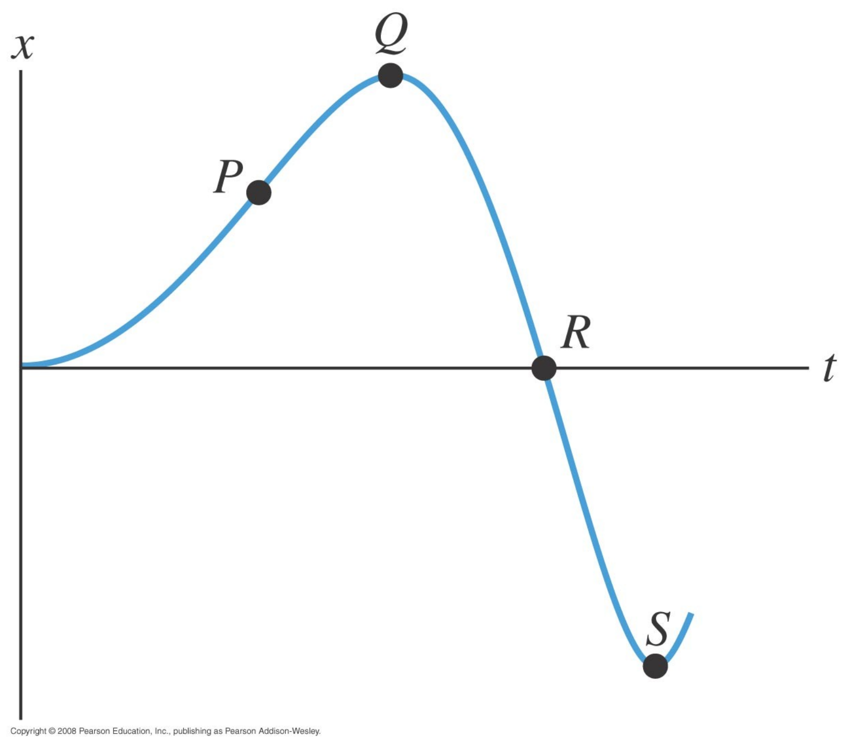


This is the x–t graph of the motion of a particle. Of the four points *P*, *Q*, *R*, and *S*, the velocity v_x is greatest (most positive) at

A. point *P*.　　B. point *Q*.　　C. point *R*.　　D. point *S*.

E. not enough information in the graph to decide

Q2.2

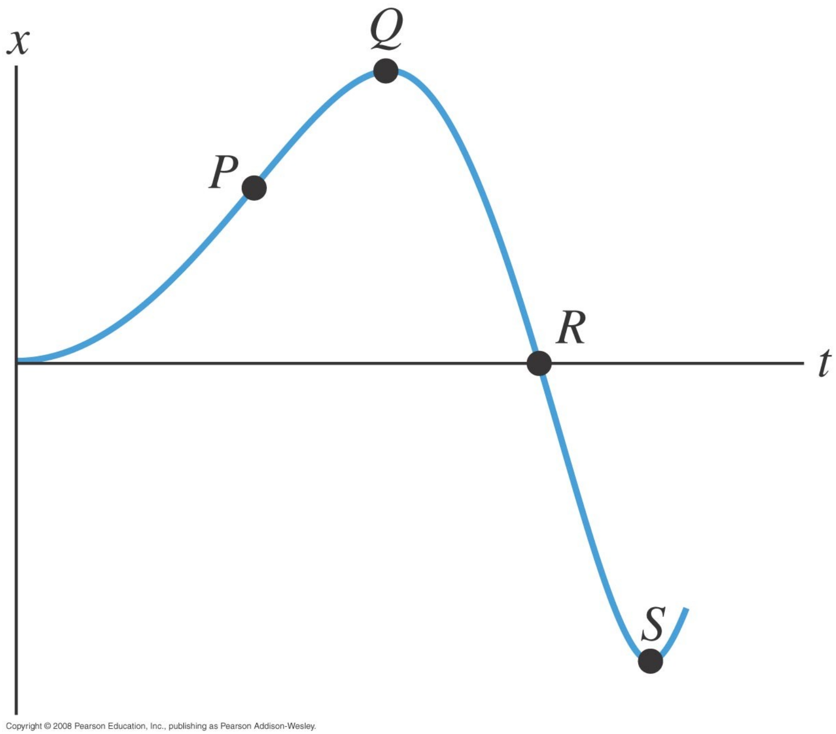


This is the *x*–*t* graph of the motion of a particle. Of the four points *P*, *Q*, *R*, and *S*, the speed is greatest at

A. point *P*. B. point *Q*. C. point *R*. D. point *S*.

E. not enough information in the graph to decide

A2.2

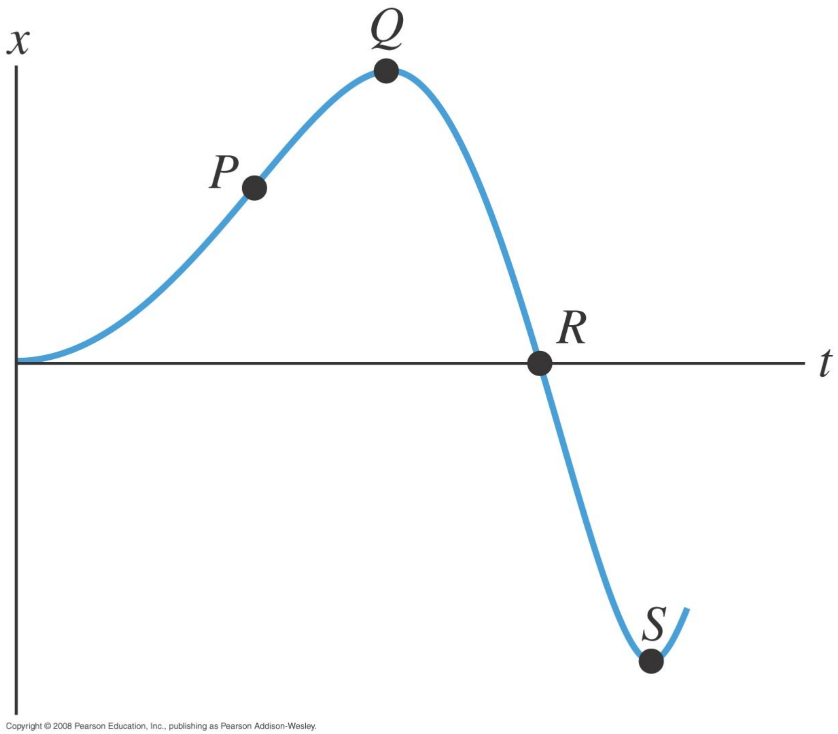


This is the *x*–*t* graph of the motion of a particle. Of the four points *P*, *Q*, *R*, and *S*, the speed is greatest at

A. point *P*. B. point *Q*. ✓ C. point *R*. D. point *S*.

E. not enough information in the graph to decide

Q2.3

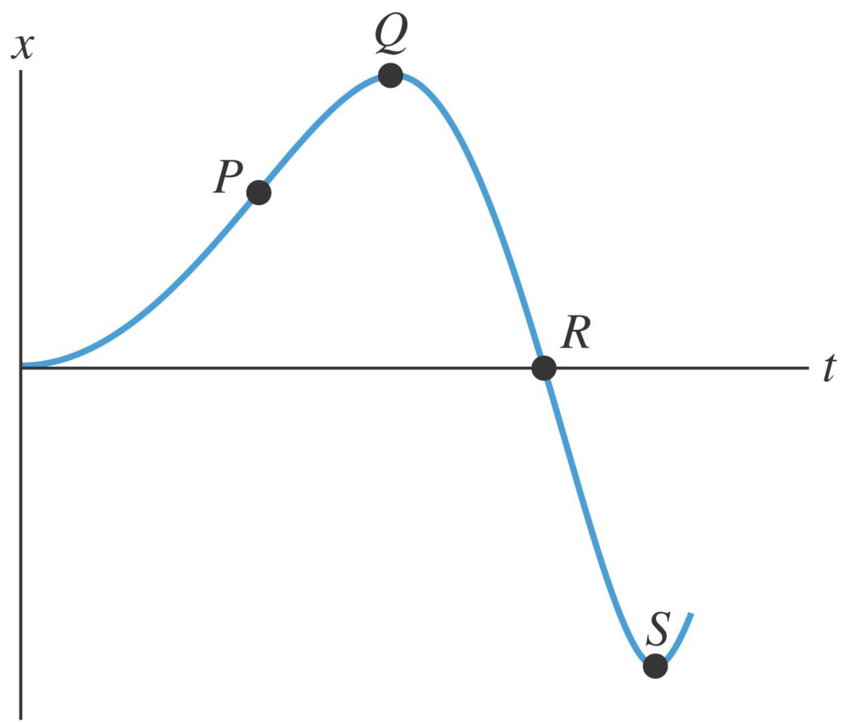


This is the *x*–*t* graph of the motion of a particle. Of the four points *P*, *Q*, *R*, and *S*, the acceleration a_x is greatest (most positive) at

A. point *P*. B. point *Q*. C. point *R*. D. point *S*.

E. not enough information in the graph to decide

A2.3



This is the x–t graph of the motion of a particle. Of the four points P, Q, R, and S, the acceleration a_x is greatest (most positive) at

A. point P. B. point Q. C. point R. ✔ D. point S.

E. not enough information in the graph to decide

Q2.4

You toss a ball straight upward, in the positive direction. The ball falls freely under the influence of gravity.

At the highest point in the ball's motion,

A. its velocity is zero and its acceleration is zero.

B. its velocity is zero and its acceleration is positive (upward).

C. its velocity is zero and its acceleration is negative (downward).

D. its velocity is positive (upward) and its acceleration is zero.

E. its velocity is positive (upward) and its acceleration is zero.

A2.4

You toss a ball straight upward, in the positive direction. The ball falls freely under the influence of gravity.

At the highest point in the ball's motion,

A. its velocity is zero and its acceleration is zero.

B. its velocity is zero and its acceleration is positive (upward).

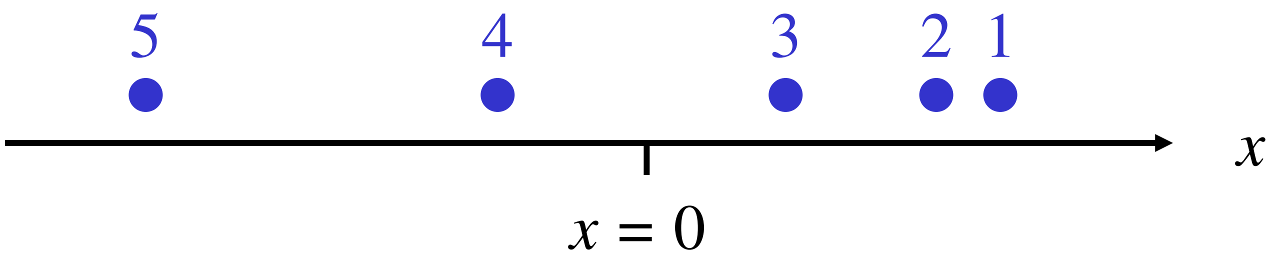
✓ C. its velocity is zero and its acceleration is negative (downward).

D. its velocity is positive (upward) and its acceleration is zero.

E. its velocity is positive (upward) and its acceleration is zero.

Q2.5

This is a motion diagram of an object moving along the x–direction with constant acceleration. The dots 1, 2, 3, … show the position of the object at equal time intervals Δt.



At the time labeled 3, what are the signs of the object's velocity v_x and acceleration a_x?

A. $v_x < 0$, $a_x = 0$

B. $v_x < 0$, $a_x > 0$

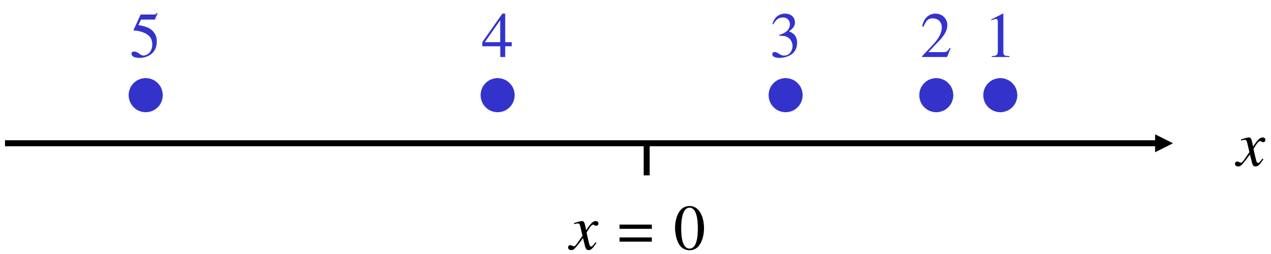
C. $v_x < 0$, $a_x < 0$

D. $v_x > 0$, $a_x > 0$

E. $v_x > 0$, $a_x < 0$

A2.5

This is a motion diagram of an object moving along the x–direction with constant acceleration. The dots 1, 2, 3, … show the position of the object at equal time intervals Δt.



At the time labeled 3, what are the signs of the object's velocity v_x and acceleration a_x?

A. $v_x < 0$, $a_x = 0$

B. $v_x < 0$, $a_x > 0$

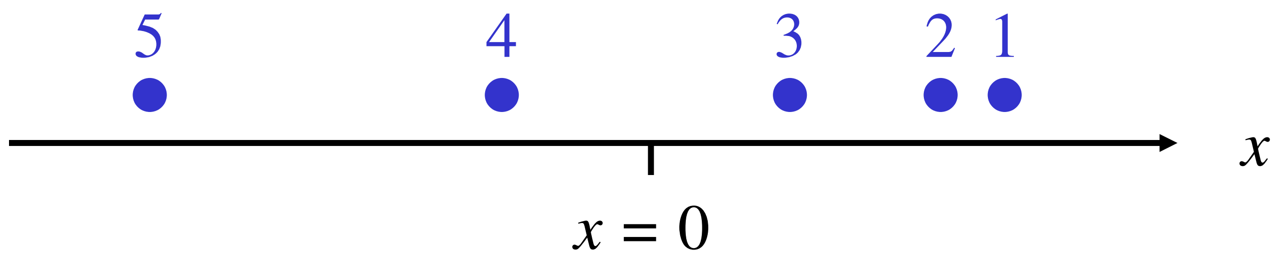
✔ C. $v_x < 0$, $a_x < 0$

D. $v_x > 0$, $a_x > 0$

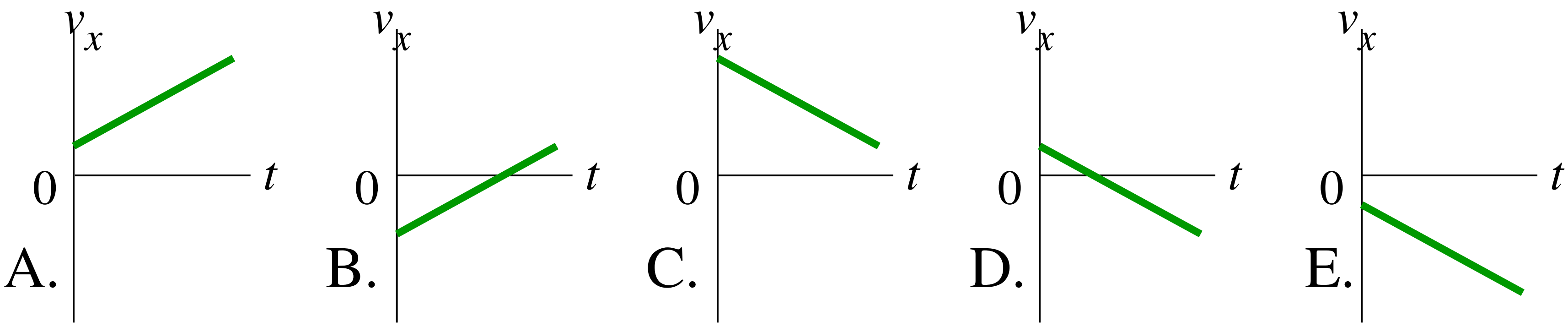
E. $v_x > 0$, $a_x < 0$

Q2.6

This is a motion diagram of an object moving along the x–direction with constant acceleration. The dots 1, 2, 3, … show the position of the object at equal time intervals Δt.

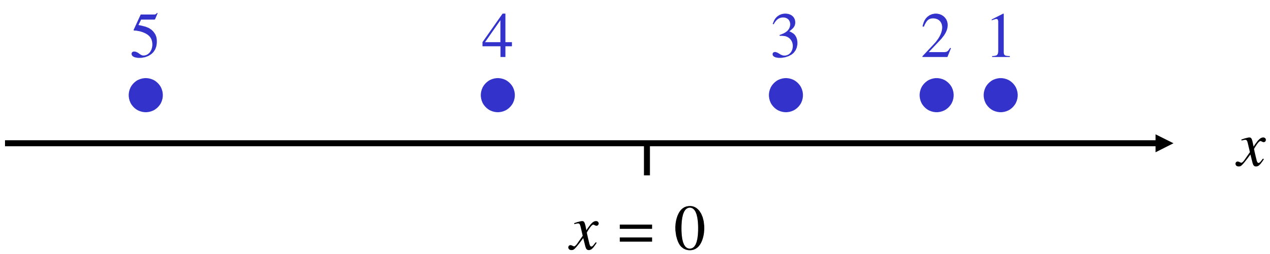


Which of the following v_x–t graphs best matches the motion shown in the motion diagram?

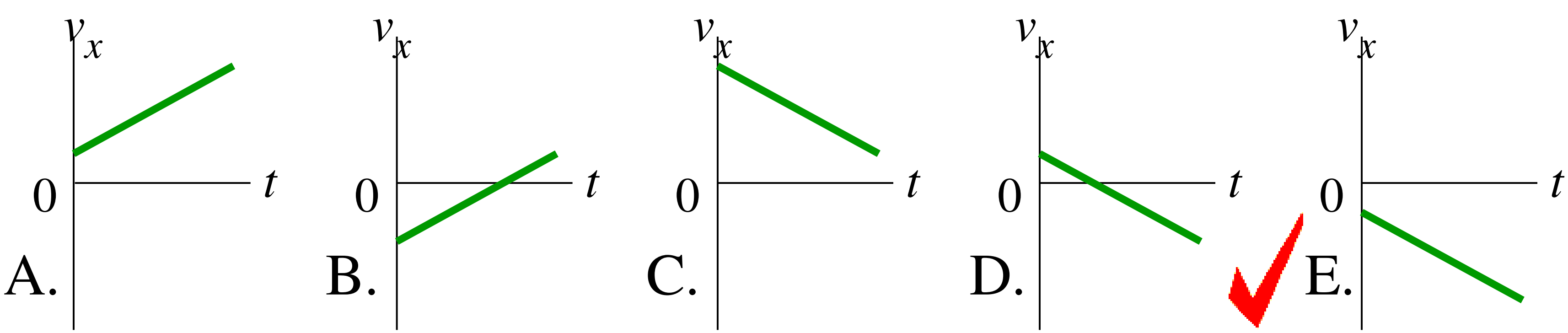


A2.6

This is a motion diagram of an object moving along the x–direction with constant acceleration. The dots 1, 2, 3, … show the position of the object at equal time intervals Δt.

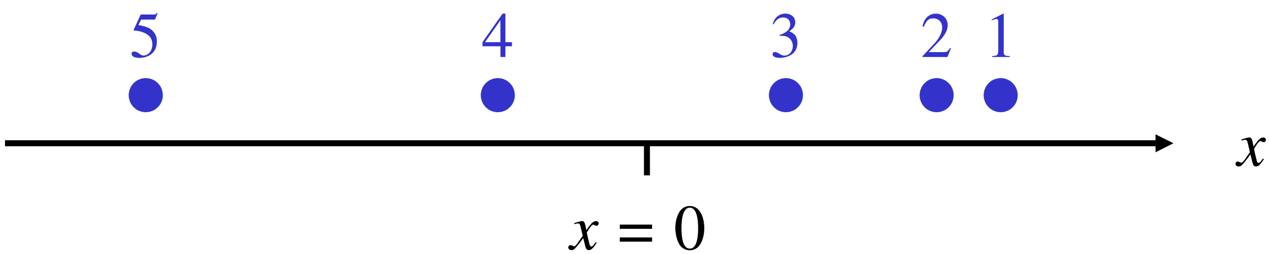


Which of the following v_x–t graphs best matches the motion shown in the motion diagram?

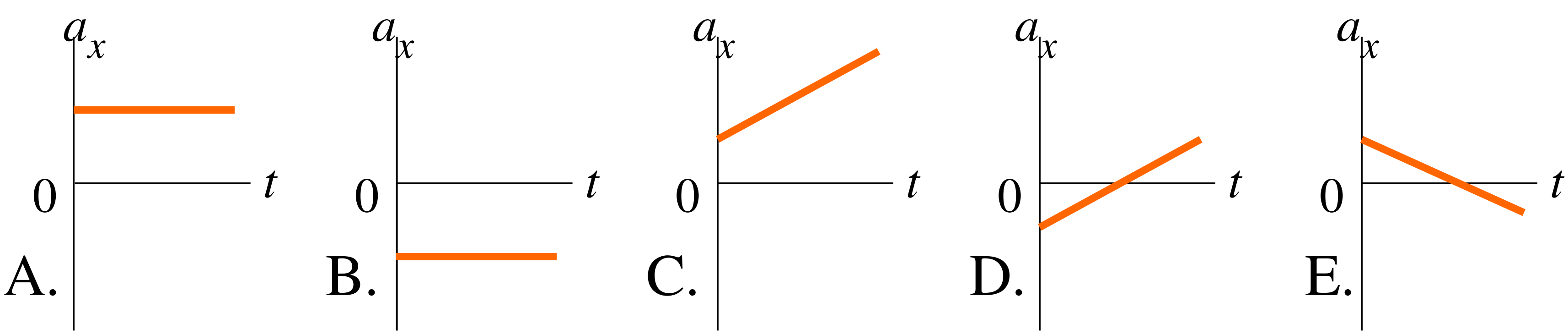


Q2.7

This is a motion diagram of an object moving along the x–direction with constant acceleration. The dots 1, 2, 3, … show the position of the object at equal time intervals Δt.

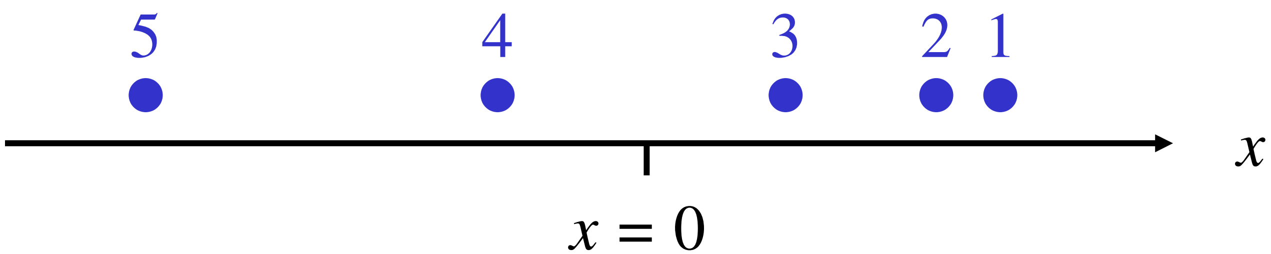


Which of the following a_x–t graphs best matches the motion shown in the motion diagram?

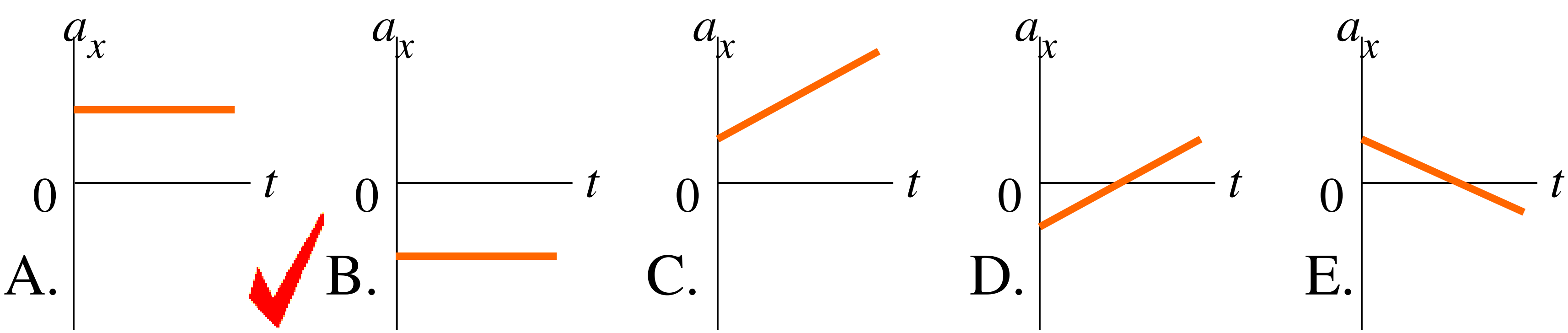


A2.7

This is a motion diagram of an object moving along the x–direction with constant acceleration. The dots 1, 2, 3, … show the position of the object at equal time intervals Δt.



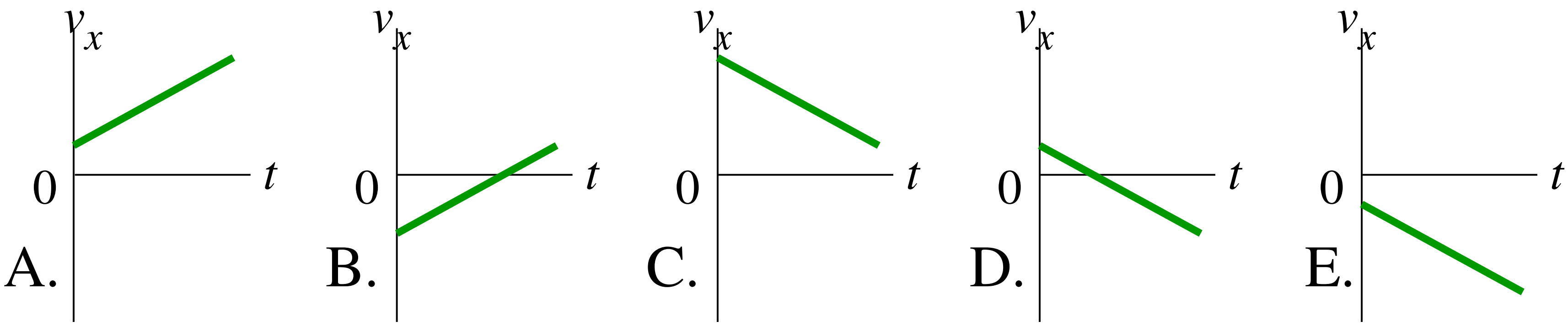
Which of the following a_x–t graphs best matches the motion shown in the motion diagram?



Q2.8

An object moves along the x–axis with constant acceleration. The initial position x_0 is positive, the initial velocity is negative, and the acceleration is positive.

Which of the following v_x–t graphs best describes this motion?



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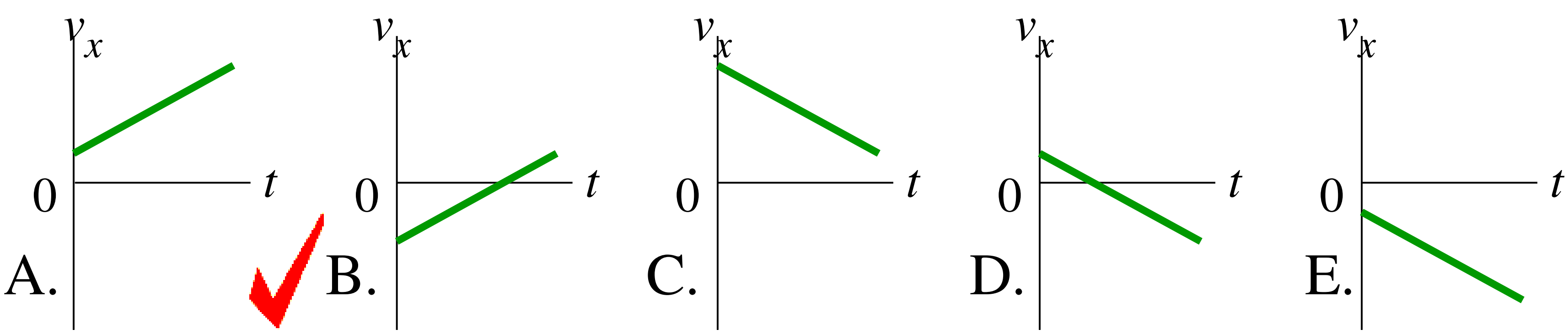
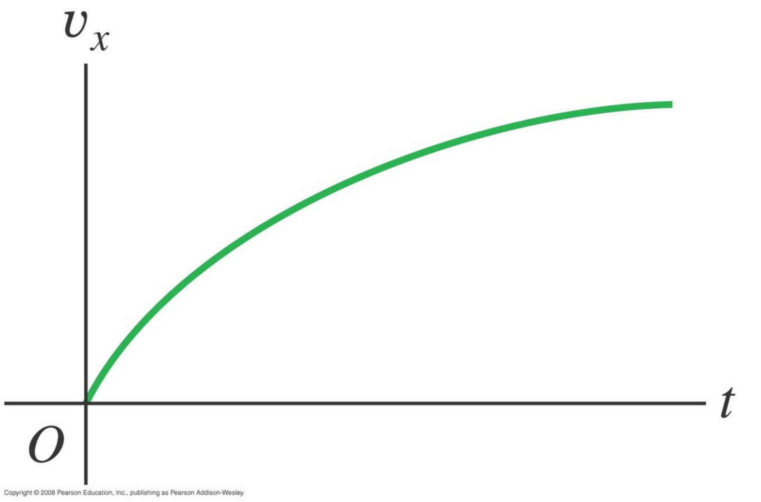


Q2.9

This is the v_x–t graph for an object moving along the x-axis.



Which of the following descriptions of the motion is most accurate?

A. The object is slowing down at a decreasing rate.

B. The object is slowing down at an increasing rate.

C. The object is speeding up at a decreasing rate.

D. The object is speeding up at an increasing rate.

E. The object's speed is changing at a steady rate.

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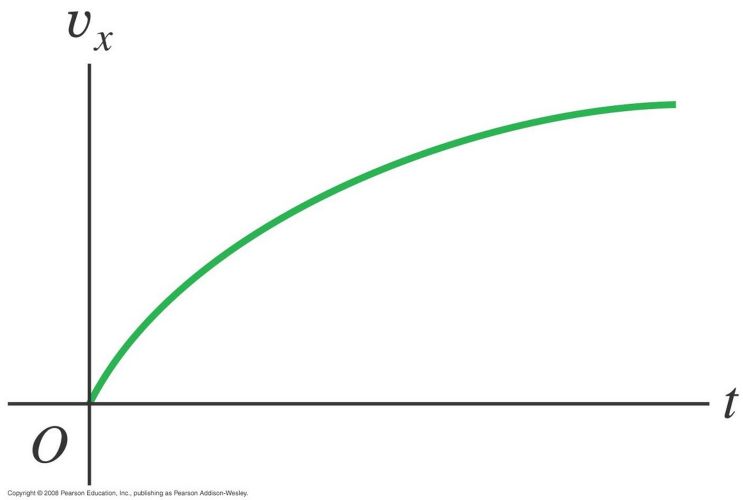

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Q2.10

You are given the v_x–t graph for an object moving along the x-axis with constant acceleration. Which of the following could you ***not*** determine from the information given in this graph alone?

A. the object's x–acceleration at any time t

B. the object's x–velocity at any time t

C. the object's position at any time t

D. more than one of the above

E. misleading question — you could determine all of these from the v_x–t graph alone

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Q2.11

The position of an object moving along the *x*-axis is given by

$$x = 5.0\text{ m} - (4.0\text{ m/s})t + (2.0\text{ m/s}^2)t^2$$

Which statement about this object is *correct*?

A. For $t > 0$, the object is never at rest.

B. The object is at rest at $t = 0.5$ s.

C. The object is at rest at $t = 1.0$ s.

D. The object is at rest at $t = 2.0$ s.

E. More than one of B., C., and D. is correct.

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Q2.12

The position of an object moving along the x-axis is given by

$$x = 5.0\ \text{m} - (4.0\ \text{m/s})t + (2.0\ \text{m/s}^2)t^2$$

How many times does this object pass through the point $x = 0$?

A. twice, first moving in the positive x-direction, then moving in the negative x-direction

B. twice, first moving in the negative x-direction, then moving in the positive x-direction

C. only once, moving in the positive x-direction

D. only once, moving in the negative x-direction

E. never

A2.12

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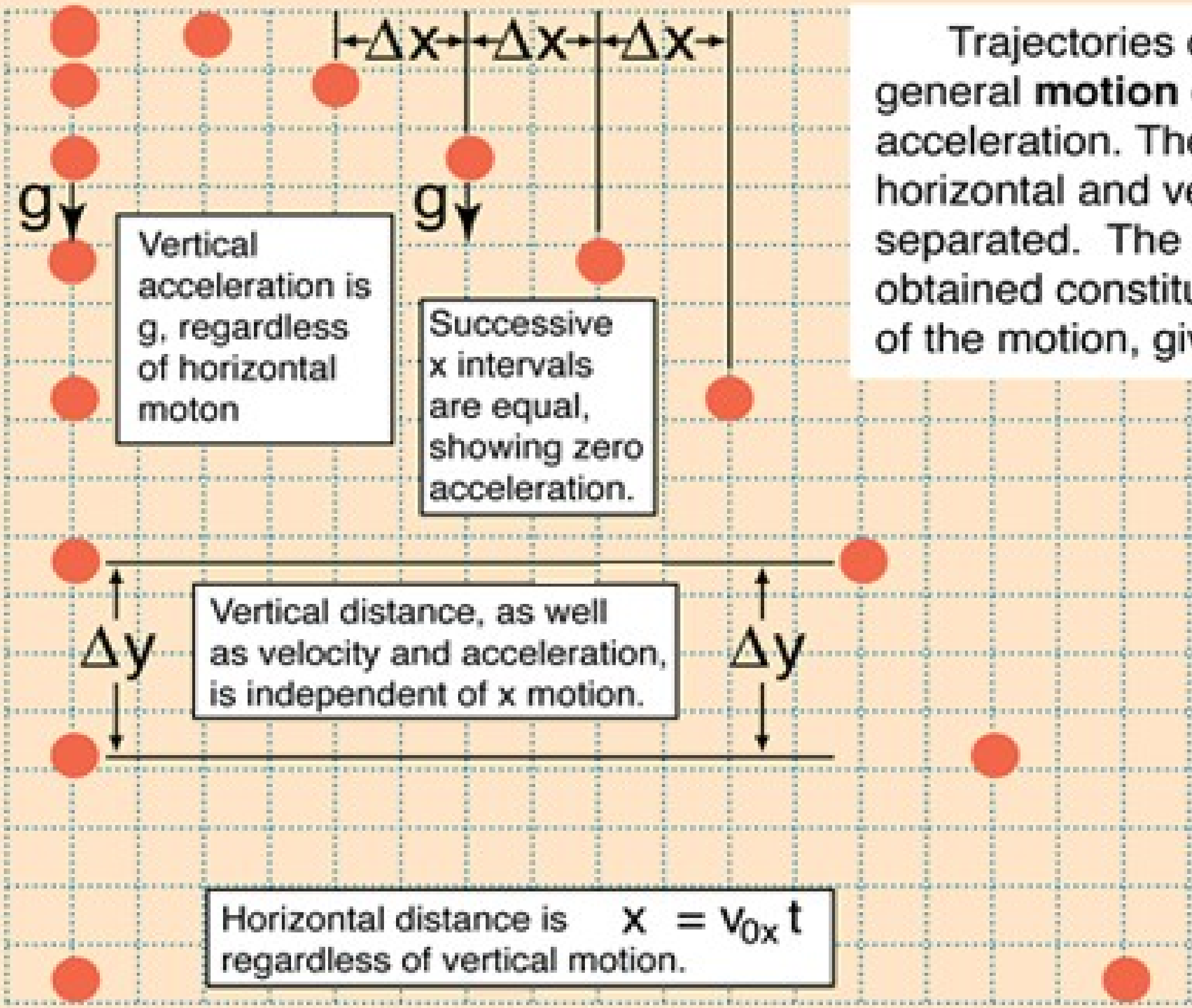
D. only once, moving in the negative *x*-direction

✓ E. never

MOTION EQUATIONS



Trajectories can be described by the general **motion equations** for constant acceleration. The key idea is that the horizontal and vertical motions can be separated. The motion equations obtained constitute a complete description of the motion, given the initial conditions.

Horizontal Motion →

$$a_x = 0$$
$$v_x = v_{0x}$$
$$x = v_{0x}t$$

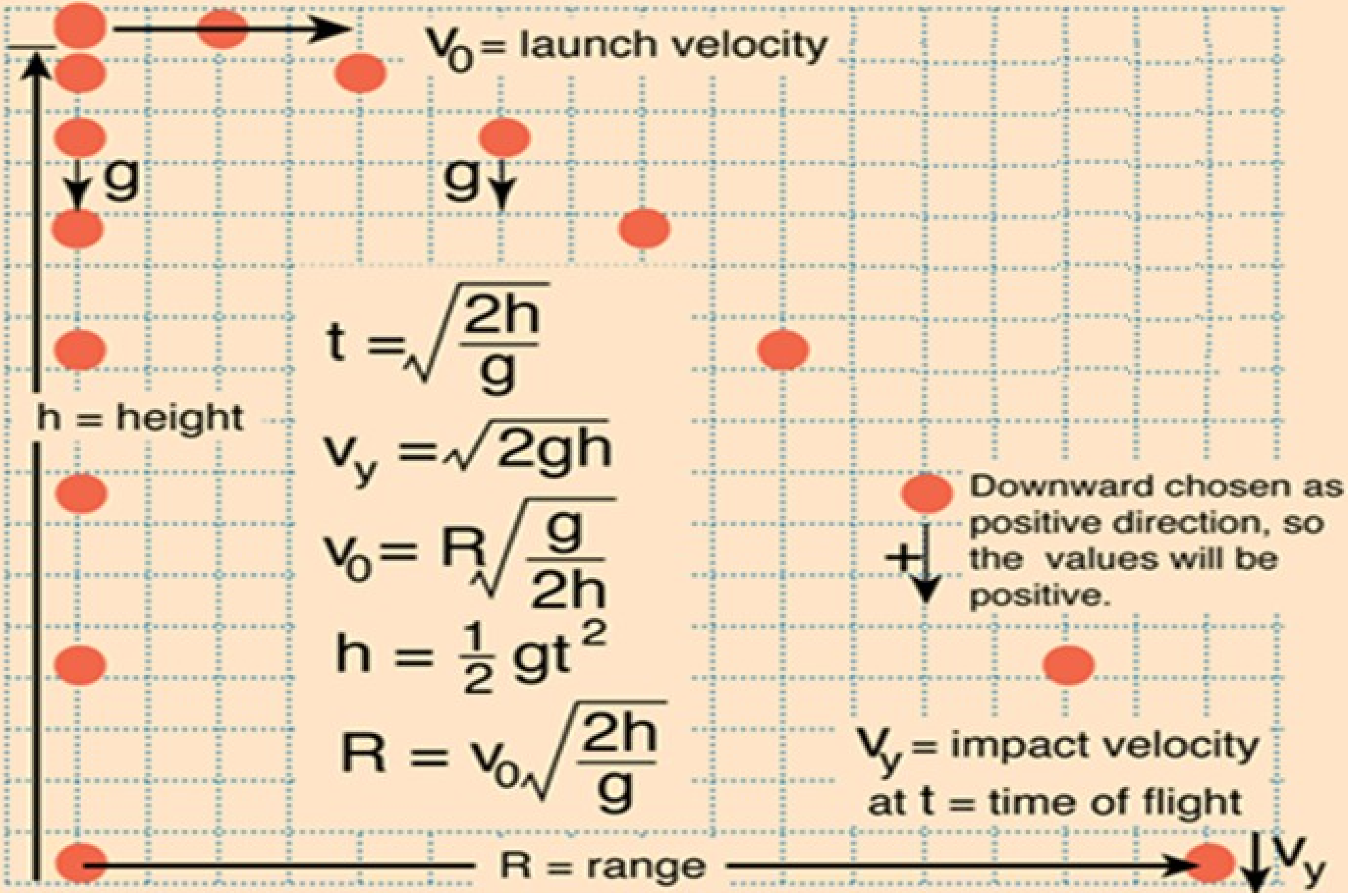
Vertical Motion ↓

$$a_y = -g$$
$$v_y = v_{0y} - gt$$
$$y = v_{0y}t - \frac{1}{2}gt^2$$

+↑ Upward chosen as positive direction, so the y values will be negative.

Horizontal Motion ($a_x=0$, $v_0=v_x=v_{0x}$, $v_{0y}=0$)

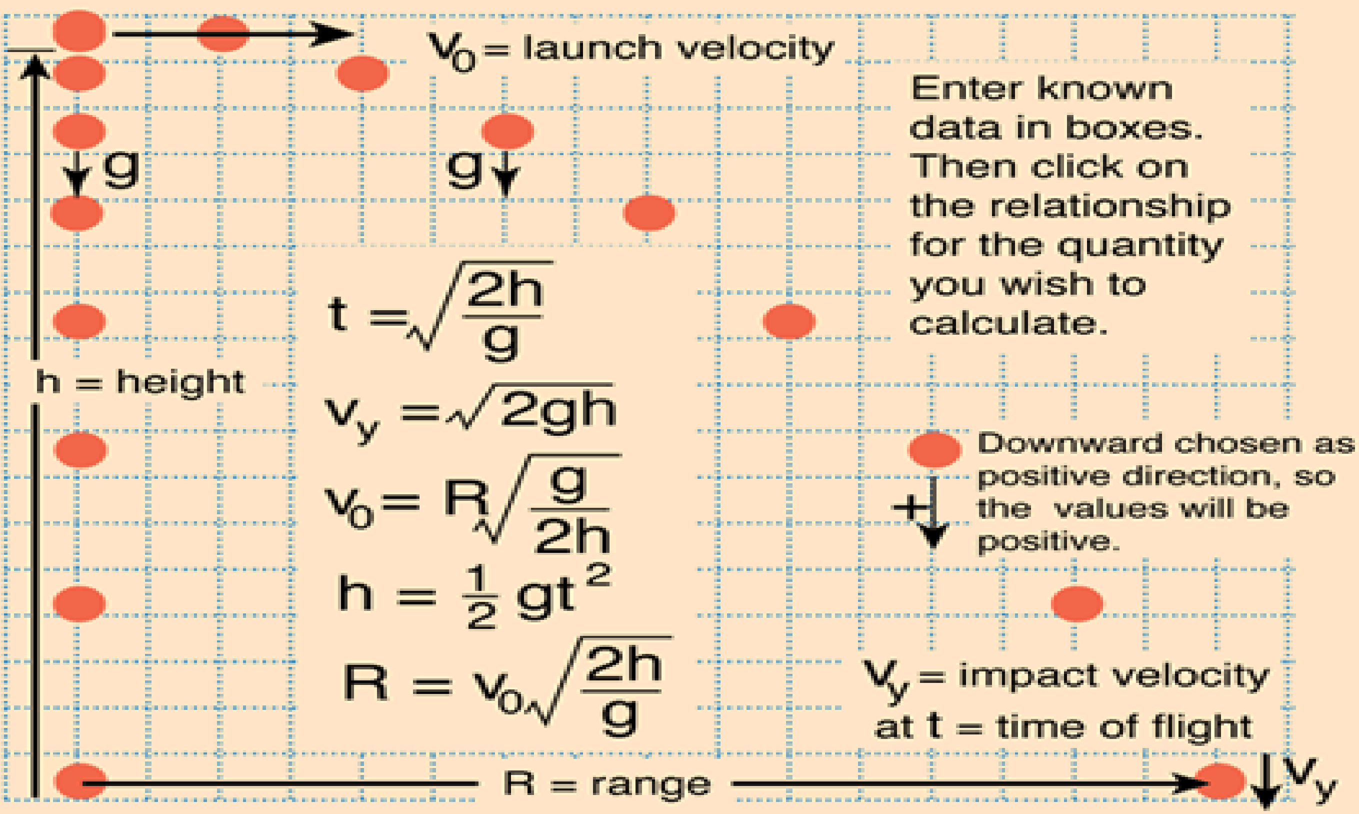
All the parameters of a horizontal launch can be calculated with the motion equations, assuming a downward acceleration of gravity of 9.8 m/s^2.



Example: Horizontal Motion ($v_0=v_x=v_{0x}$=12 m/s, h=20 m)

Horizontal Launch

All the parameters of a horizontal launch can be calculated with the motion equations, assuming a downward acceleration of gravity of 9.8 m/s^2.



Time of flight

t = 2.020305089 s

Vertical impact velocity

v_y = 19.79898987 m/s

Launch velocity

v_0 = 12 m/s

Height of launch

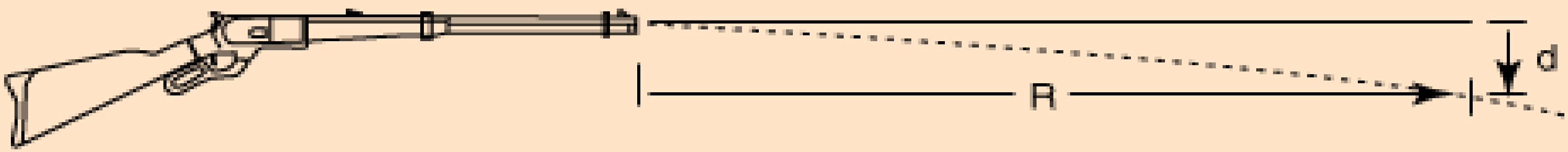
h = 20.0000000(m

Horizontal range

R = 24.2436610(m

Calculation is initiated by clicking on the formula in the illustration for the quantity you wish to calculate.

Example: Vertical distance for v=500 m/s, R=1000 m



If air friction is neglected, then the drop of a bullet fired horizontally can be treated as an ordinary horizontal trajectory. The air friction is significant, so this is an idealization.

If the muzzle velocity is

v = 500 m/s = 1640.419947 ft/s = 1120 mi/hr = 1798.561151 km/hr

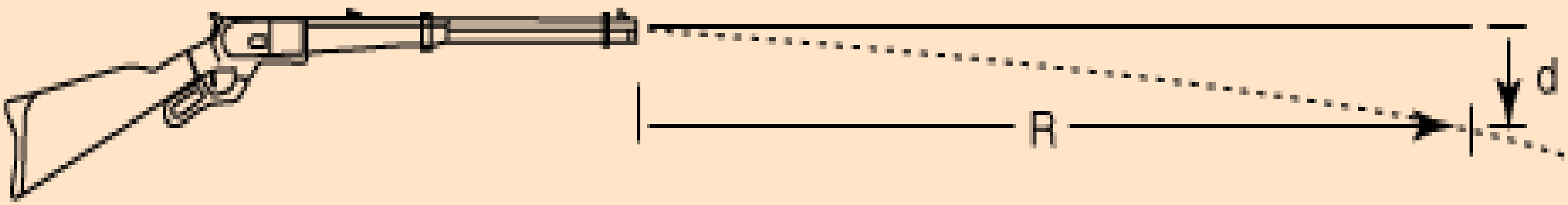
and the distance downrange is

R = 1000 m = 3280.839895 ft = 1093.613298 yards,

Then the amount of drop of the bullet below the horizontal would be

d = 19.6 m = 1960.000000 cm = 771.653543 inches

Example: Horizontal range and time for v= 500 m/s,d=20 m

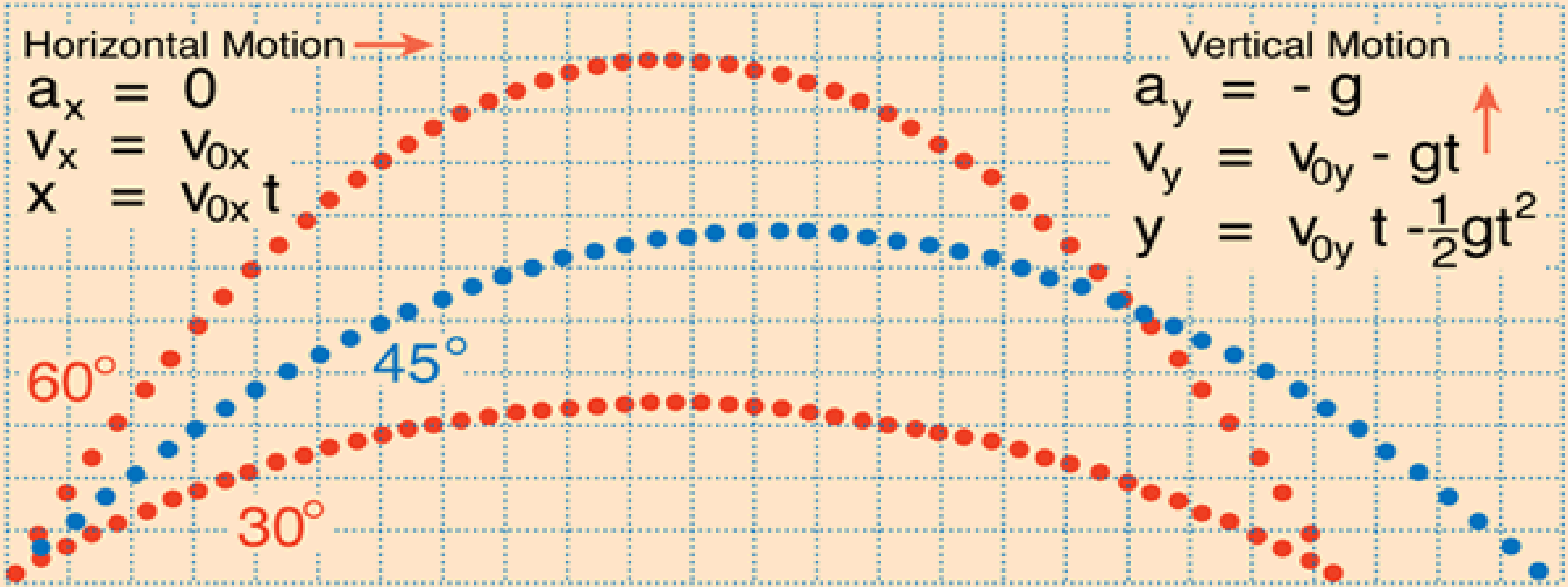


If air friction is neglected, then the drop of a bullet fired horizontally can be treated as an ordinary horizontal trajectory. The air friction is significant, so this is an idealization.

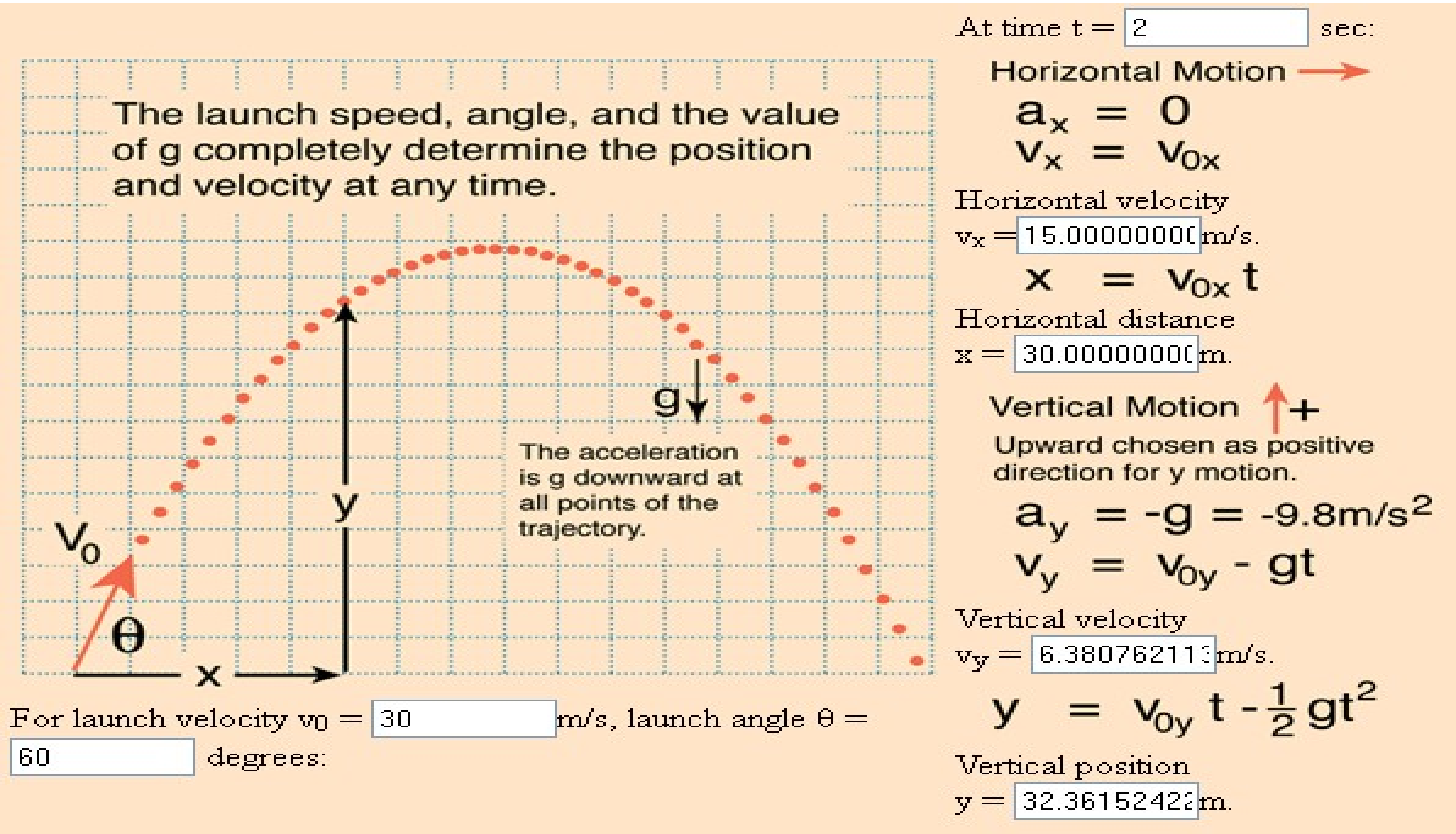
If the gun is fired on level ground at a height of 20 m = 65.61679790 ft, then the bullet will hit the ground in 2.020305089 seconds, having traveled a distance of 1010.152544 meters = 3314.148768 feet.

General Trajectory: Projectile motion

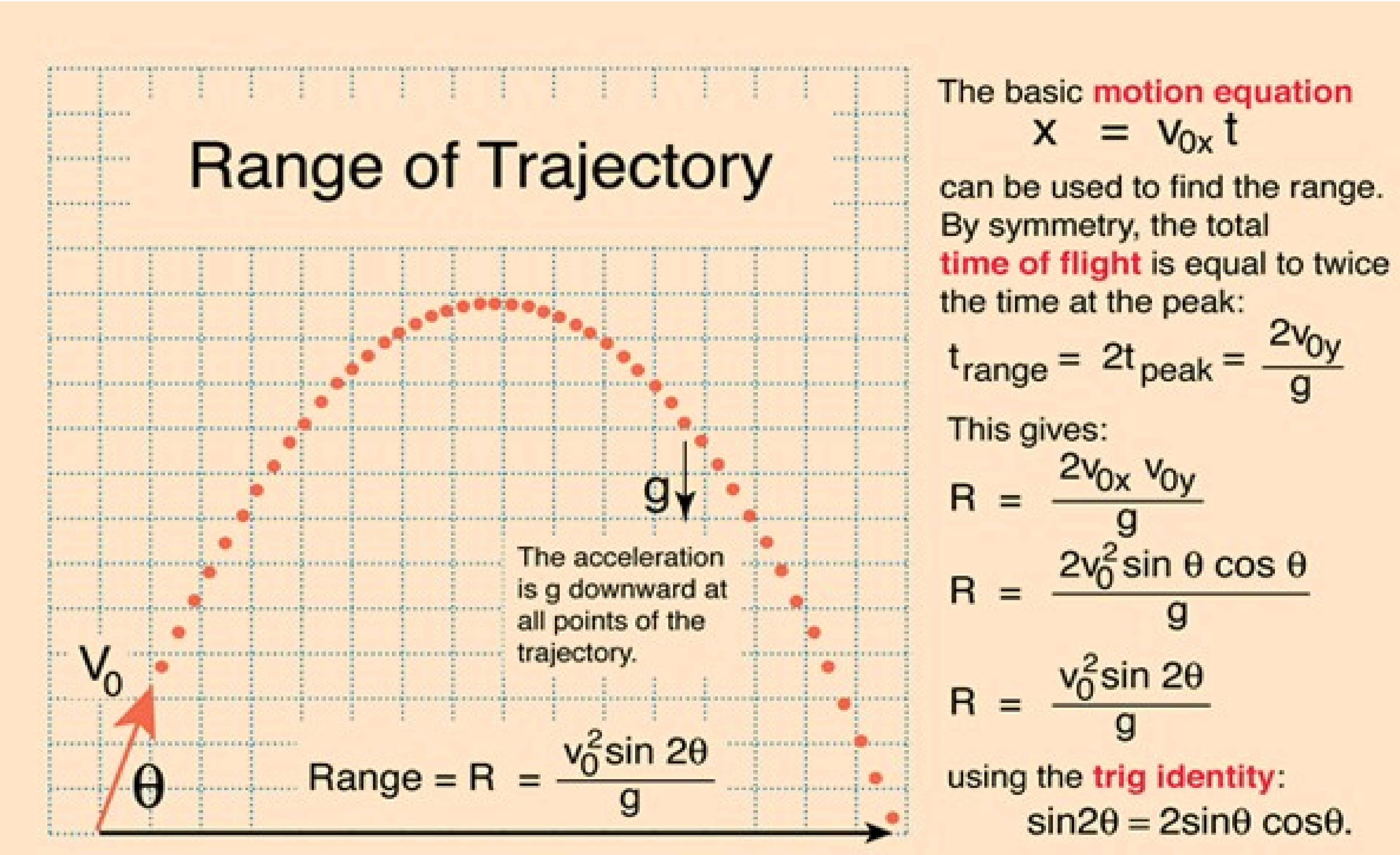
The motion of an object under the influence of gravity is determined completely by the acceleration of gravity, its launch speed, and launch angle provided air friction is negligible. The horizontal and vertical motions may be separated and described by the general motion equations for constant acceleration. The initial vector components of the velocity are used in the equations. The diagram shows trajectories with the same launch speed but different launch angles. Note that the 60 and 30 degree trajectories have the same range, as do any pair of launches at complementary angles. The launch at 45 degrees gives the maximum range.



Example: Particular horizontal velocity, horizontal distance, vertical velocity and vertical position for v_0=30 m/s and θ=60 at t=2

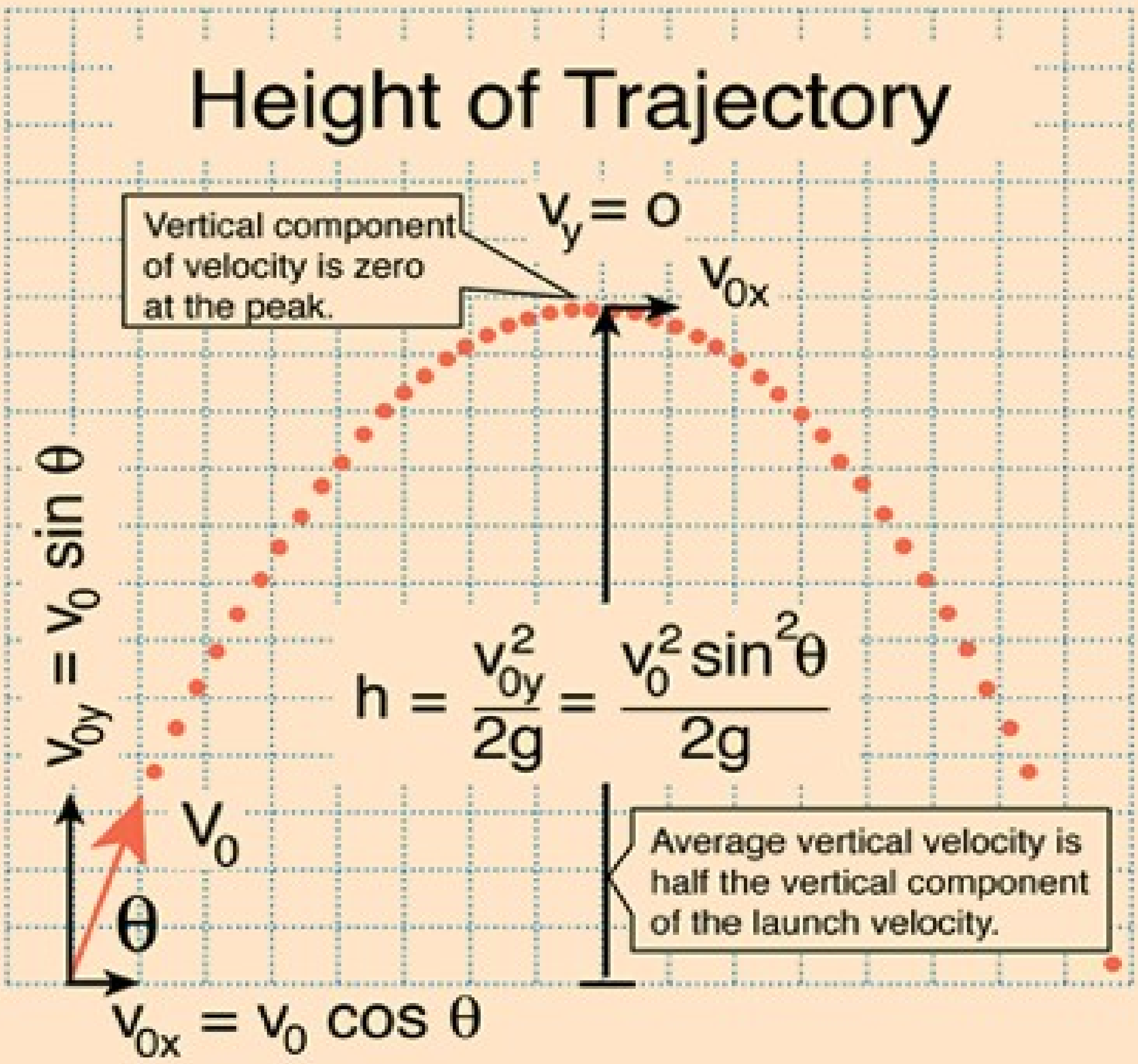


Range



Range of Trajectory
V_0
θ
g
The acceleration is g downward at all points of the trajectory.
Range = R = $\frac{v_0^2 \sin 2\theta}{g}$
The basic motion equation
$x = v_{0x} t$
can be used to find the range. By symmetry, the total time of flight is equal to twice the time at the peak:
$t_{range} = 2t_{peak} = \frac{2v_{0y}}{g}$
This gives:
$R = \frac{2v_{0x} v_{0y}}{g}$
$R = \frac{2v_0^2 \sin\theta \cos\theta}{g}$
$R = \frac{v_0^2 \sin 2\theta}{g}$
using the trig identity:
$\sin 2\theta = 2\sin\theta \cos\theta$.

Peak Height



The basic **motion equation**

$$y = \bar{v}_y t$$

can be used to find the height. The average vertical speed is:

$$\bar{v}_y = \frac{v_{0y} + 0}{2} = \frac{v_{0y}}{2}$$

The time at the peak is obtained by solving for the time at zero vertical speed:

$$0 = v_{0y} - gt_{peak}$$

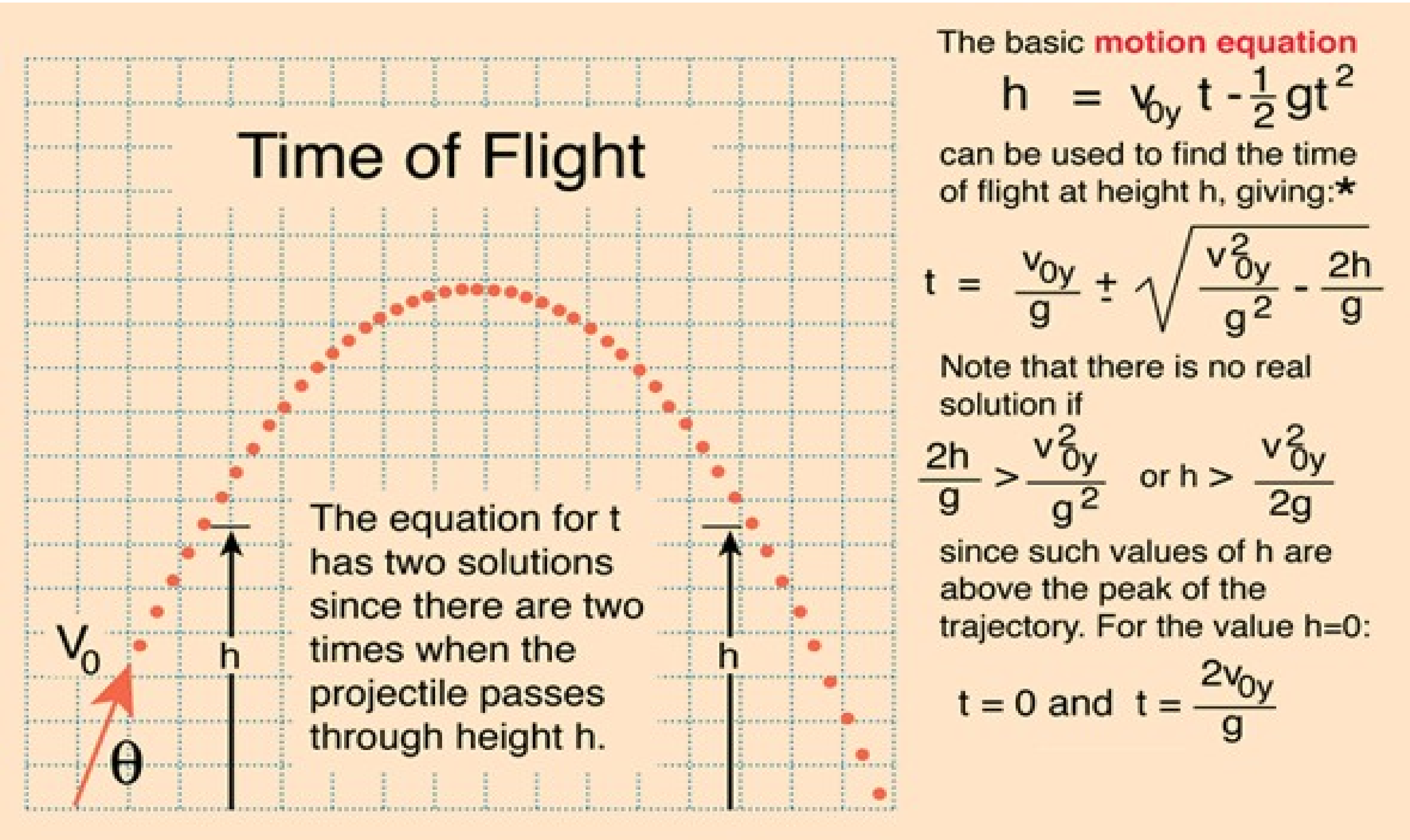
This gives:

$$t_{peak} = \frac{v_{0y}}{g}$$

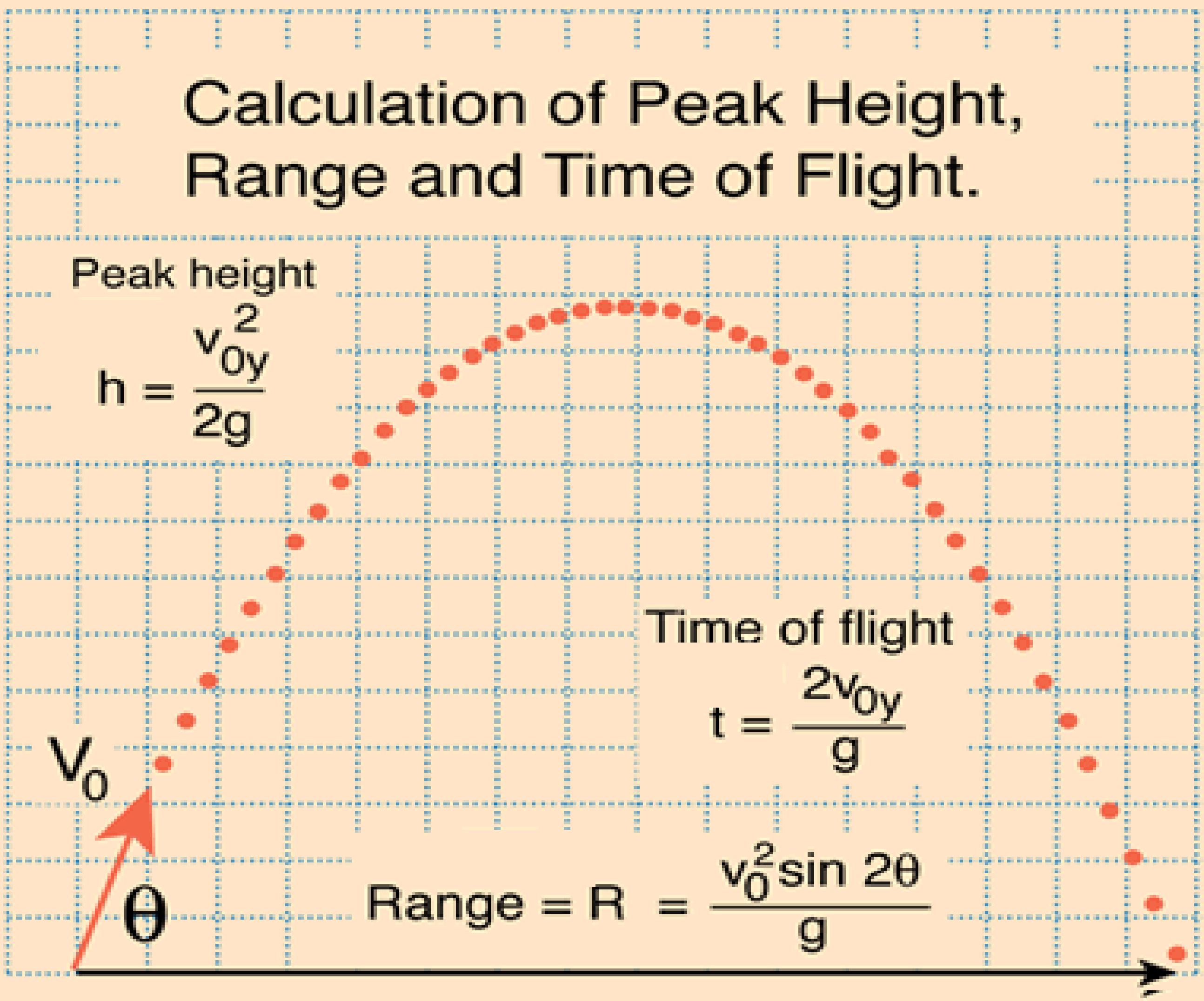
and substituting:

$$h = y_{peak} = \frac{v_{0y}^2}{2g}$$

Time of Flight



Example: Peak Height, Range and Time of Flight for v_0=10 m/s and θ=30



For launch velocity

$v_0 =$ 10 m/s,

launch angle

$\theta =$ 30 degrees,

The horizontal range is

R = 8.83699391(m.

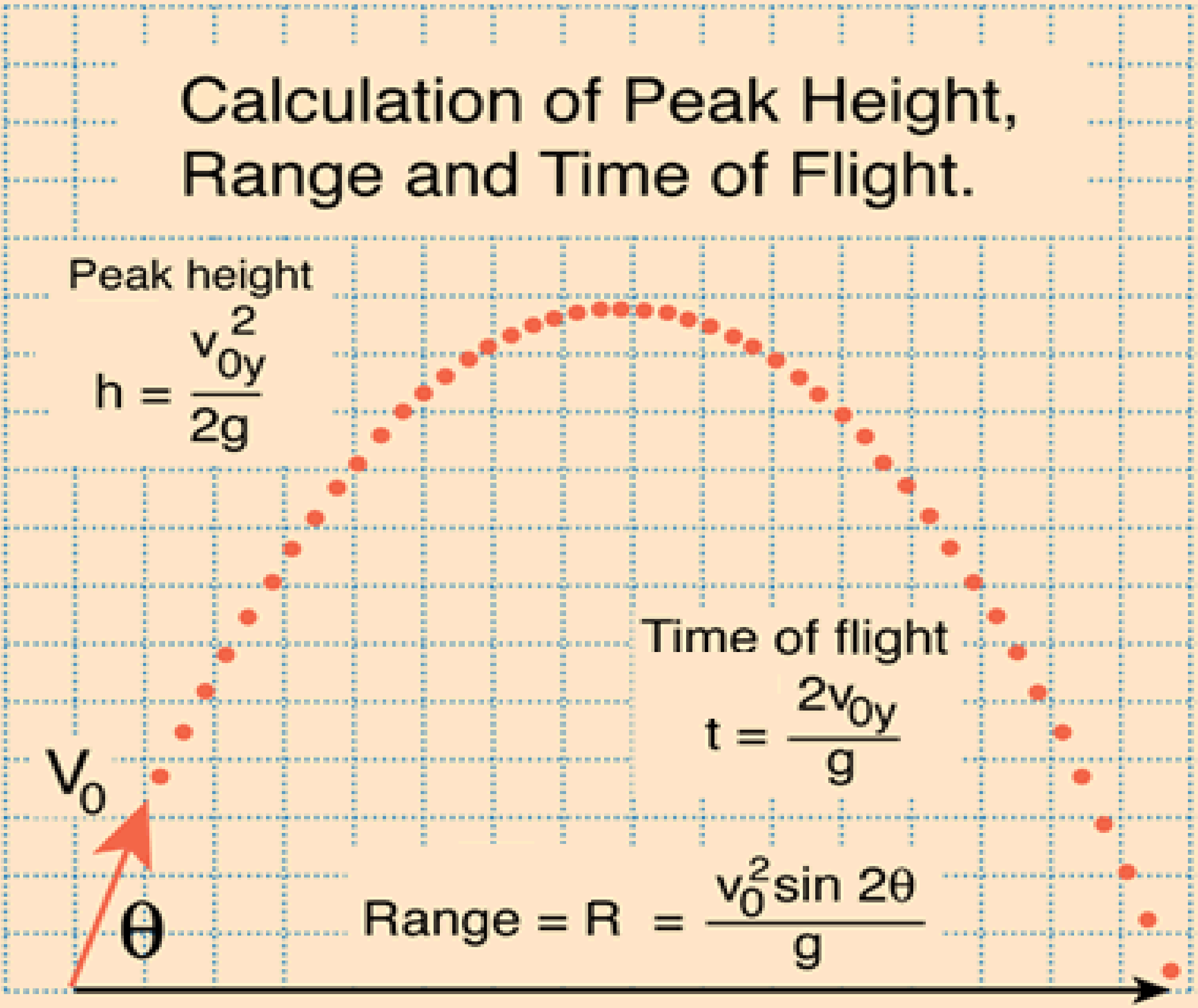
The total time of flight is

t = 1.02040816: s.

The peak height is

h = 1.27551020‹ m.

Example: Peak Height, Range and Time of Flight for v_0=10 m/s and θ=45



For launch velocity

$v_0 =$ 10 m/s,

launch angle

$\theta =$ 45 degrees,

The horizontal range is

$R =$ 10.2040816 m.

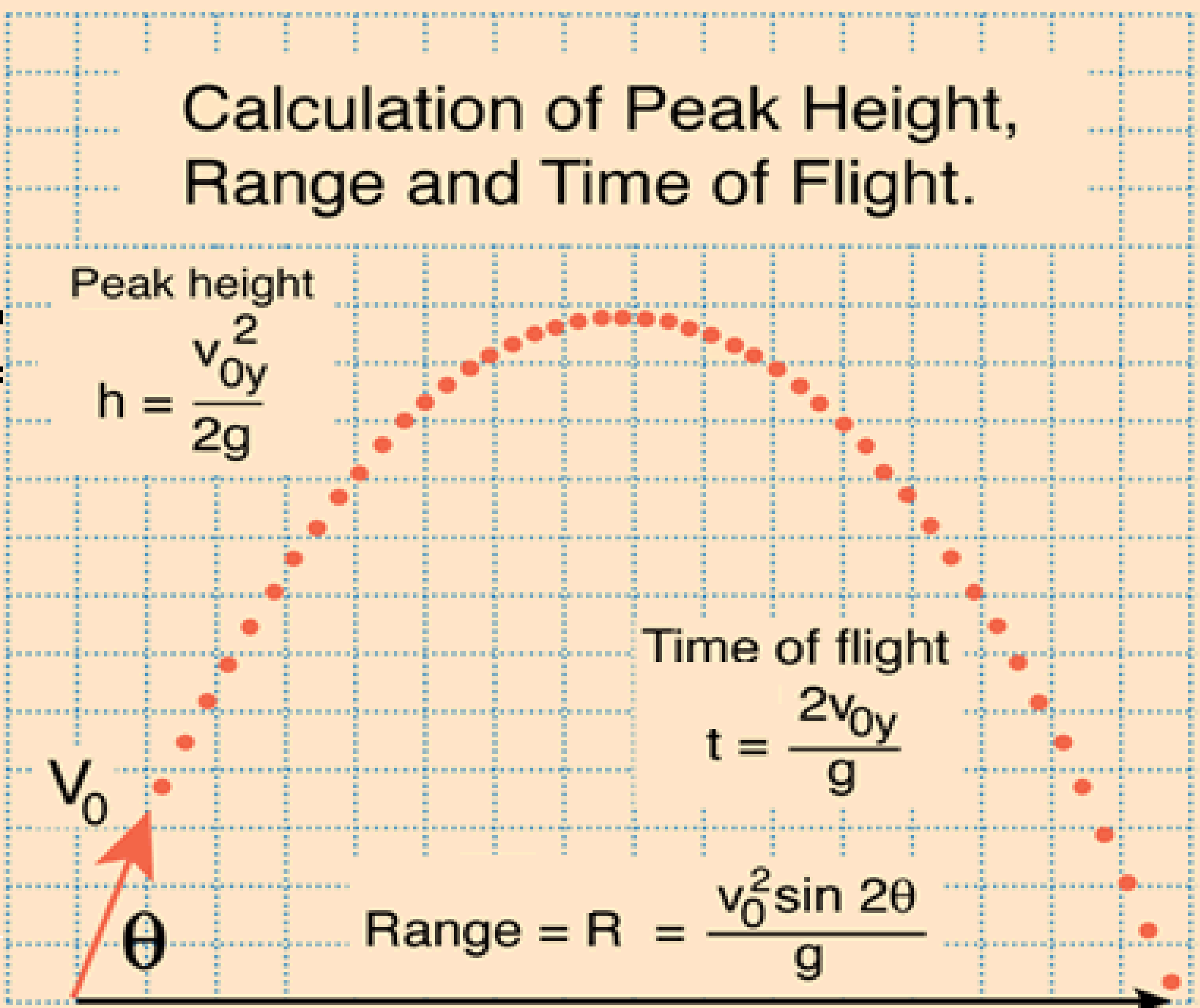
The total time of flight is

$t =$ 1.44307506 s.

The peak height is

$h =$ 2.55102040 m.

Example: Peak Height, Range and Time of Flight for v_0=10 m/s and θ=60



For launch velocity

v_0 = 10 m/s,

launch angle

θ = 60 degrees,

The horizontal range is

R = 8.83699391€ m.

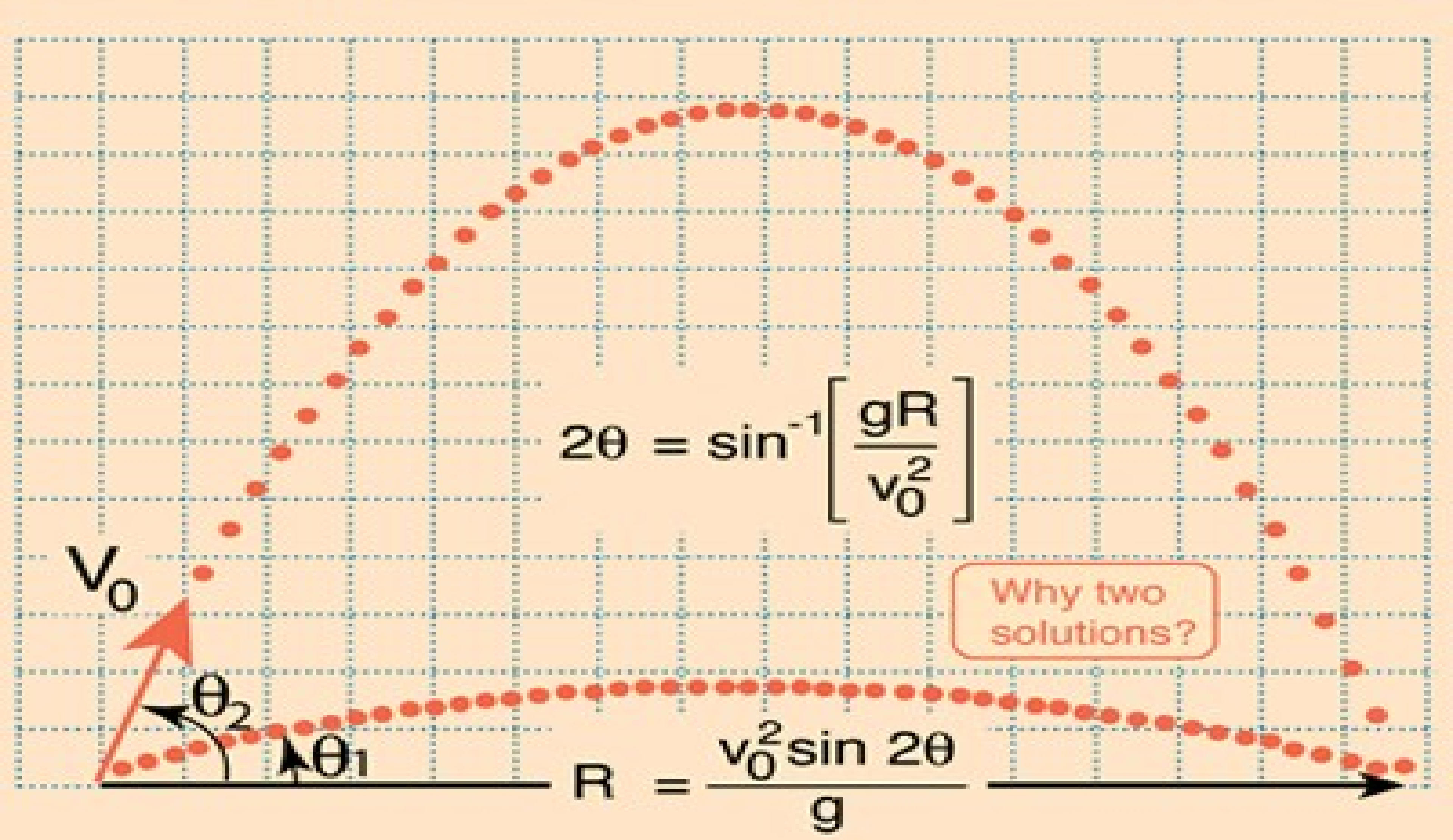
The total time of flight is

t = 1.76739878: s.

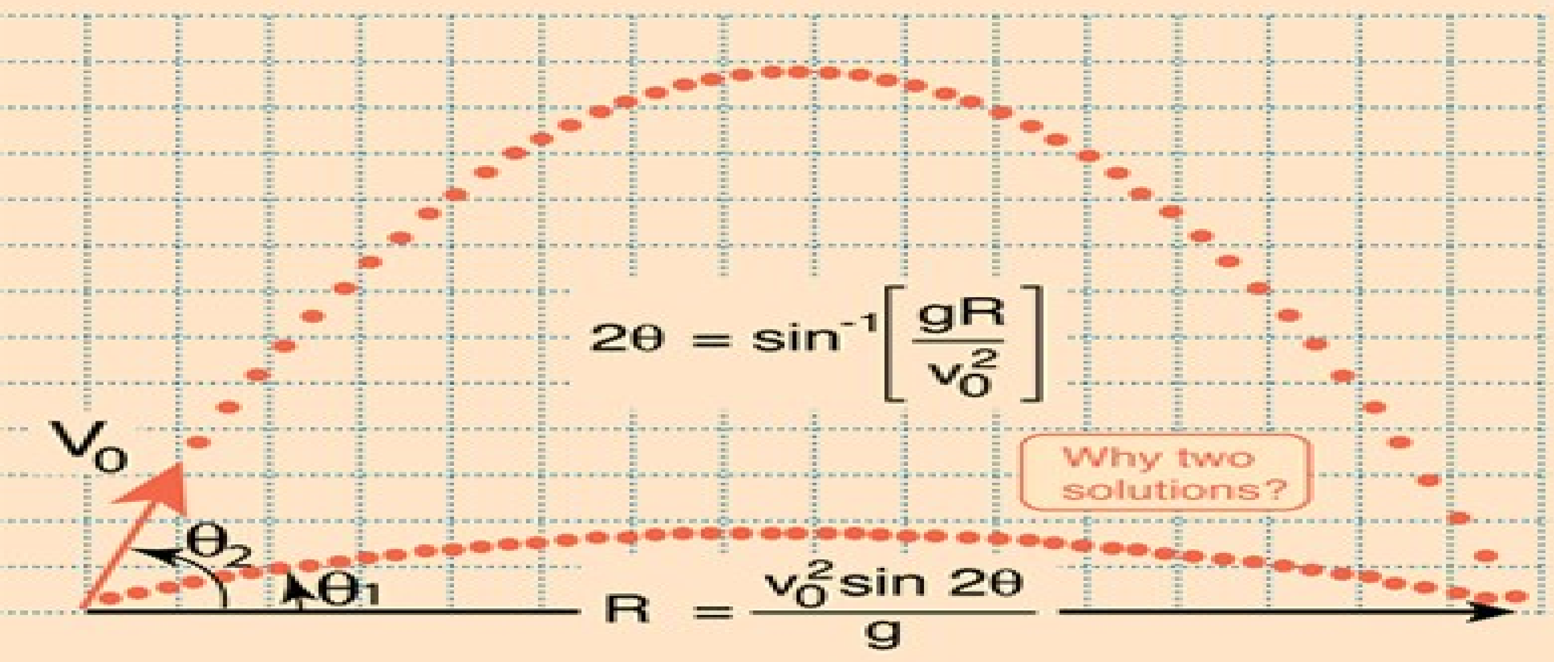
The peak height is

h = 3.82653061: m.

Angle of Launch



Example: Angle of Launch for R=20 m and v_0=20 m/s



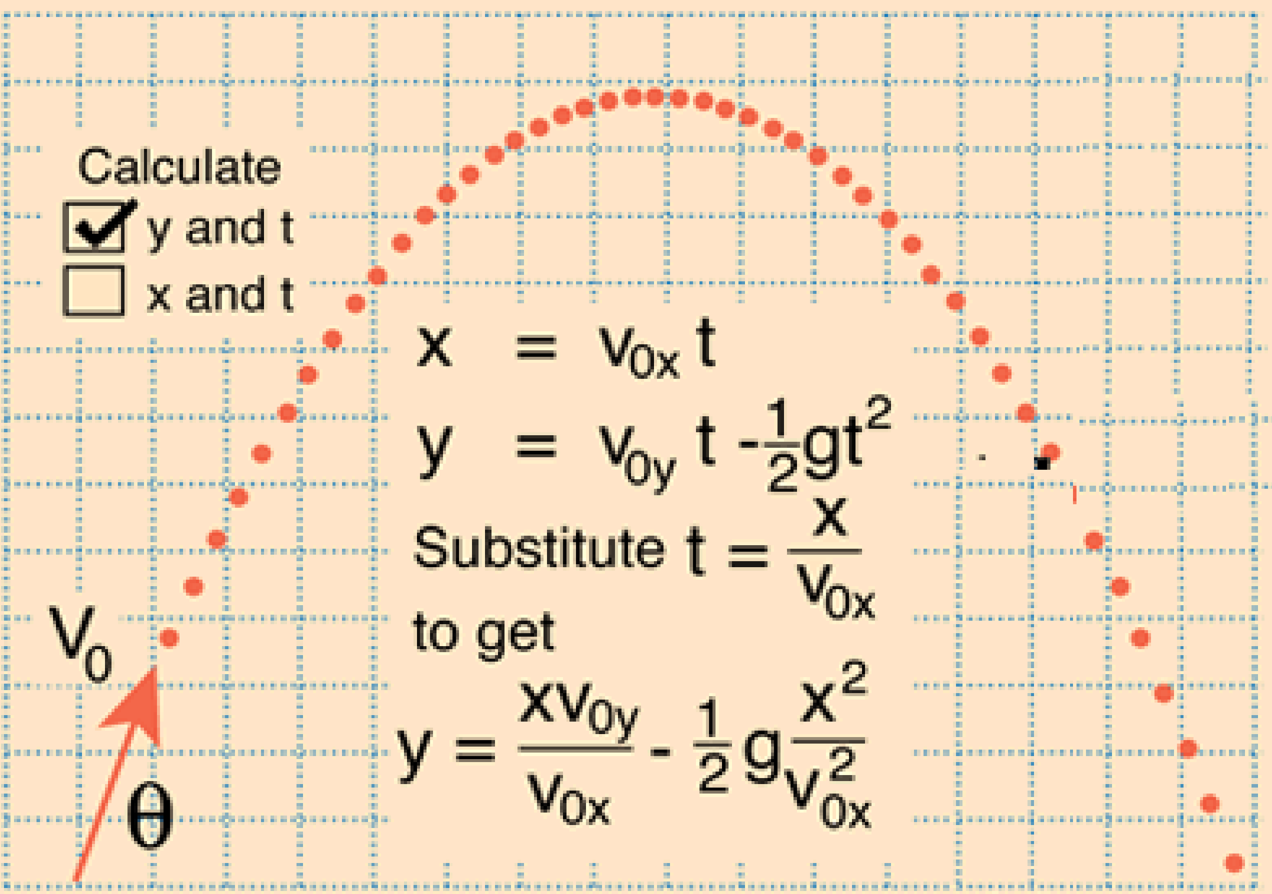
For range R = 20 m = 65.6167979(ft, and launch velocity v_0 = 20 m/s = 65.6167979(ft/s. there are two solutions for the launch angle.

θ_1 = 14.6702907£ degrees,

θ_2 = 75.32970921 degrees.

Peak Height and Time of Flight for v_0=20 m/s x=25 m and θ=30

The basic motion equations can be solved simultaneously to express y in terms of x.



For launch velocity

v_0 = 20 m/s = 65.6167979(ft/s, launch angle

θ = 30 degrees,

and horizontal range

x = 25 m = 82.0209973 ft,

the calculated height is

y = 4.22542339(m = 13.8629376(ft.

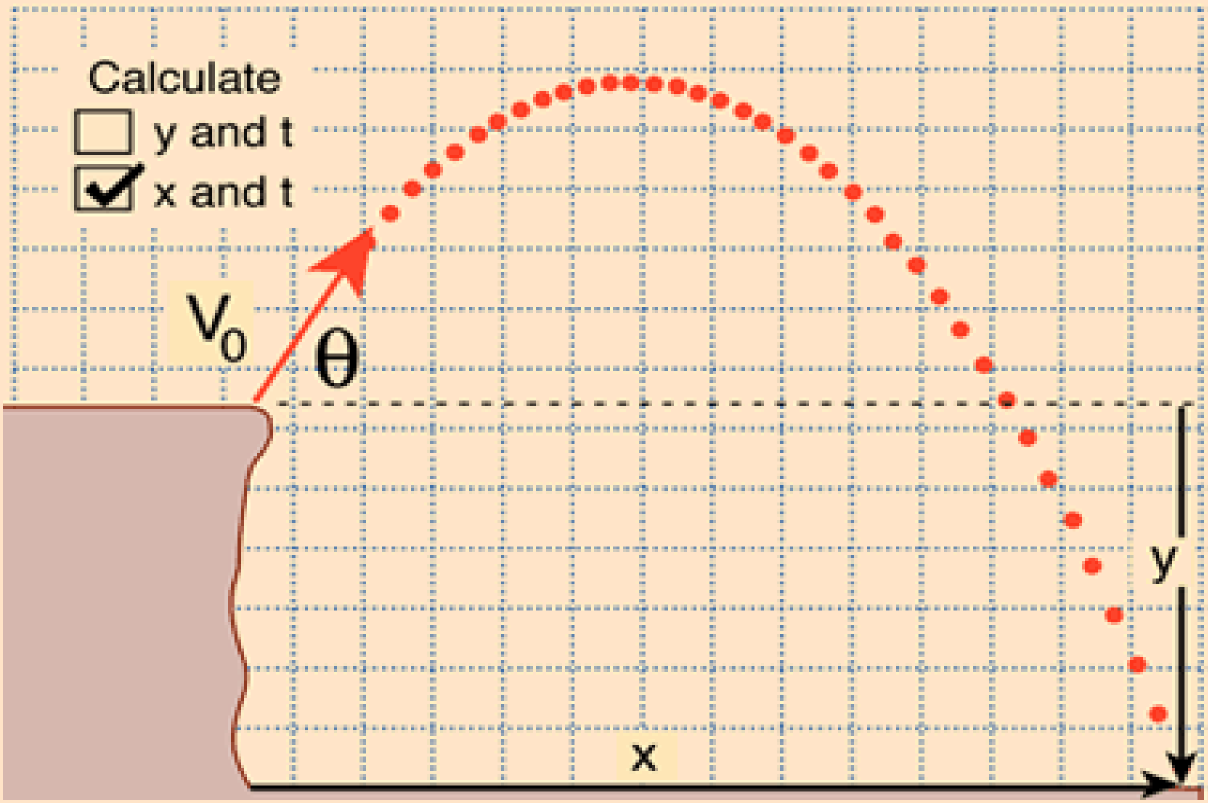
The time of flight is

t = 1.44337567(s.

More on Horizontal Distance

Where will it land?

The basic motion equations give the position components x and y in terms of the time. Solving for the horizontal distance in terms of the height y is useful for calculating ranges in situations where the launch point is not at the same level as the landing point.



$$x = v_{0x}t$$

$$y = v_{0y}t - \frac{1}{2}gt^2$$

Using the **quadratic formula** to solve for t gives two values of time for a given value of y:

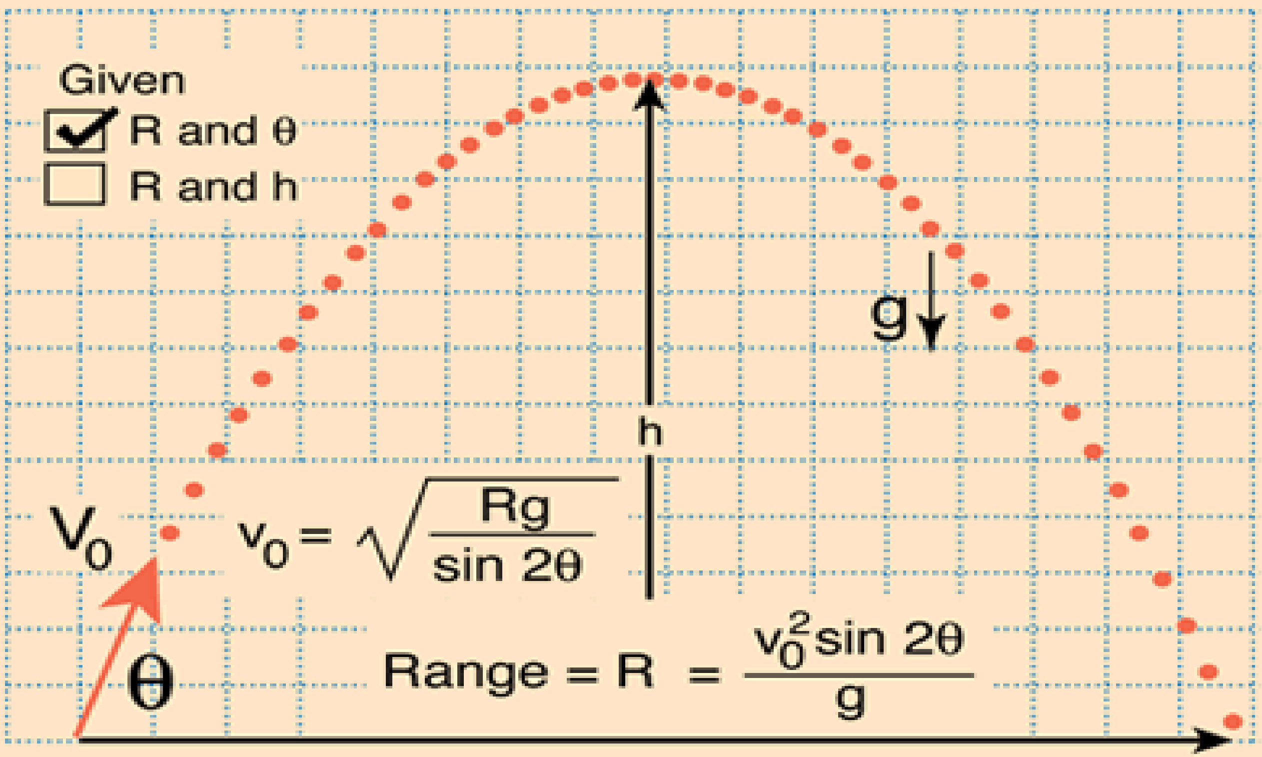
$$t = \frac{v_{0y}}{g} \pm \sqrt{\frac{v_{0y}^2}{g^2} - \frac{2y}{g}}$$

Substitution of the two time values gives the two values of x corresponding to a given height y.

Example: Launch Velocity for R=40 m and θ=30

Launch Velocity

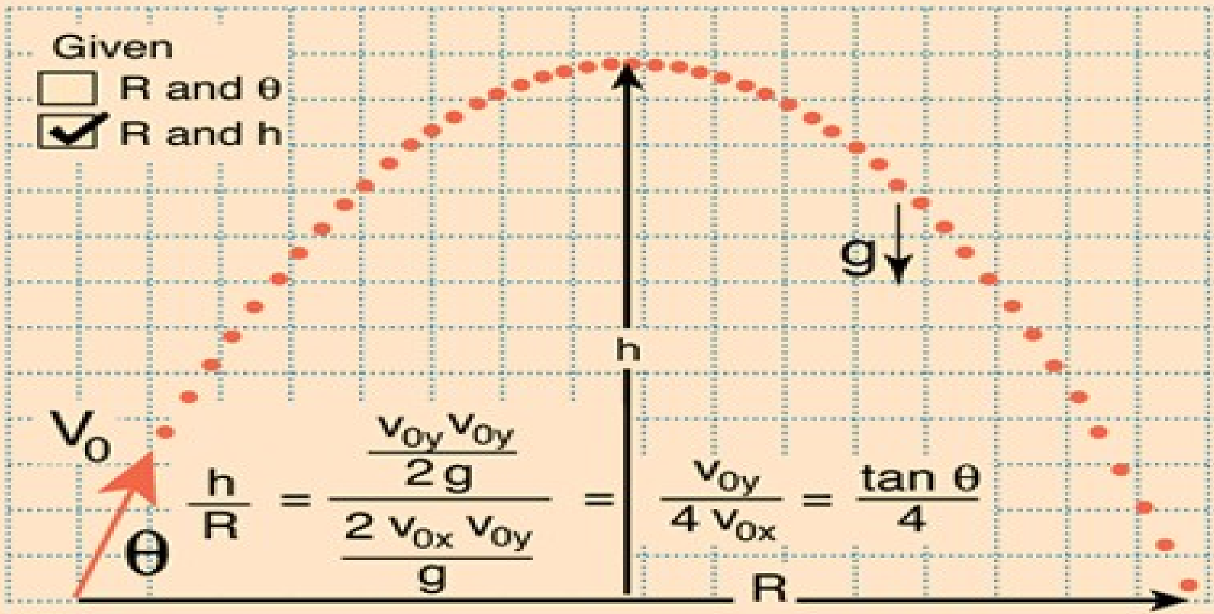
The launch velocity of a projectile can be calculated from the range if the angle of launch is known. It can also be calculated if the maximum height and range are known, because the angle can be determined.



From the range relationship, the launch velocity can be calculated. For range R = 40 m = 131.2335958 ft, and launch angle θ = 30 degrees,

the launch velocity is v_0 = 21.27539919 m/s = 69.80117847 ft/s.

Example: Launch Velocity and Launch Angle for R=40 m and h=20m



From the **range** and **peak** relationships:

$$R = \frac{v_0^2 \sin 2\theta}{g} \text{ and } h = \frac{v_{0y}^2}{2g}$$

the angle of launch can be determined, leading to:

$$v_0 = \sqrt{\frac{Rg}{\sin 2\theta}}$$

For range

R = 40 m = 131.2335958 ft,

and peak height

h = 20 m = 65.6167979(ft,

the launch velocity is

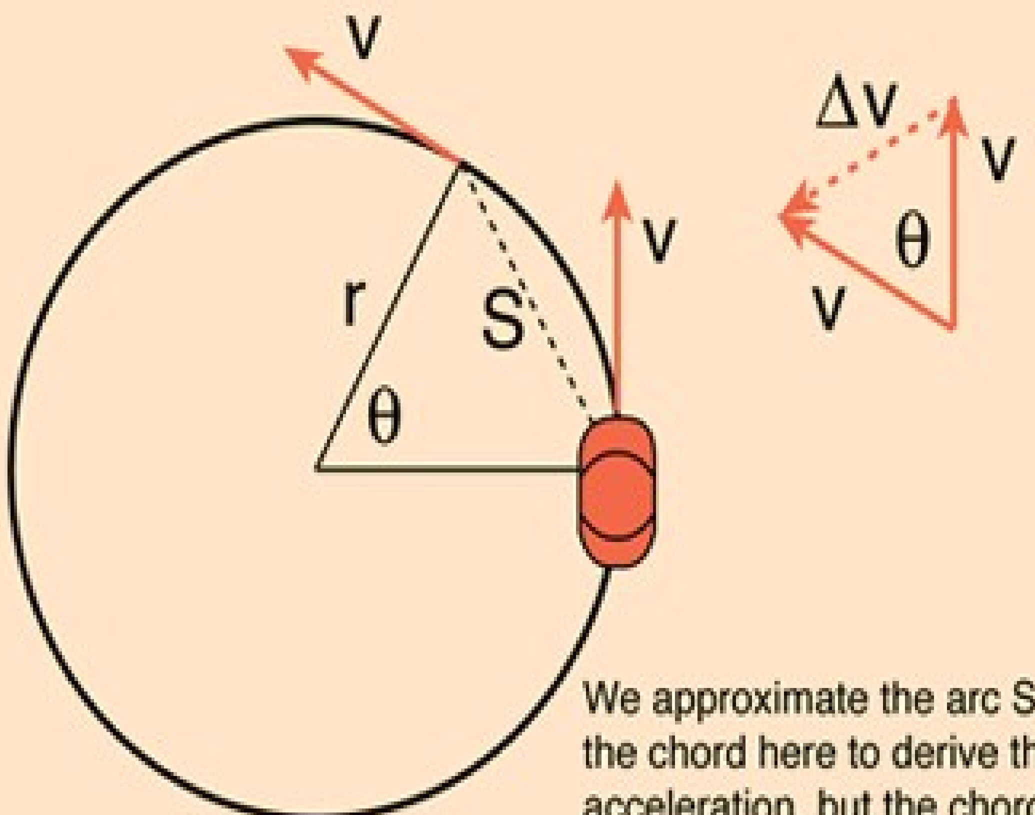
v_0 22.1359436: m/s = 72.6244869‹ ft/s.

The required launch angle is

θ = 63.4349488: degrees.

Circular Motion

For circular motion at a constant speed v, the centripetal acceleration of the motion can be derived. Since in radian measure,



$$\theta = \frac{S}{r} = \frac{v\Delta t}{r}$$

we can draw a similar triangle with the velocities and conclude

$$\theta = \frac{\Delta v}{v} .$$

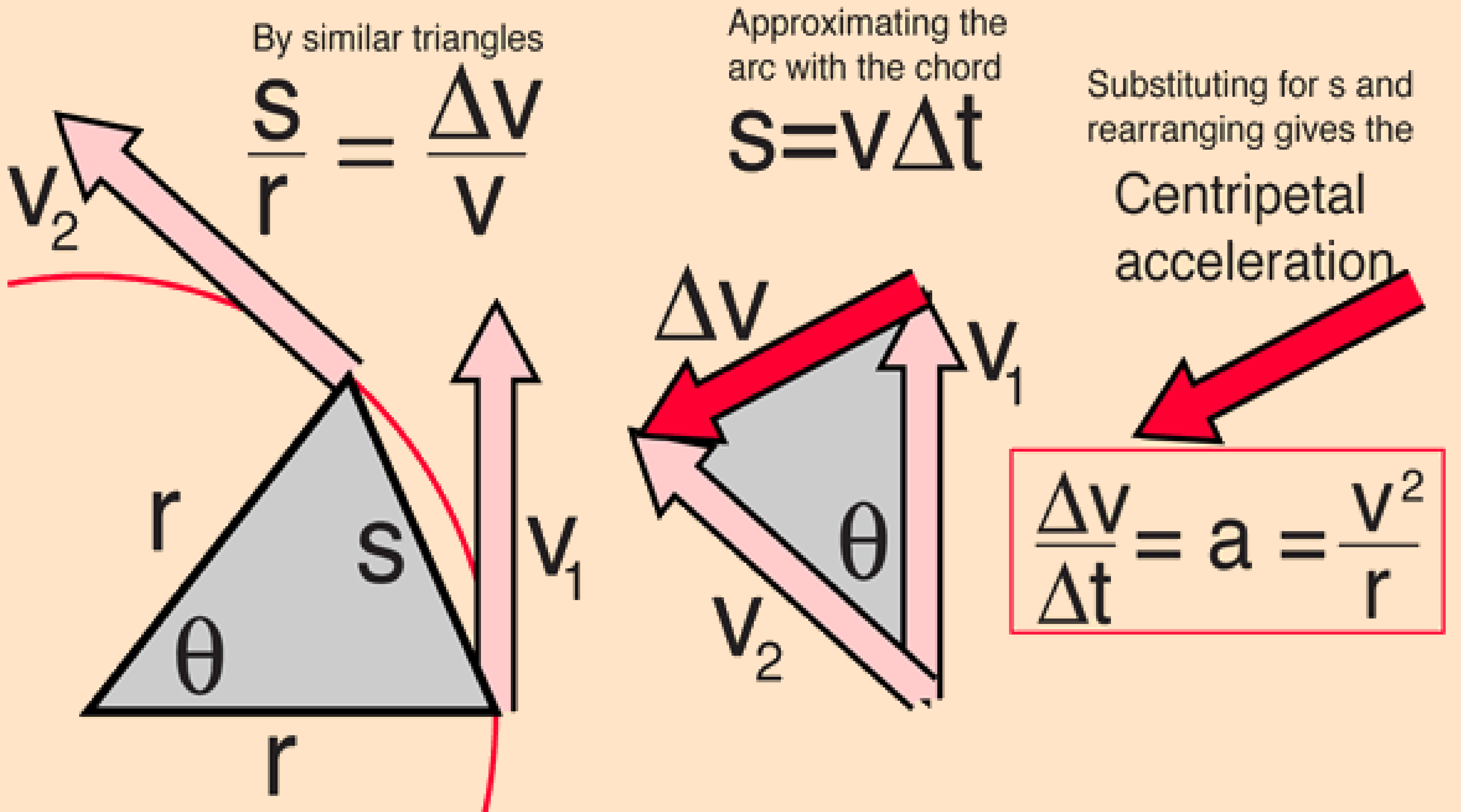
Setting the two expressions for θ equal and solving for the acceleration gives:

$$a_{centripetal} = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

We approximate the arc S by the chord here to derive the acceleration, but the chord approaches the arc for small angles and in the limit, the result we get is exact.

Centripetal Acceleration

The centripetal acceleration expression is obtained from analysis of constant speed circular motion by the use of similar triangles. From the ratio of the sides of the triangles:



Centripetal Acceleration

$$\frac{\Delta v}{\Delta t} = a = \frac{v^2}{r}$$

For a velocity of [5] m/s and radius [2] m, the centripetal acceleration is [12.5] m/s^2.

Note that if the velocity is doubled to [10] m/s at the same radius, the acceleration is **quadrupled** to [50] m/s^2.

Centripetal Acceleration

$$\frac{\Delta v}{\Delta t} = a = \frac{v^2}{r}$$

For a velocity of 1 m/s and radius 2 m, the centripetal acceleration is 0.5 m/s^2.

Note that if the velocity is doubled to 2 m/s at the same radius, the acceleration is **quadrupled** to 2 m/s^2.

Centripetal Force

Any motion in a curved path represents accelerated motion, and requires a force directed toward the center of curvature of the path. This force is called the centripetal force which means "center seeking" force. The force has the magnitude

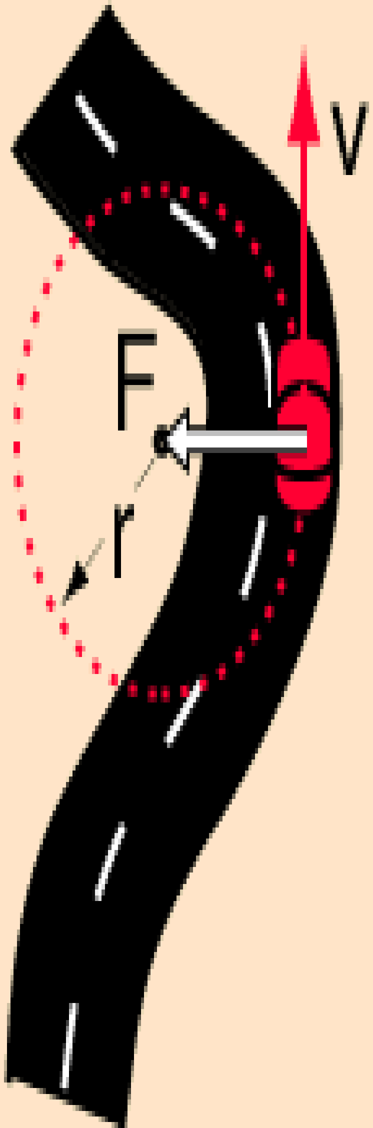
$$F_{centripetal} = m\frac{v^2}{r}$$

Swinging a mass on a string requires string tension, and the mass will travel off in a tangential straight line if the string breaks.

The centripetal acceleration can be derived for the case of circular motion since the curved path at any point can be extended to a circle.

$$F_{centripetal} = m\frac{v^2}{r}$$

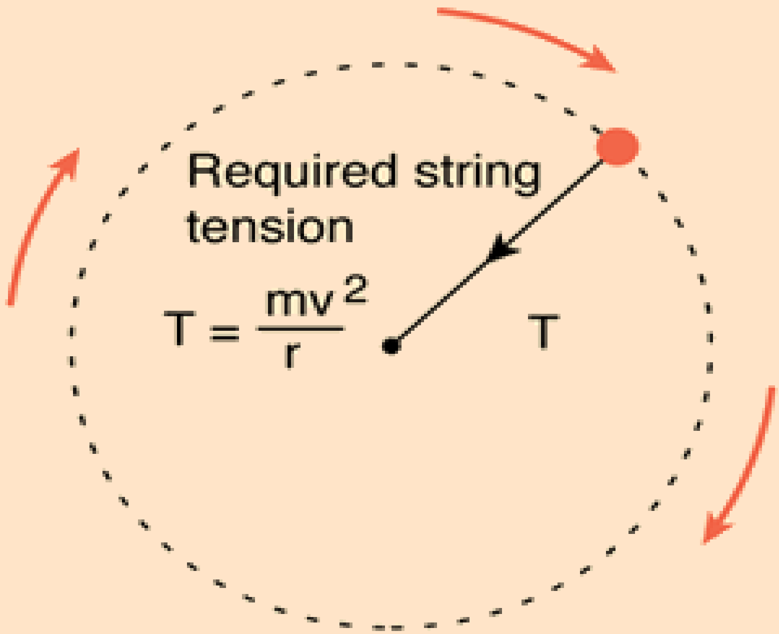
$\frac{v^2}{r}$ is the centripetal acceleration



Note that the centripetal force is proportional to the square of the velocity, implying that a doubling of speed will require **four times** the centripetal force to keep the motion in a circle. If the centripetal force must be provided by friction alone on a curve, an increase in speed could lead to an unexpected skid if friction is insufficient.

Centripetal Force Calculation

Centripetal force = mass x velocity² / radius



Any of the data values may be changed. When finished with data entry, click on the quantity you wish to calculate in the formula above. Unit conversions will be carried out as you enter data, but values will not be forced to be consistent until you click on the desired quantity.

Calculation for:

Radius **r** = 1 m = 3.2808398 ft

Mass = **m**= 2 kg = 0.1370503 slugs

Weight = **W**= 19.6 N = 4.4064748 lbs

Velocity = **v**= 5 m/s = 16.404199 ft/s

or in common highway speed units,

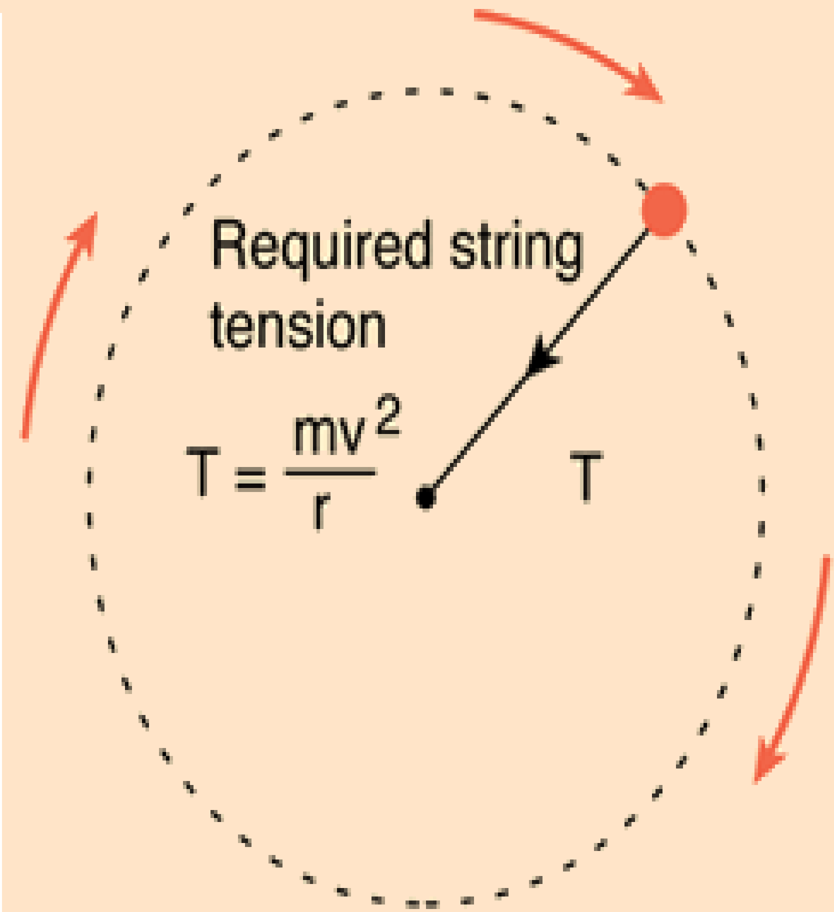
velocity = 18 km/h = 11.184681 mi/h

Centripetal force= **F**= 50 N = 11.241007 lbs

Note that the conditions here assume no additional forces, like a horizontal circle on a frictionless surface. For a vertical circle, the speed and tension must vary.

Centripetal Force Calculation

Centripetal force = mass x velocity^2 / radius



Calculation for:

Radius **r** = 10 m = 32.808398 ft

Mass = **m** = 2 kg = 0.1370503 slugs

Weight = **W** = 19.6 N = 4.4064748 lbs

Velocity = **v** = 20 m/s = 65.616797 ft/s

or in common highway speed units,

velocity = 72 km/h = 44.738725 mi/h

Centripetal force = **F** = 80 N = 17.985611 lbs

Relative Motion

The laws of physics which apply when you are at rest on the earth also apply when you are in any reference frame which is moving at a constant velocity with respect to the earth. For example, you can toss and catch a ball in a moving bus if the motion is in a straight line at constant speed.

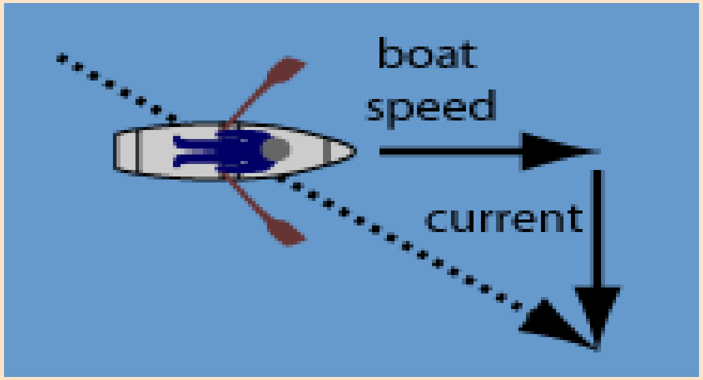
The motion may have a different appearance as viewed from a different reference frame, but this can be explained by including the relative velocity of the reference frame in the description of the motion.

Relative Velocity

One must take into account relative velocities to describe the motion of an airplane in the wind or a boat in a current. Assessing velocities involves vector addition and a useful approach to such relative velocity problems is to think of one reference frame as an "intermediate" reference frame in the form:

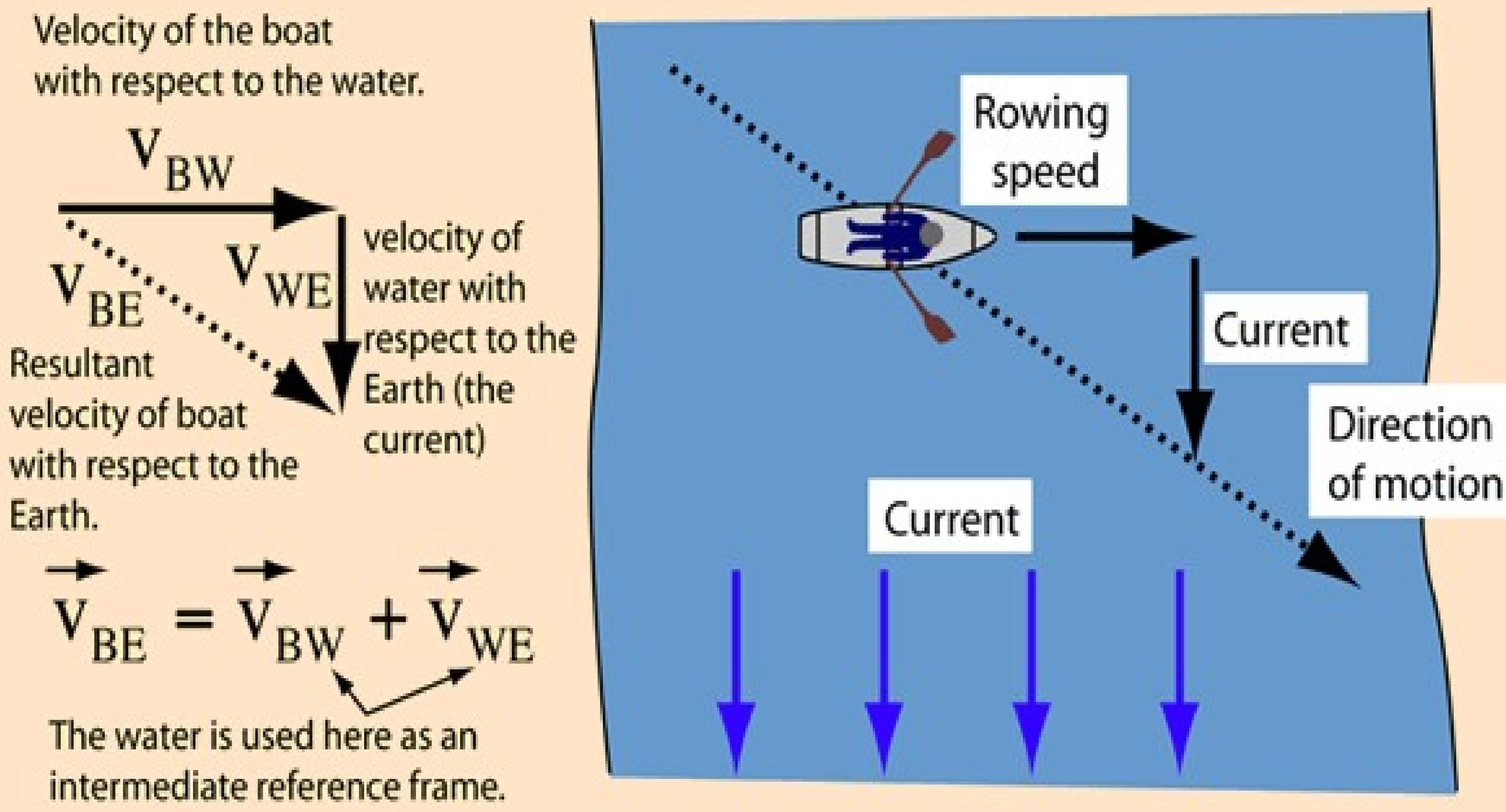
$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$

Put into words, the velocity of A with respect to C is equal to the velocity of A with respect to B plus the velocity of B with respect to C. Reference frame B is the intermediate reference frame. This approach can be used with the boat examples



Boat in Current

A boat in current is a good example of relative velocity.



Boat in Current: Resultant Speed and Bearing

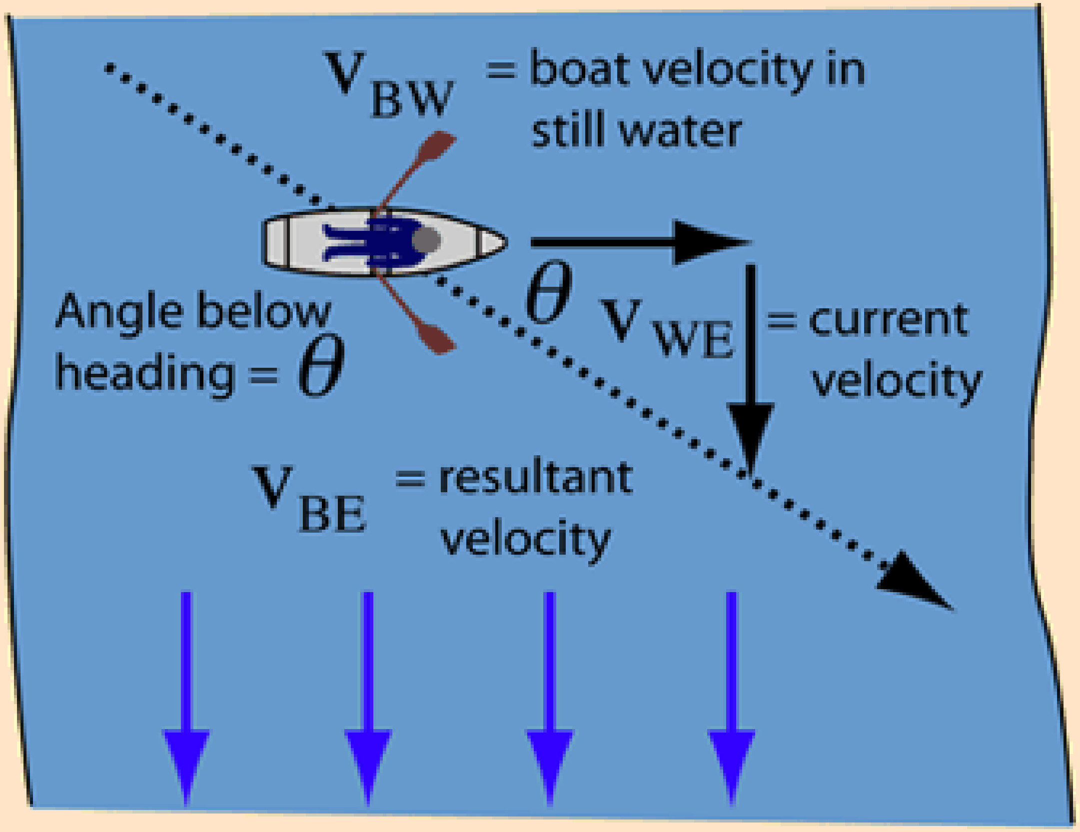
Boat in current as example of relative velocity. Assume the rower heads straight across the river and is carried downstream by the current.

$$\vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE}$$

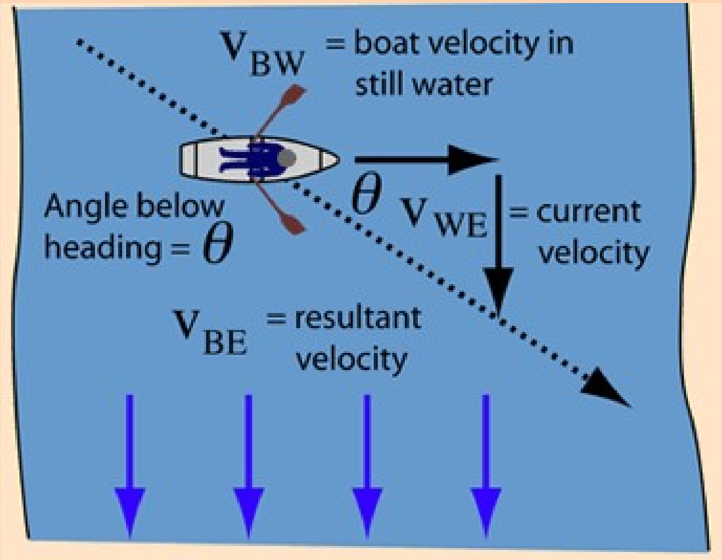
Since the velocities form a right triangle, any velocity can be found from the triangle relationships if the other two are known.

$$v_{BE} = \sqrt{v_{BW}^2 + v_{WE}^2}$$

$$\theta = \tan^{-1}\frac{v_{WE}}{v_{BW}}$$



Resultant velocity?



Velocity unit: m/s

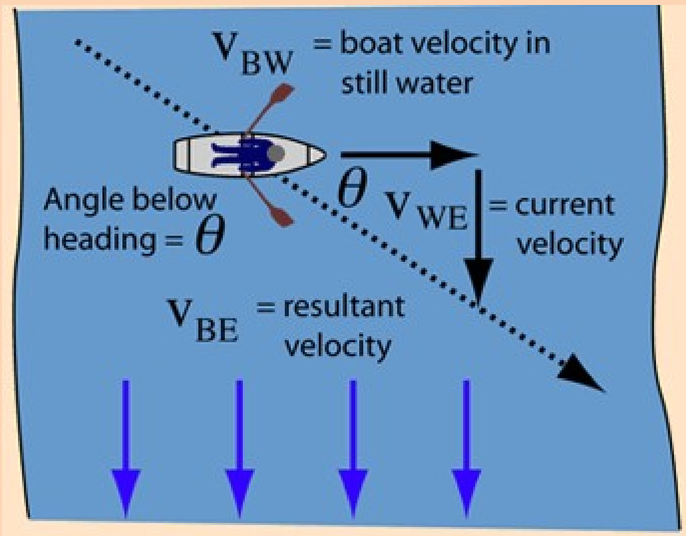
If the rowing speed is $\vec{V}_{BW}$ = 12 m/s

and the current speed is $\vec{V}_{WE}$ = 5 m/s

then the resultant speed is $\vec{V}_{BE}$ = 13 m/s

at an angle θ = 22.619864° downstream

Resultant velocity?



Velocity unit: m/s

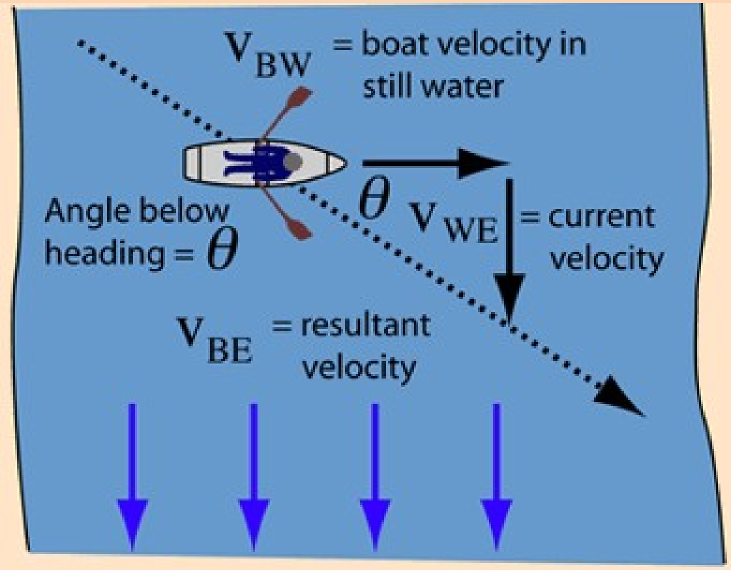
If the rowing speed is $\vec{V}_{BW}$ = 5 m/s

and the current speed is $\vec{V}_{WE}$ = 5 m/s

then the resultant speed is $\vec{V}_{BE}$ = 7.0710678 m/s

at an angle θ = 45 ° downstream

Boat velocity ?



Velocity unit m/s

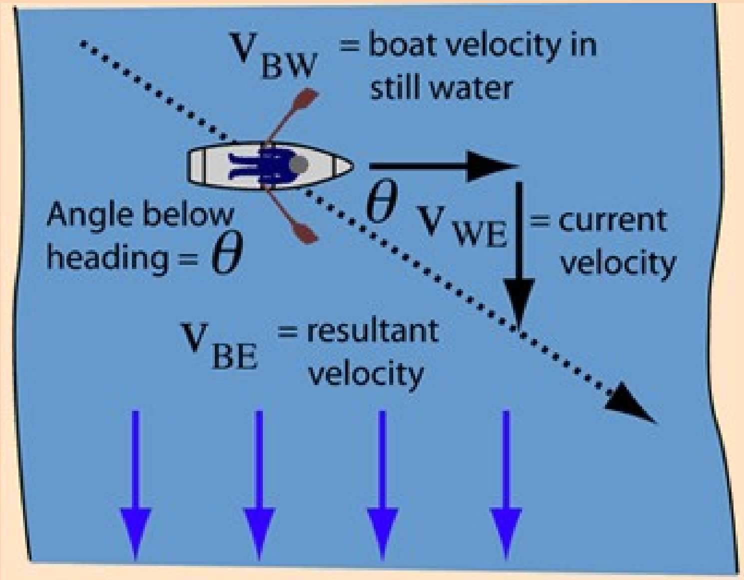
If the current speed is known to be $\vec{V}_{WE}$ = 3 m/s

and the resultant boat speed is measured to be $\vec{V}_{BE}$ = 7 m/s,

then the speed of the boat is $\vec{V}_{BW}$ = 6.3245553 m/s

and it will be angled downstream at θ = 25.376933°

Current velocity?



Velocity unit m/s

If the rowing speed is known to be $\vec{V}_{BW}$ = 10 m/s

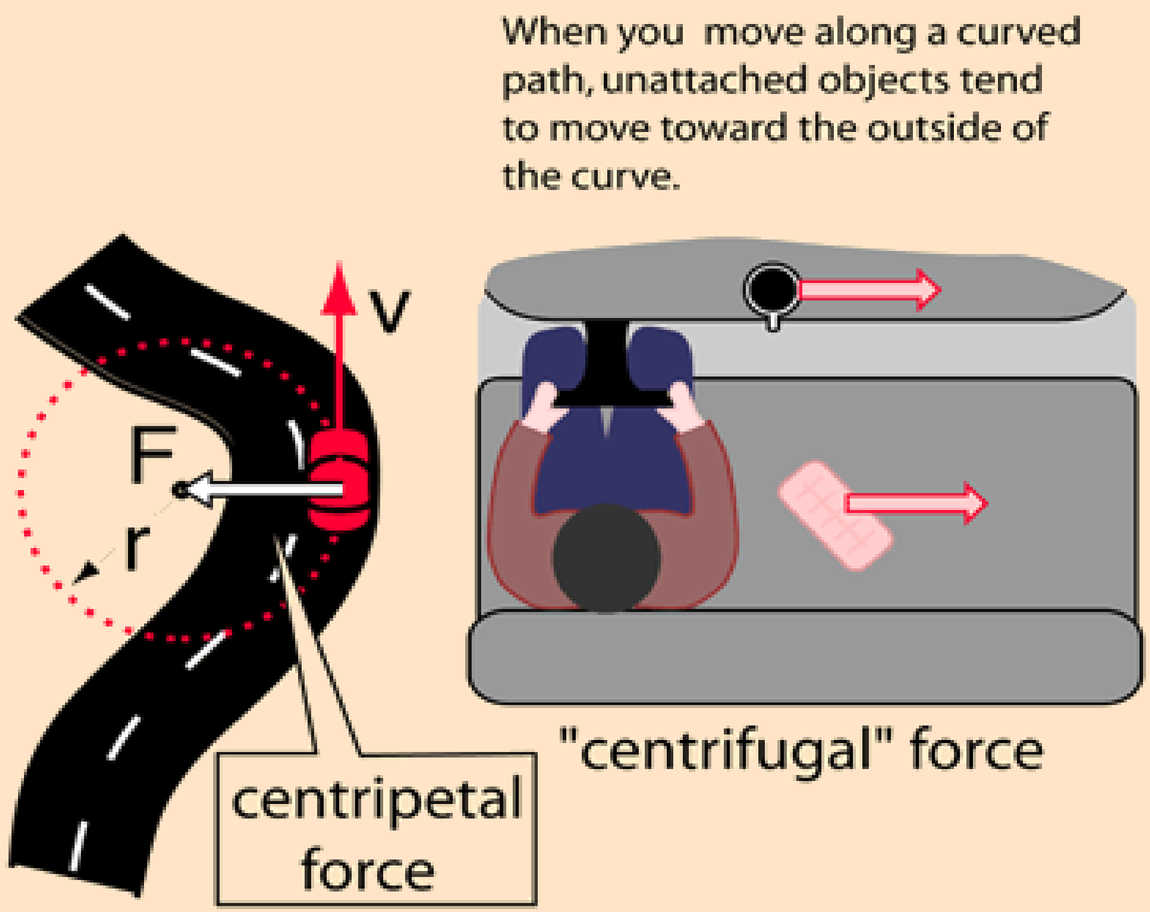
and the resultant boat speed is measured to be $\vec{V}_{BE}$ = 12 m/s,

Then the current speed is $\vec{V}_{WE}$ = 6.6332495 m/s

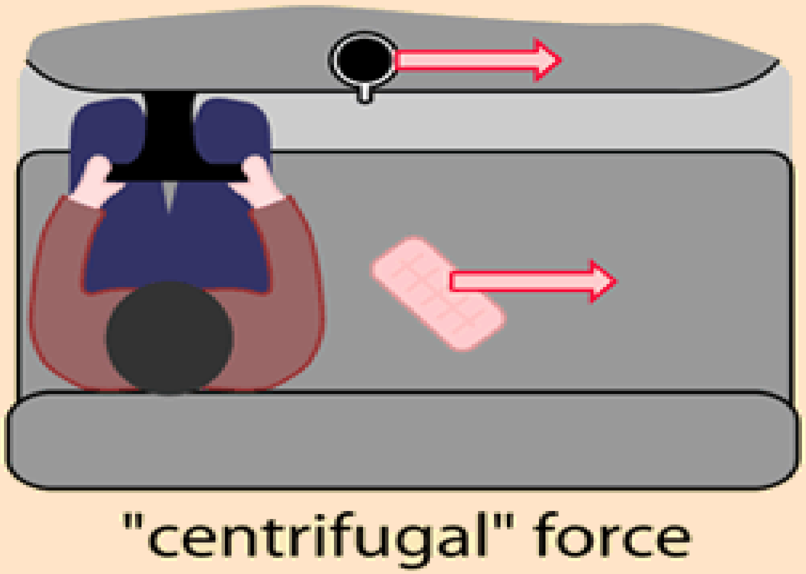
and the boat will be angled downstream at θ = 33.557309°

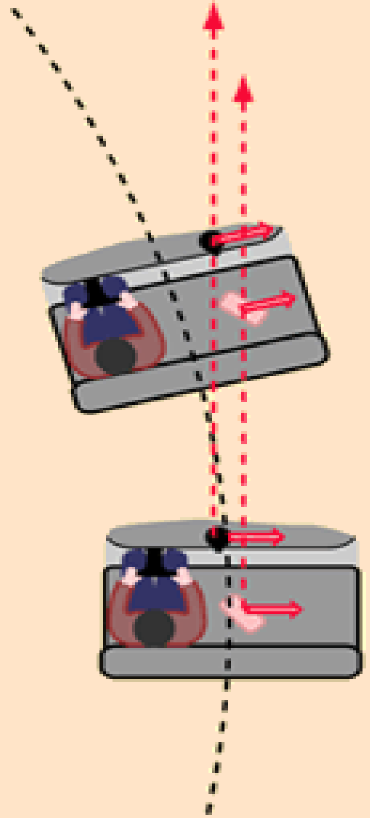
Centrifugal Force

Whereas the centripetal force is seen as a force which must be applied by an external agent to force an object to move in a curved path, the centrifugal and coriolis forces are "effective forces" which are invoked to explain the behavior of objects from a frame of reference which is rotating.



The driver of a car on a curve is in a rotating reference frame and he could invoke a "centrifugal" force to explain why his coffee cup and the carton of eggs he has on the seat beside him tend to slide sideways. The friction of the seat or dashboard may not be sufficient to accelerate these objects in the curved path.

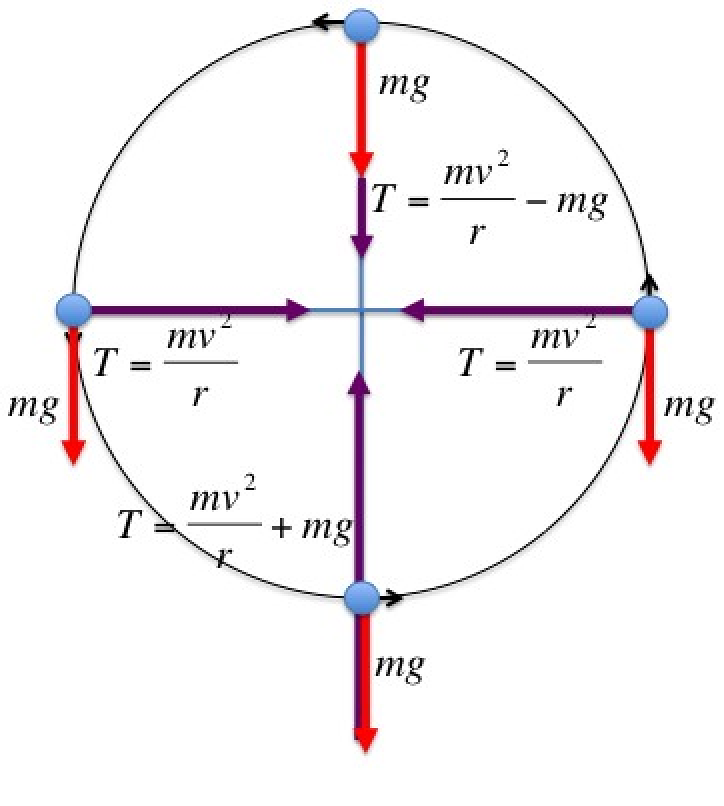


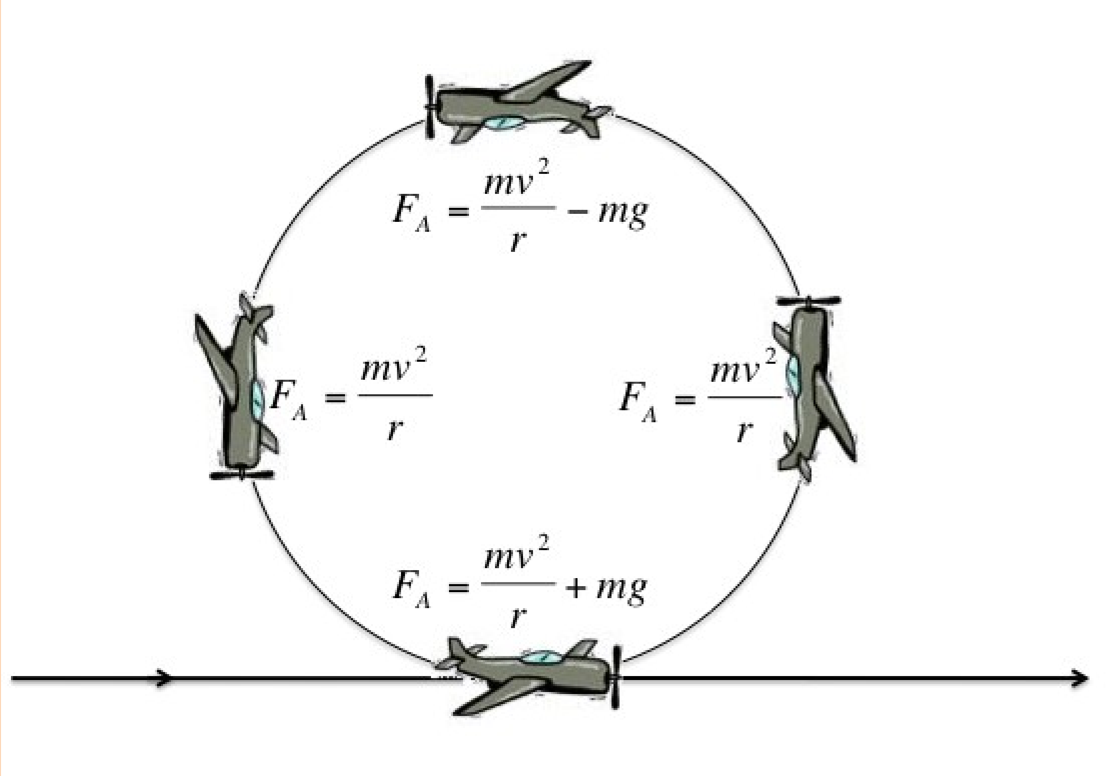


A person in a hovering helicopter above the car could describe the movement of the cup and the egg carton as just going straight while the car travels in a curved path. It is similar to the "broken string" example.

The centrifugal force is a useful concept when the most convenient reference frame is one which is moving in a curved path, and therefore experiencing a centripetal acceleration. Since the car above will be experiencing a centripetal acceleration v^2/r, then an object of mass m on the seat will require a force mv^2/r toward the center of the circle to stay at the same spot on the seat. From the reference frame of a person in the car, there seems to be an outward centrifugal force mv^2/r acting to move the mass radially outward. In practical descriptive terms, you would say that your carton of eggs is more likely to slide outward if you have a higher speed around the curve (the velocity squared factor) and more likely to slide outward if you go around a sharper curve (the inverse dependence upon r).

Motion in a Vertical Circle





Motion in a Vertical Circle

Consider a mass m performing circular motion under gravity, the circle with radius r .
The centripetal force on the mass varies at different positions on the circle.

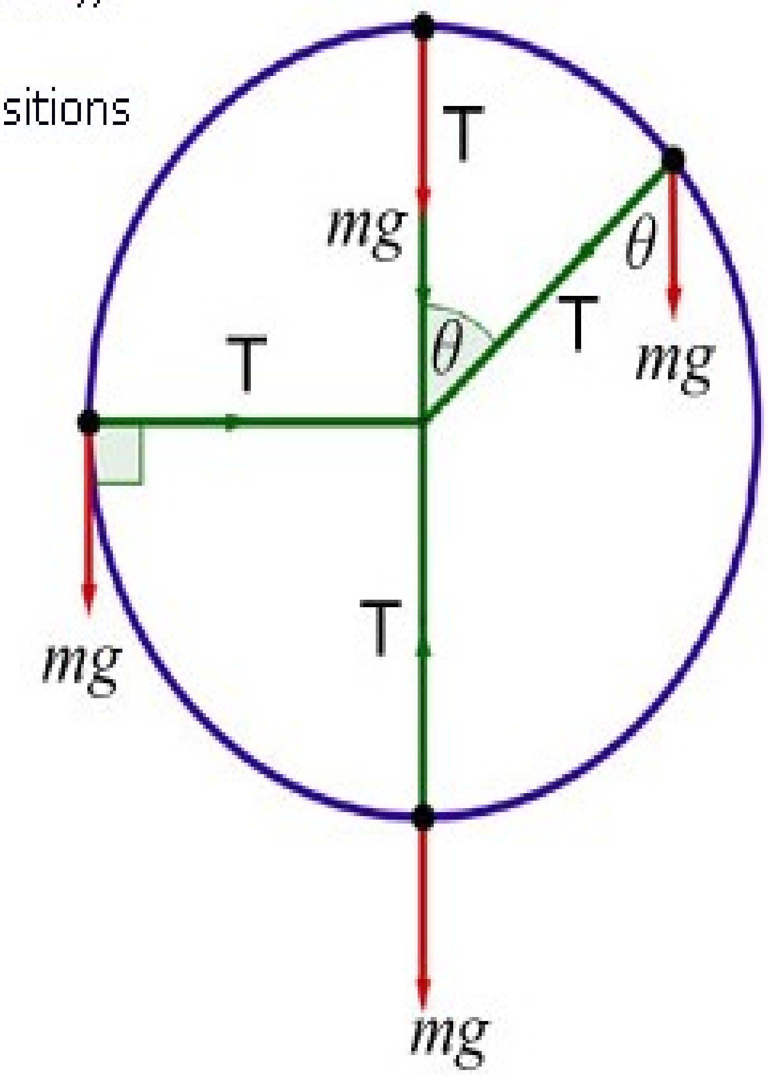
top $\quad mg + T = \dfrac{mv^2}{r}$

middle $\quad T = \dfrac{mv^2}{r}$

bottom $\quad T - mg = \dfrac{mv^2}{r}$

string at an angle θ to the vertical

$$mg\cos\theta + T = \frac{mv^2}{r}$$



Example

<u>Example</u>

A 50g mass suspended at the end of a light inextensible string performs vertical motion of radius 2m.
if the mass has a speed of 5 ms^{-1} when the string makes an angle of 30º with the vertical, what is the tension?
(assume g =10 ms^{-2} , answer to 1 d.p.)

$m = 50\text{g} \equiv 0.05\text{kg} \quad v = 5\text{ms}^{-1} \quad \theta = 30^\circ \quad r = 2\text{m}$

$g = 10\text{ms}^{-2}$

the centripetal force is the sum of the tension in the string and the component of the weight along the string

$$\Rightarrow \quad mg\cos\theta + T = \frac{mv^2}{r}$$

$$\Rightarrow \quad T = \frac{mv^2}{r} - mg\cos\theta$$

$$= \frac{(0.05)(5)^2}{2} - (0.05)(10)\cos 30^\circ$$

$$= 0.625 - 0.433 = 0.192$$

<u>Ans. tension in string is 0.2N</u>

Example

<u>Example</u>

A 5kg mass performs circular motion at the end of a light inextensible string of length 3m.
If the speed of the mass is 2 ms^{-1} when the string is horizontal, what is its speed at the bottom of the circle? (assume g =10 ms^{-2})

$v_H = 2\text{ms}^{-1}$ $\quad r = 3\text{m}$ $\quad g = 10\text{ms}^{-1}$

v_B speed at bottom of circle

PE is measured relative to the bottom of the circle

KE + PE string horizontal = KE + PE at bottom

$$\frac{1}{2}mv_H^2 + mgr = \frac{1}{2}mv_B^2 + 0$$

$$v_H^2 + 2gr = v_B^2$$

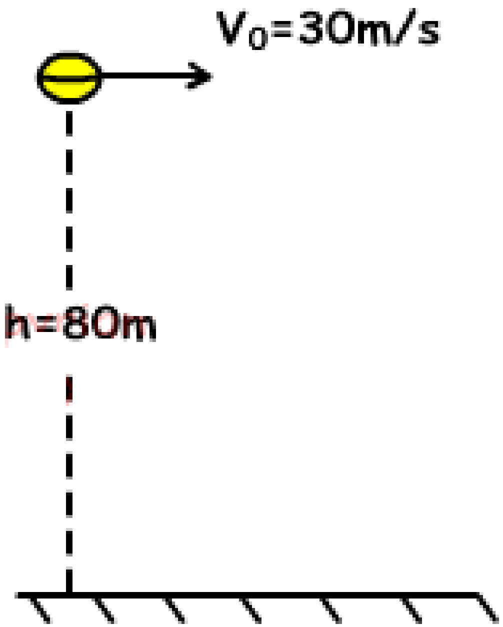
$$v_B = \sqrt{v_H^2 + 2gr}$$

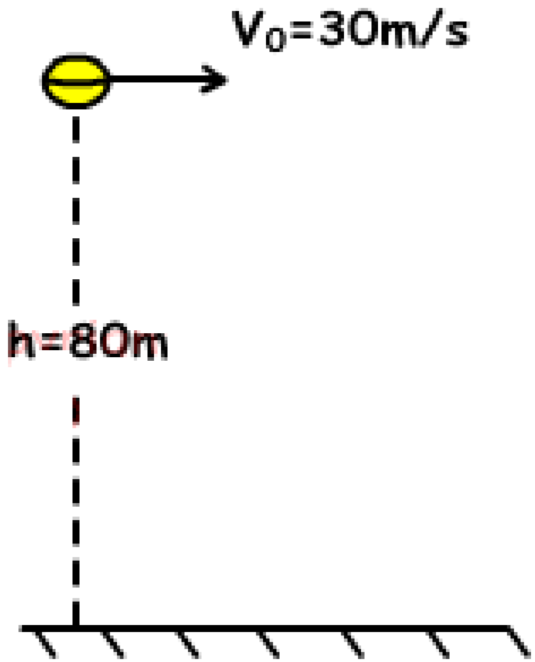
$$= \sqrt{(2)^2 + 2 \times (10) \times (3)}$$

$$= \sqrt{4 + 60} = \sqrt{64} = 8$$

<u>Ans. speed at bottom of circle is 8 ms^{-1}</u>

As you can see from the given picture, ball is thrown horizontally with an initial velocity. Find the time of motion. ($g=10m/s^2$)





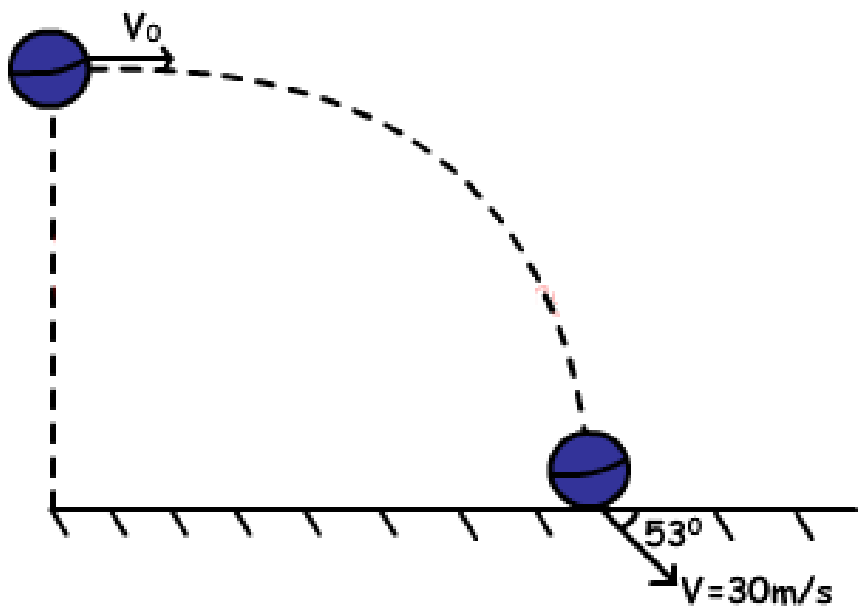
Ball does projectile motion in other words it does free fall in vertical and linear motion in horizontal. Time of motion for horizontal and vertical is same. Thus in vertical;

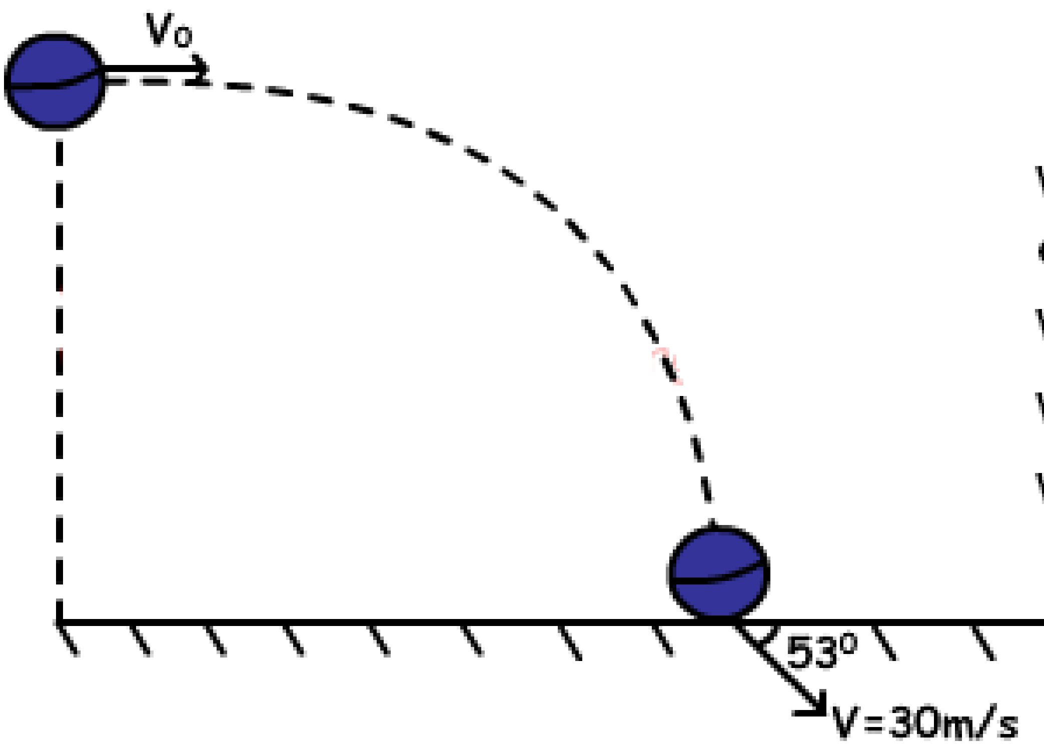
$$h=1/2g.t^2$$

$$80=1/2.10.t^2$$

$$t=4s$$

An object hits the ground as given in the picture below. Find the initial velocity of the object.





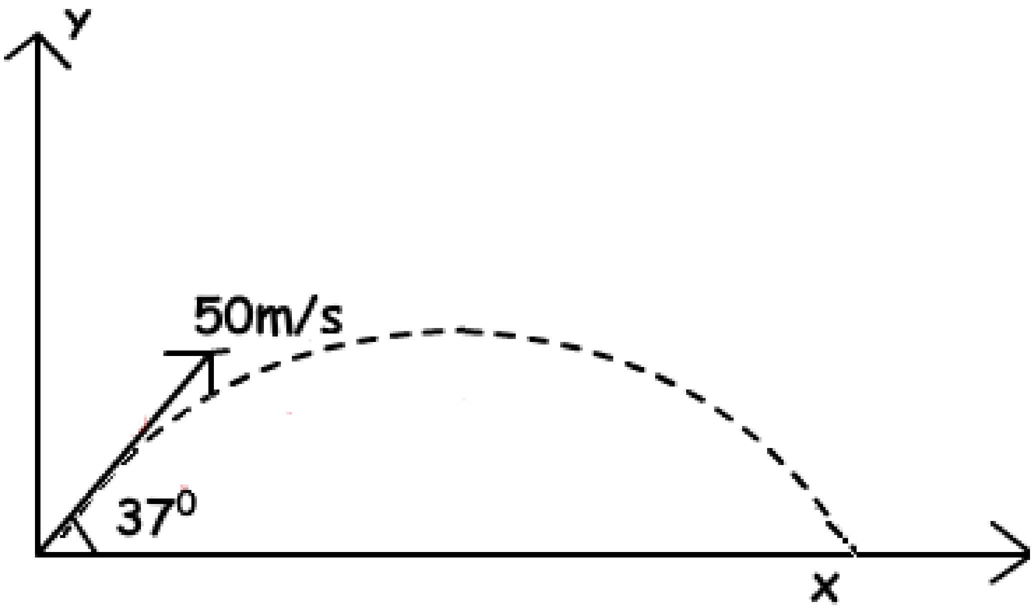
Velocity of horizontal motion is constant. So;

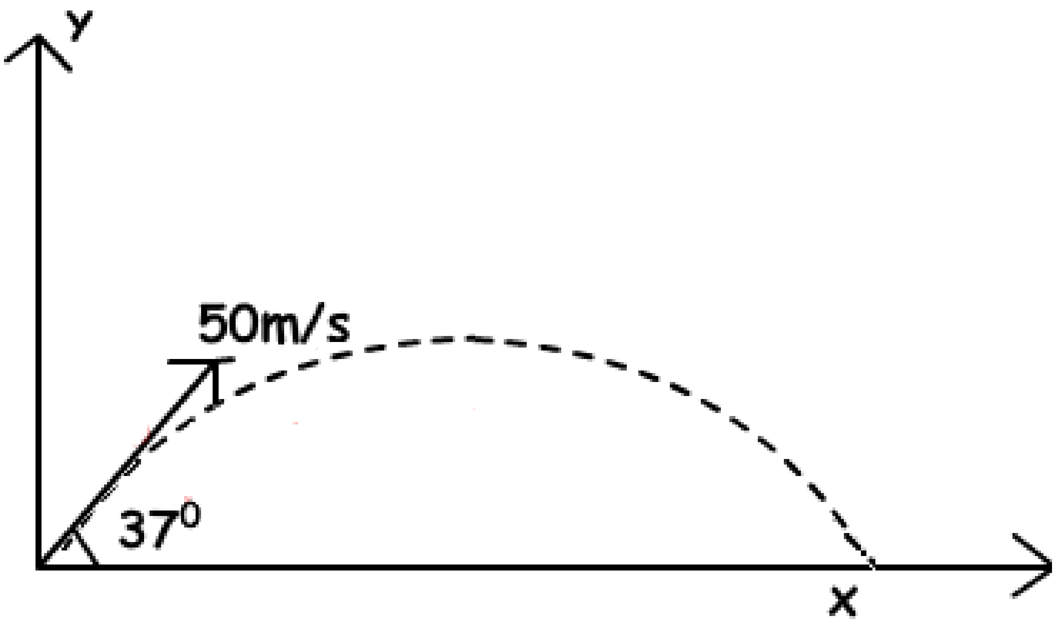
$V_0 = V_x = V\cos 53^0$

$V_x = V_0 = 30 m/s . 0,6$

$V_0 = V_x = 18 m/s$

An object is thrown with an angle 37^0 with horizontal. If the initial velocity of the object is 50m/s, find the time of motion, maximum height it can reach, and distance in horizontal.





$V_{0x}=V_0 cos53^0=50.0,8=40m/s$

$V_{0y}=V_{0y}.sin53^0=50.0,6=30m/s$

a) $V-V_{0y}=0-g.t$ at the maximum height

t=30/10=3s

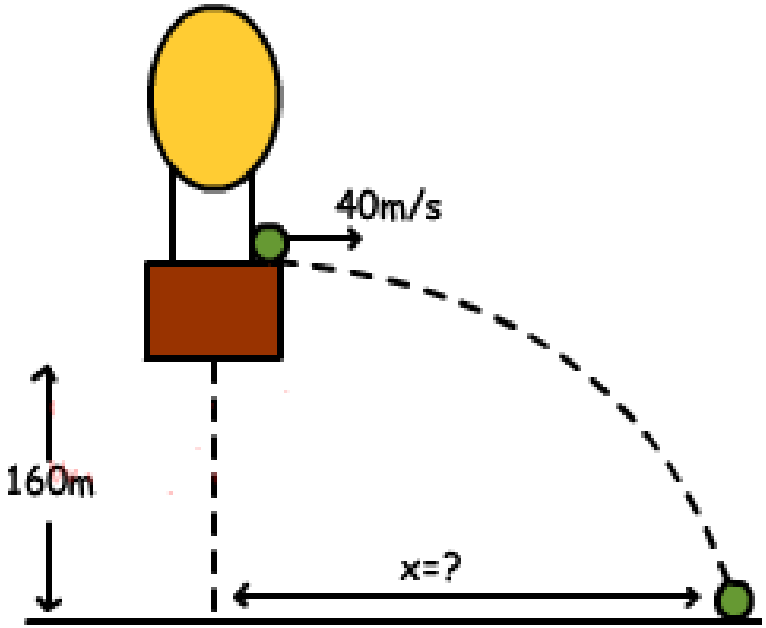
2.t=time of motion=2.3=6s

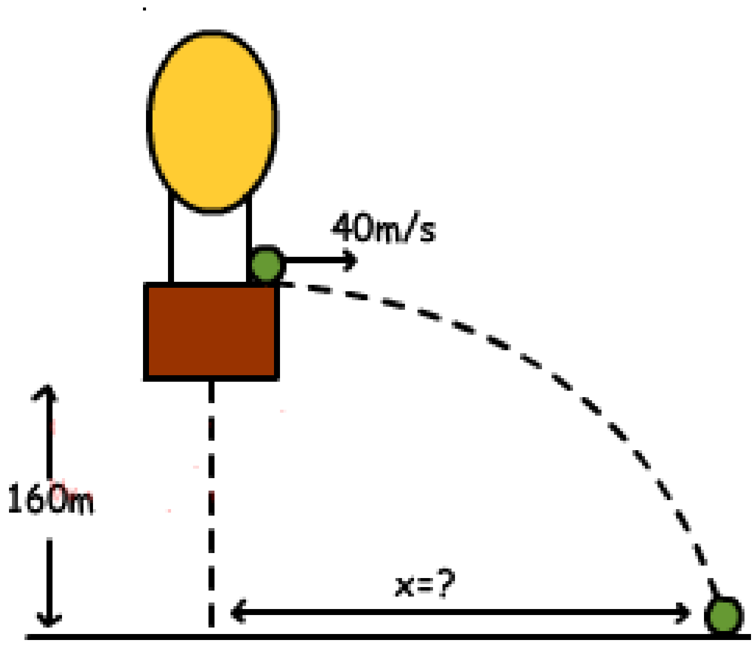
b) $V_{0y}^2=h_{max}.2.g$

$h_{max}=302/2.10=45m$

c) $X=V_{0x}.t_{total}=40.6=240m$

A balloon having 20 m/s constant velocity is rising from ground to up. When the balloon reaches 160 m height, an object is thrown horizontally with a velocity of 40m/s with respect to balloon. Find the horizontal distance travelled by the object.





Object has velocity 40m/s in horizontal, 20m/s in vertical and its height is 160m. We can find time of motion with following formula;

$h=V_{0y}.t-1/2.g.t^2$

$-160=20.t-1/2.10.t^2$

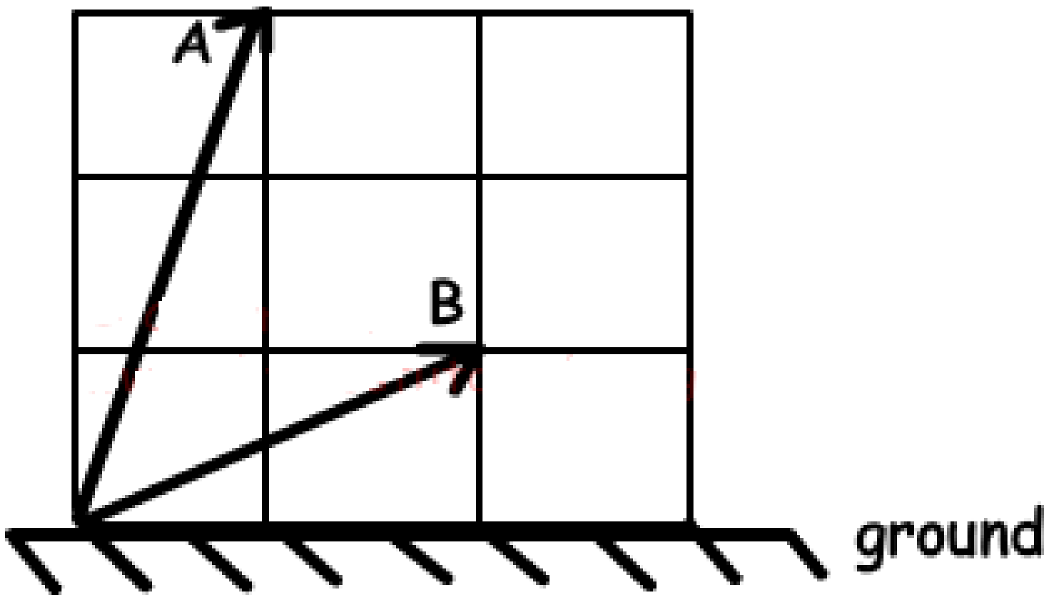
$t^2=4t-32$

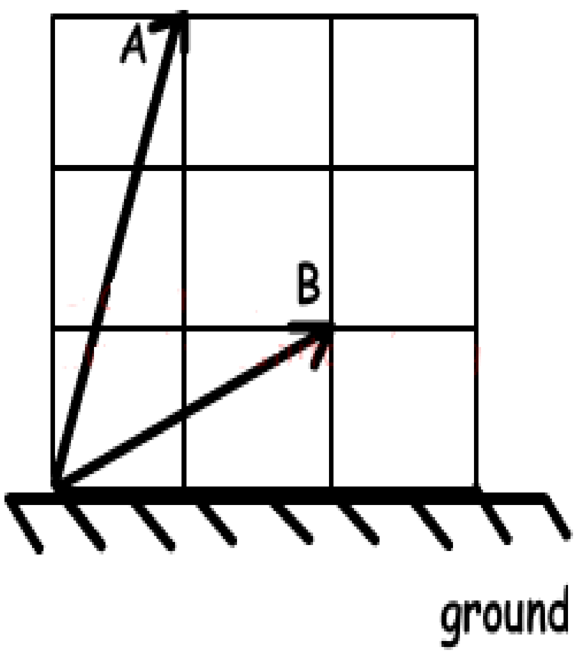
$(t-8).(t+8)=0$

$t=8s$

$X=V_{0x}.t=40.8=320m.$

Objects **A** and **B** are thrown with velocities as shown in the figure below. Find the ratio of horizontal distances taken by objects.





Time of flight is directly proportional to vertical component of velocity. Vertical velocity component of A is three times bigger than vertical velocity component of B.

$t_A/t_B=3$ $t_B=t_A/3$

Horizontal distance traveled by the object is found by the following formula;

$X_A=V_A.t_A$

$X_B=V_B.t_B$

Horizontal component of V_A is half of V_B, so we can write following equation;

$V_A=V_B/2$

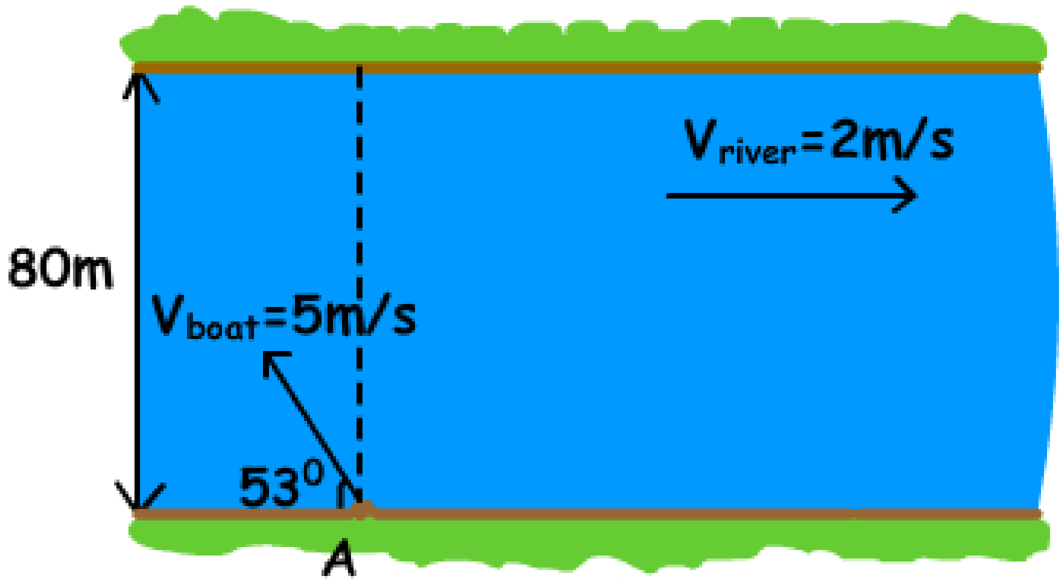
$V_B=2.V_A$

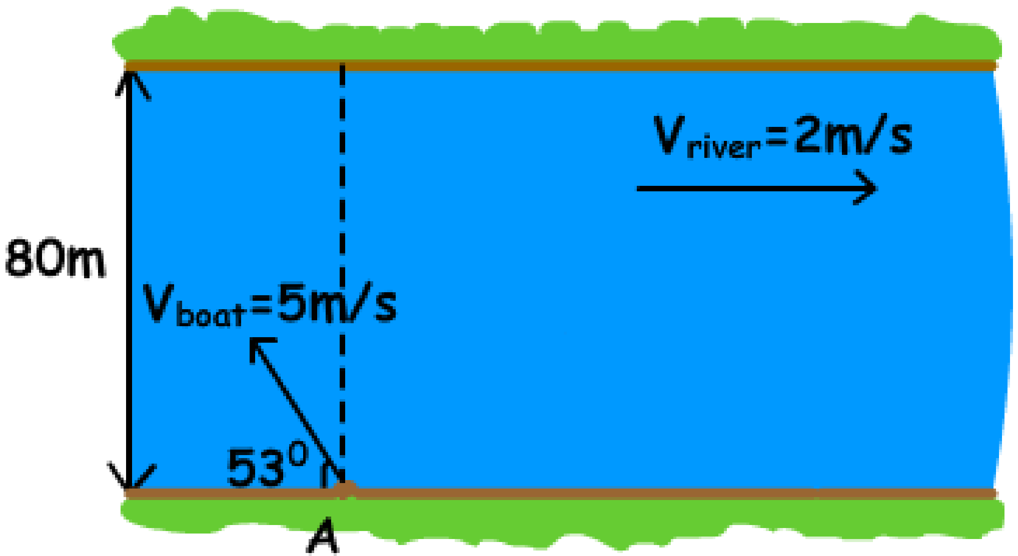
$X_A=V_A.t_A$

$X_B=2.V_A.t_A/3$

$X_A/X_B=3/2$

Velocity of the river with respect to ground is 2m/s to the east. Width of the river is 80m. One boat starts its motion on this river at point A with a velocity shown in the figure below. Find the time of the motion and horizontal distance between the arrival point and point A.





Components of boat velocity;

Vx=5.cos53^0=-3m/s to the west

Vy=5.sin53^0=4m/s to the north

Time for passing the river is;

t=X/V=80m/4m/s=20s

Resultant velocity in horizontal is;

V_R=V_X+Vriver

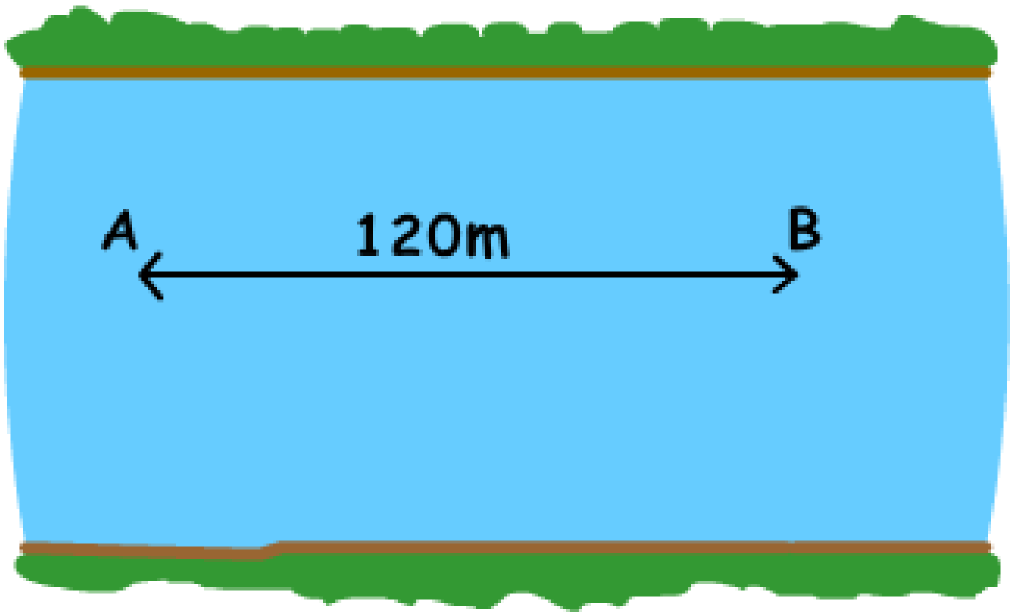
V_R=-3+2=-1m/s to the west

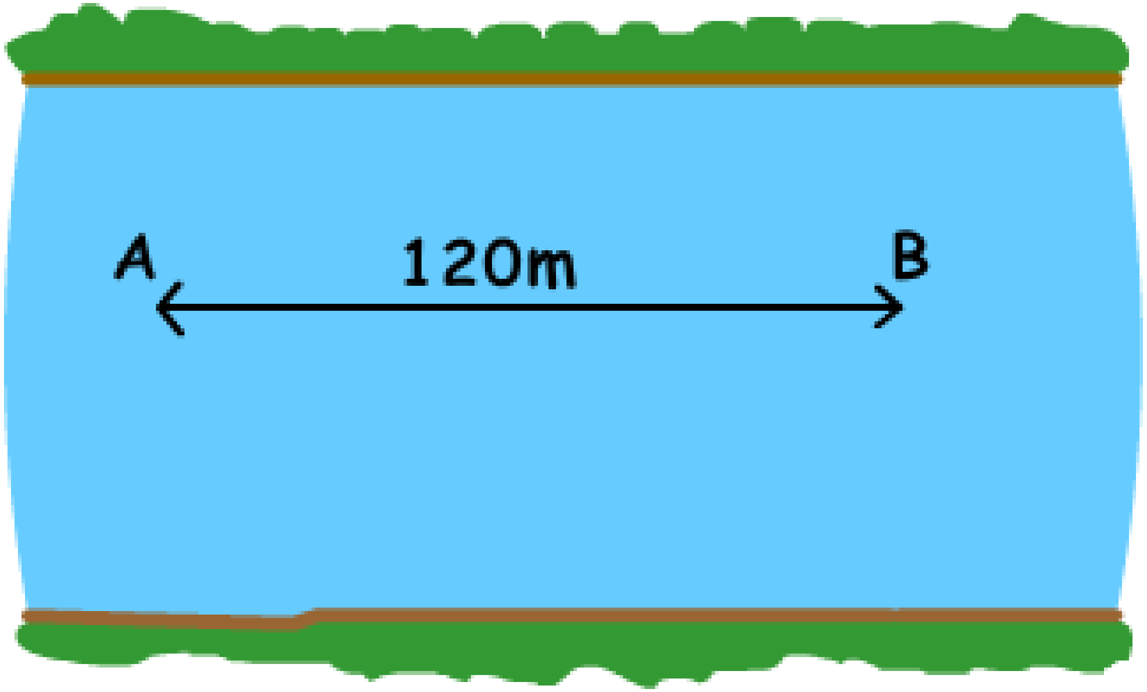
Distance taken in horizontal is;

X=V.t

X=1m/s.20s=20m

A river boat in a river having constant velocity travels 120m distance from point A to B in 20 s and turns back from B to A in 12 s. If the velocity of the river is zero, find the time of this trip.





Since the time of trip from B to A is longer than the time of trip from A to B, direction of river velocity is to the west.

Velocity of river with respect to ground is V_{river}, and velocity of boat with respect to river is $V_{boatriver}$.

Velocity of boat with respect to ground when it travels from A to B becomes;

$$V_b=V_{boatriver}-V_{river}$$

and when it travels from B to A;

$$V_b=V_{boatriver}+V_{river}$$

We can find velocities using following formula;

1. $V_{boatriver}-V_{river}=120/20=6m/s$

and

2. $V_{boatriver}+V_{river}=120/12=10m/s$

Solving equations 1. and 2. we find the velocities of river and boat.

$V_{boatriver}=8m/s$ **and** $V_{river}=2m/s$

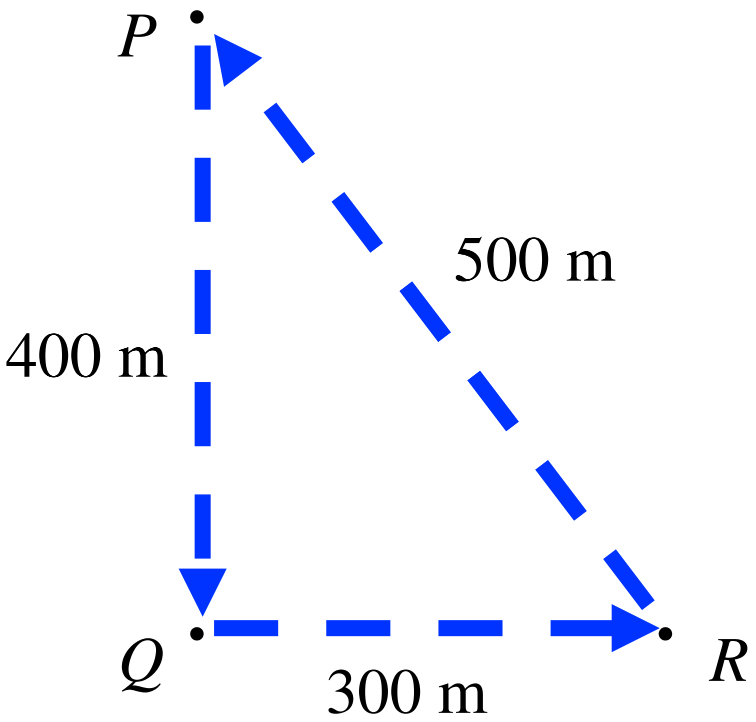
If the velocity of river is zero, boat travels 240m distance in;

240=8m/s.t

t=30s

Q3.1

A bicyclist starts at point P and travels around a triangular path that takes her through points Q and R before returning to point P. What is the magnitude of her net displacement for the entire round trip?



A. 100 m

B. 200 m

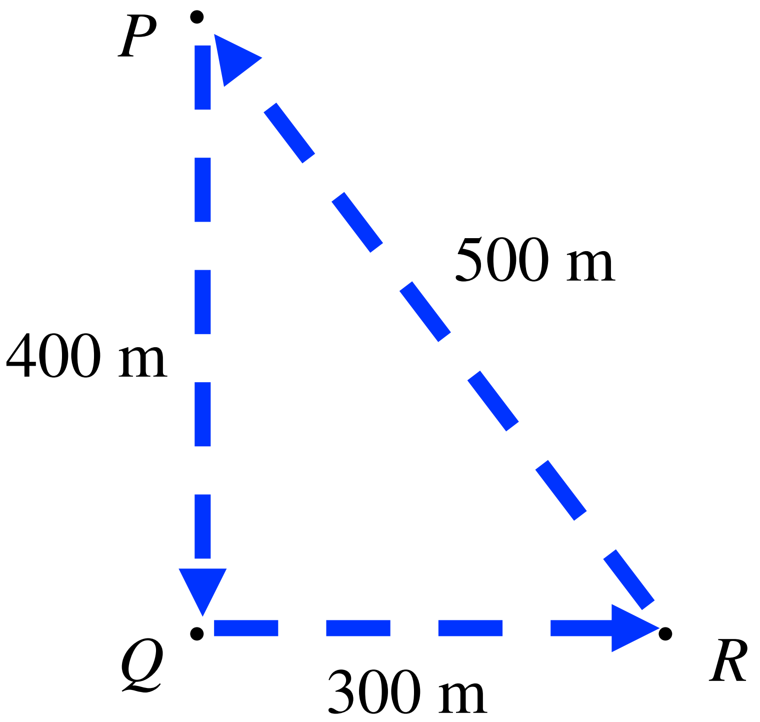
C. 600 m

D. 1200 m

E. zero

A3.1

A bicyclist starts at point *P* and travels around a triangular path that takes her through points *Q* and *R* before returning to point *P*. What is the magnitude of her net displacement for the entire round trip?



A. 100 m

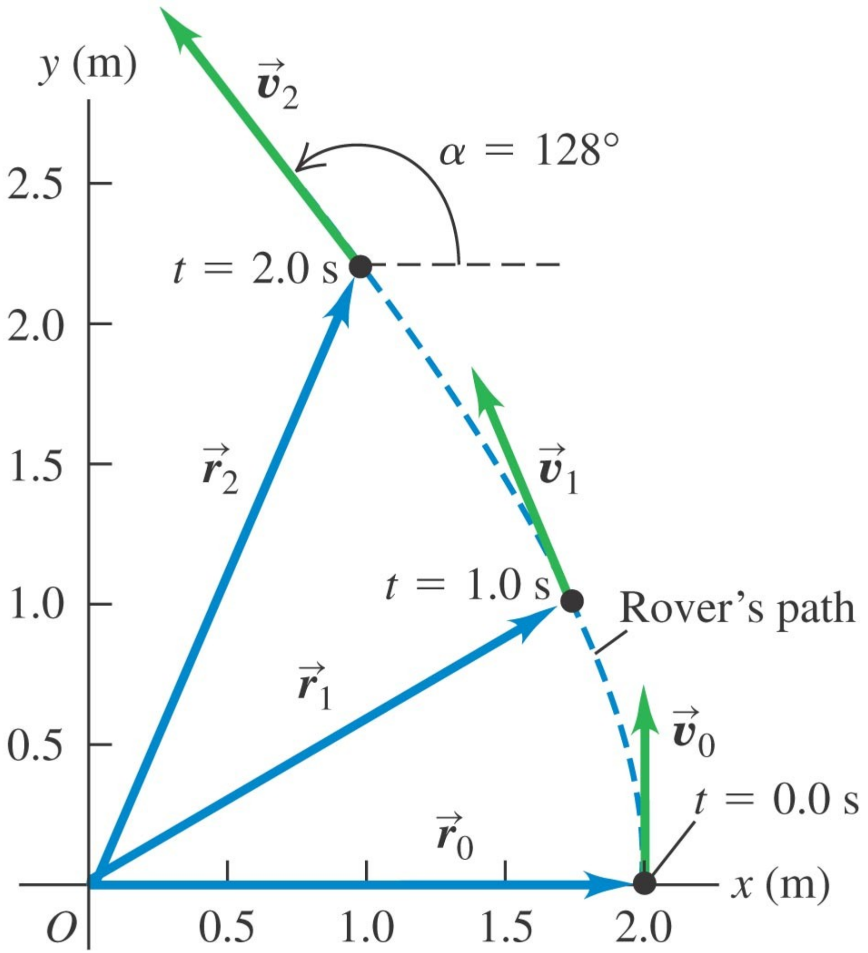
B. 200 m

C. 600 m

D. 1200 m

✓ E. zero

Q3.2



This illustration shows the path of a robotic vehicle, or rover. What is the direction of the rover's average acceleration vector for the time interval from $t = 0.0$ s to $t = 2.0$ s?

A. up and to the left

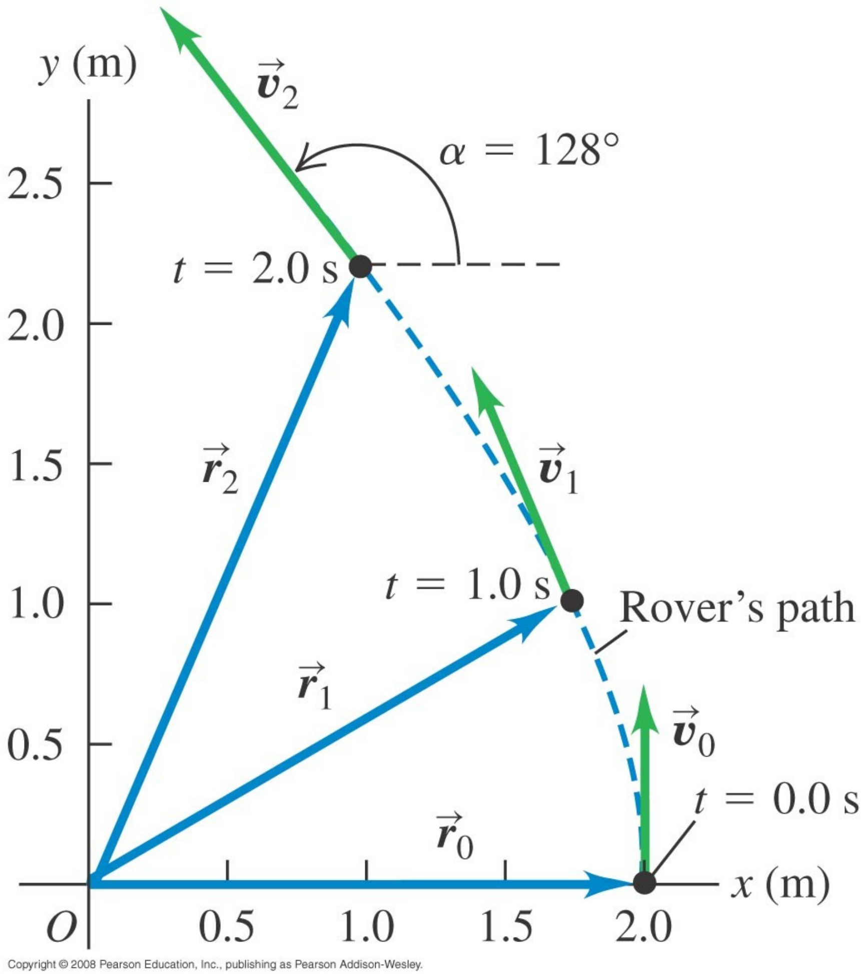
B. up and to the right

C. down and to the left

D. down and to the right

E. none of the above

A3.2



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✓ A. up and to the left

B. up and to the right

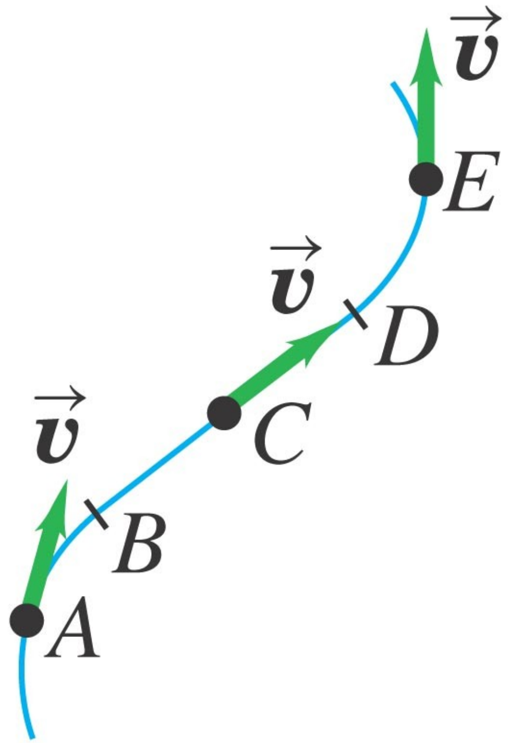
C. down and to the left

D. down and to the right

E. none of the above

Q3.3

The motion diagram shows an object moving along a curved path at constant speed. At which of the points *A*, *C*, and *E* does the object have *zero* acceleration?



A. point *A* only

B. point *C* only

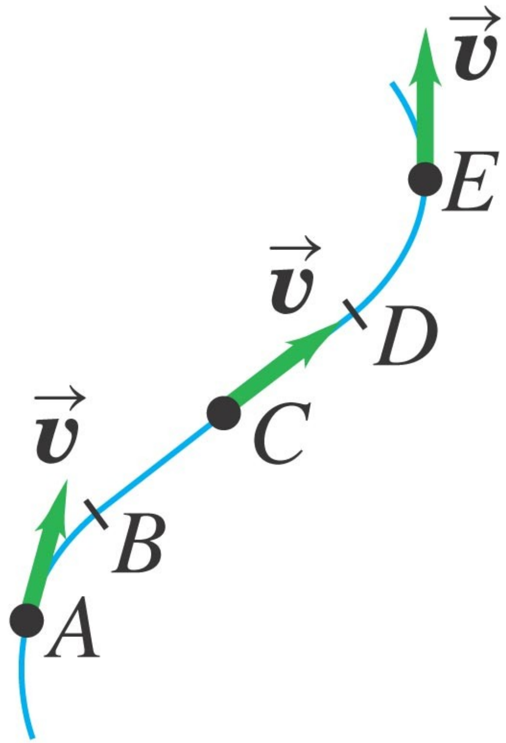
C. point *E* only

D. points *A* and *C* only

E. points *A*, *C*, and *E*

A3.3

The motion diagram shows an object moving along a curved path at constant speed. At which of the points *A*, *C*, and *E* does the object have *zero* acceleration?



A. point *A* only

✓ B. point *C* only

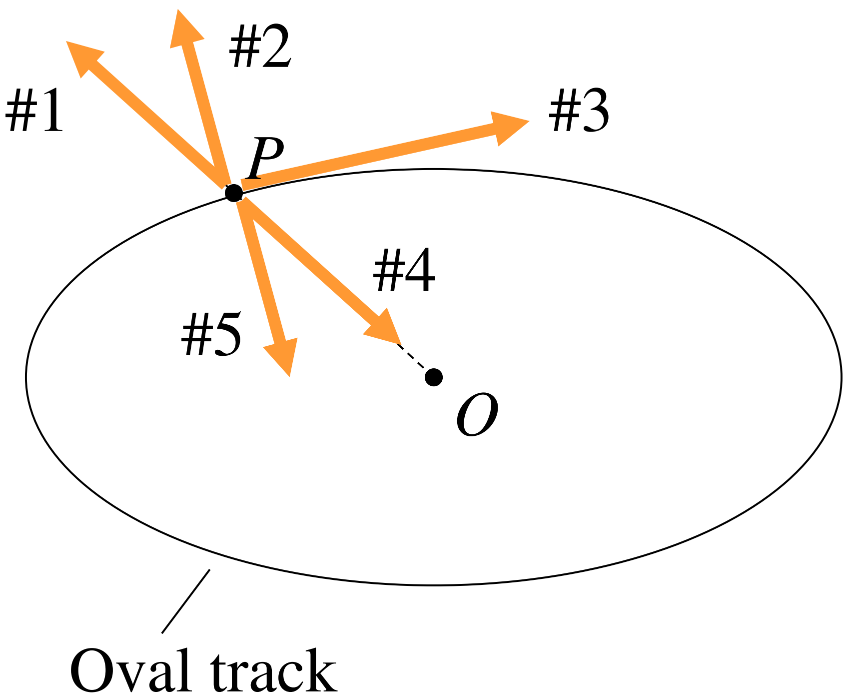
C. point *E* only

D. points *A* and *C* only

E. points *A*, *C*, and *E*

Q3.4

An object moves at a constant speed in a clockwise direction around an oval track. The geometrical center of the track is at point *O*. When the object is at point *P*, which arrow shows the direction of the object's acceleration vector?



A. #1 (directly away from *O*)

B. #2 (perpendicular to the track)

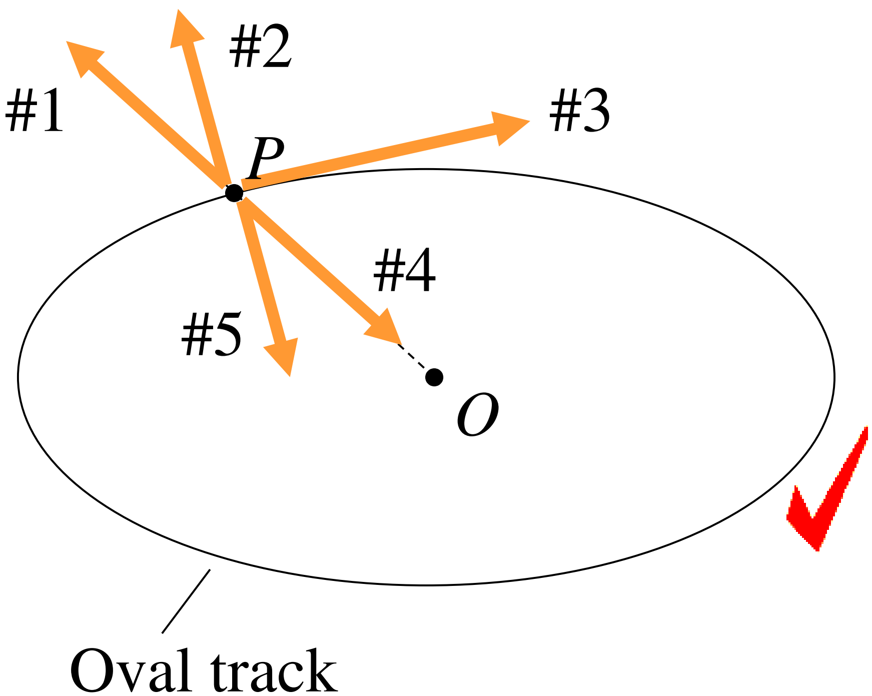
C. #3 (in the direction of motion)

D. #4 (directly toward *O*)

E. #5 (perpendicular to the track)

A3.4

An object moves at a constant speed in a clockwise direction around an oval track. The geometrical center of the track is at point *O*. When the object is at point *P*, which arrow shows the direction of the object's acceleration vector?



A. #1 (directly away from *O*)

B. #2 (perpendicular to the track)

C. #3 (in the direction of motion)

D. #4 (directly toward *O*)

E. #5 (perpendicular to the track)

Q3.5

A pendulum swings back and forth, reaching a maximum angle of 45° from the vertical. Which arrow shows the direction of the pendulum bob's acceleration as it moves from left to right through point *Q* (the low point of the motion)?

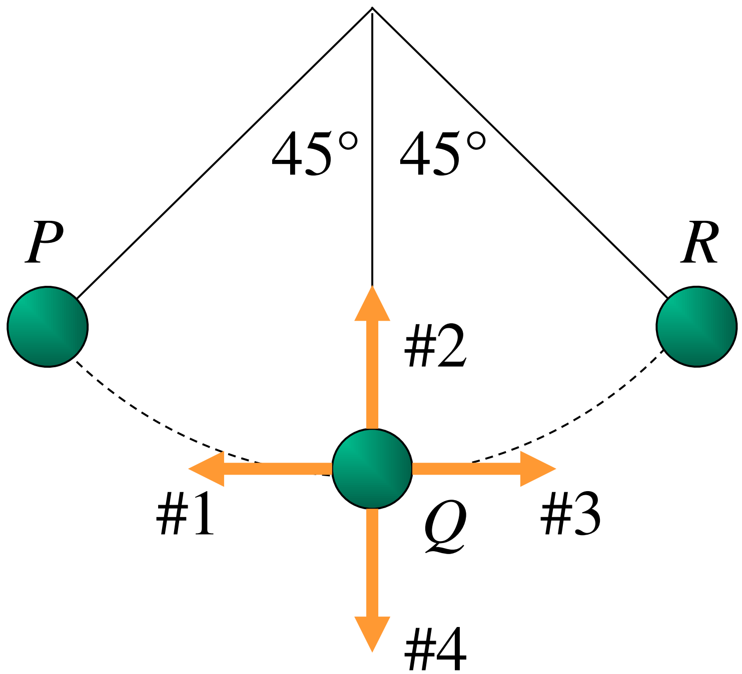
A. #1 (to the left)

B. #2 (straight up)

C. #3 (to the right)

D. #4 (straight down)

E. misleading question — the acceleration is zero at *Q*



A3.5

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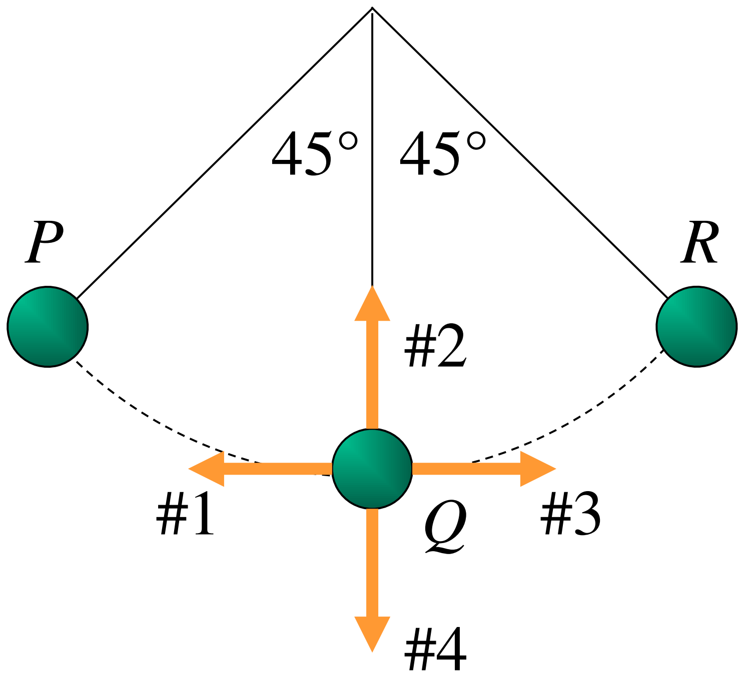
D. #4 (straight down)

E. misleading question — the acceleration is zero at *Q*



Q3.6

A pendulum swings back and forth, reaching a maximum angle of 45° from the vertical. Which arrow shows the direction of the pendulum bob's acceleration at *P* (the far left point of the motion)?

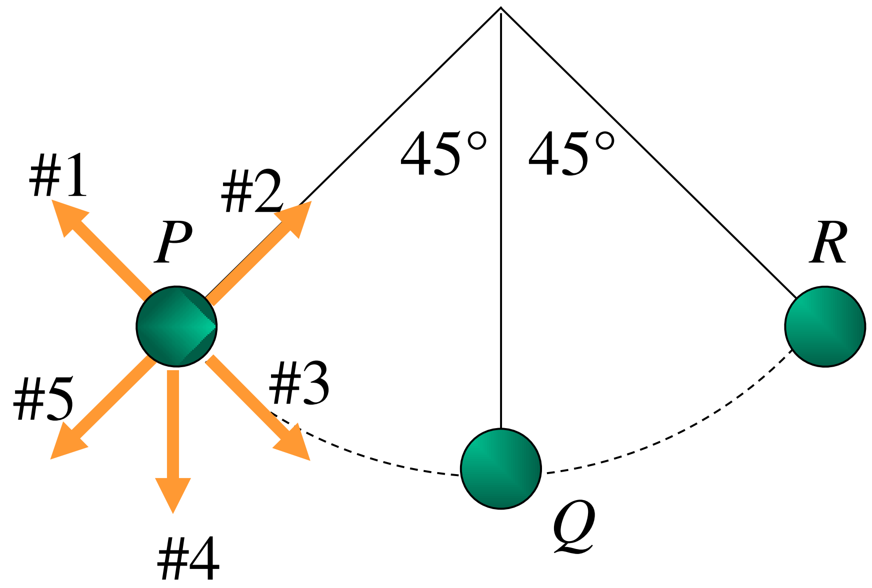
A. #1 (up and to the left)

B. #2 (up and to the right)

C. #3 (down and to the right)

D. #4 (straight down)

E. #5 (down and to the left)



A3.6

A pendulum swings back and forth, reaching a maximum angle of 45° from the vertical. Which arrow shows the direction of the pendulum bob's acceleration at *P* (the far left point of the motion)?

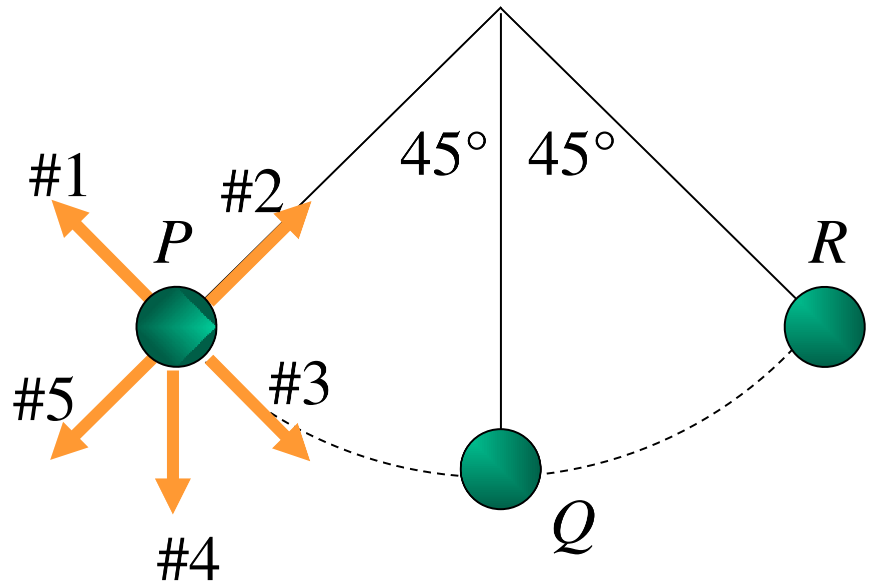
A. #1 (up and to the left)

B. #2 (up and to the right)

C. #3 (down and to the right)

D. #4 (straight down)

E. #5 (down and to the left)



Q3.7

The velocity and acceleration of an object at a certain instant are

$$\vec{v} = (3.0\ \text{m/s})\hat{j}$$

$$\vec{a} = (0.5\ \text{m/s}^2)\hat{i} - (0.2\ \text{m/s}^2)\hat{j}$$

At this instant, the object is

A. speeding up and following a curved path.

B. speeding up and moving in a straight line.

C. slowing down and following a curved path.

D. slowing down and moving in a straight line.

E. none of these is correct

A3.7

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Q3.8

The velocity and acceleration of an object at a certain instant are

$$\vec{v} = \left(2.0\ \text{m/s}^2\right)\hat{i} + \left(3.0\ \text{m/s}\right)\hat{j}$$

$$\vec{a} = \left(0.5\ \text{m/s}^2\right)\hat{i} - \left(0.2\ \text{m/s}^2\right)\hat{j}$$

At this instant, the object is

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E. none of these is correct

Q3.9

A zoo keeper fires a tranquilizer dart directly at a monkey. The monkey lets go at the same instant that the dart leaves the gun barrel. The dart reaches a maximum height *P* before striking the monkey. Ignore air resistance.

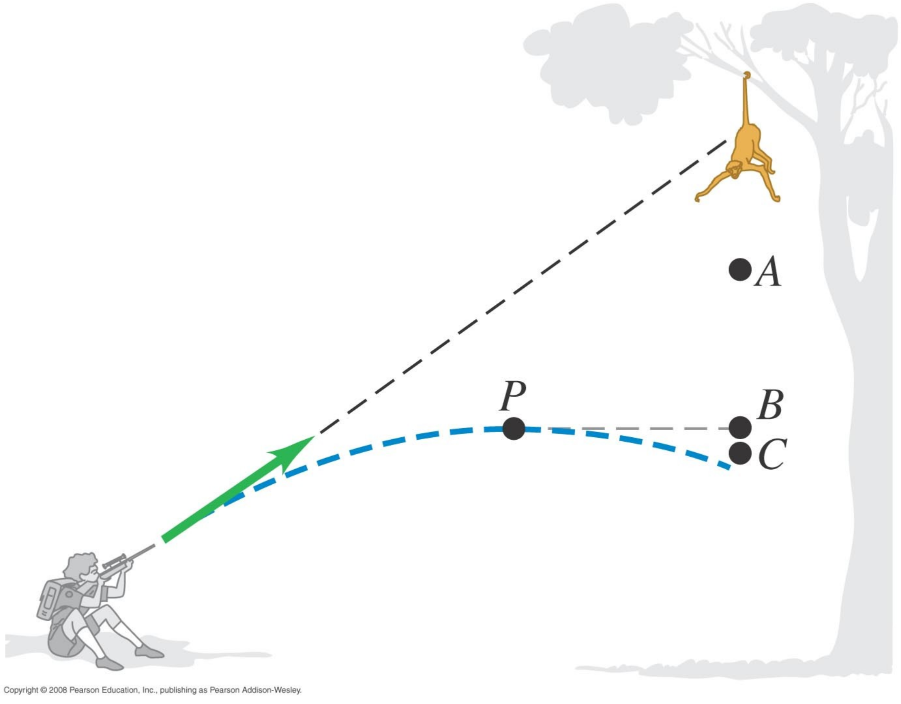
When the dart is at *P*, the monkey

A. is at *A* (higher than *P*).

B. is at *B* (at the same height as *P*).

C. is at *C* (lower than *P*).

D. not enough information given to decide



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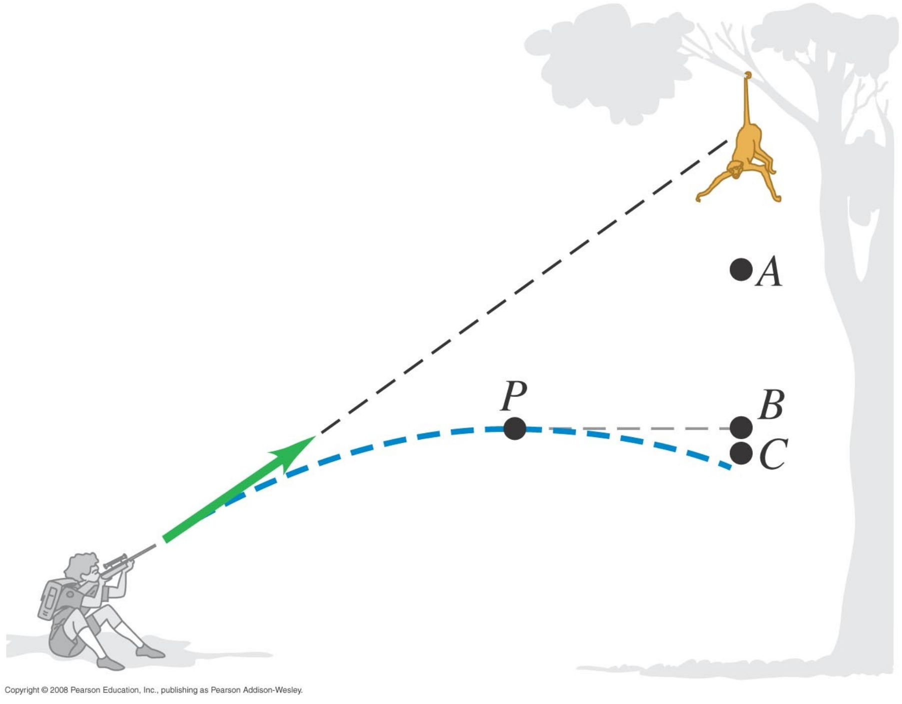

Q3.10

A projectile is launched at a 30° angle above the horizontal. Ignore air resistance. The projectile’s acceleration is greatest

A. at a point between the launch point and the high point of the trajectory.

B. at the high point of the trajectory.

C. at a point between the high point of the trajectory and where it hits the ground.

D. misleading question — the acceleration is the same (but nonzero) at all points along the trajectory

E. misleading question — the acceleration is zero at all points along the trajectory

A3.10

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Q3.11

You drive a race car around a circular track of radius 100 m at a constant speed of 100 km/h. If you then drive the same car around a different circular track of radius 200 m at a constant speed of 200 km/h, your acceleration will be

A. 8 times greater.

B. 4 times greater.

C. twice as great.

D. the same.

E. half as great.

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Newton's Laws of Motion

Newton's First Law

Newton's First Law states that an object will remain at rest or in uniform motion in a straight line unless acted upon by an external force. It may be seen as a statement about inertia, that objects will remain in their state of motion unless a force acts to change the motion. Any change in motion involves an acceleration, and then Newton's Second Law applies; in fact, the First Law is just a special case of the Second Law for which the net external force is zero.

Newton's First Law contains implications about the fundamental symmetry of the universe in that a state of motion in a straight line must be just as "natural" as being at rest. If an object is at rest in one frame of reference, it will appear to be moving in a straight line to an observer in a reference frame which is moving by the object. There is no way to say which reference frame is "special", so all constant velocity reference frames must be equivalent.

Newton's Second Law

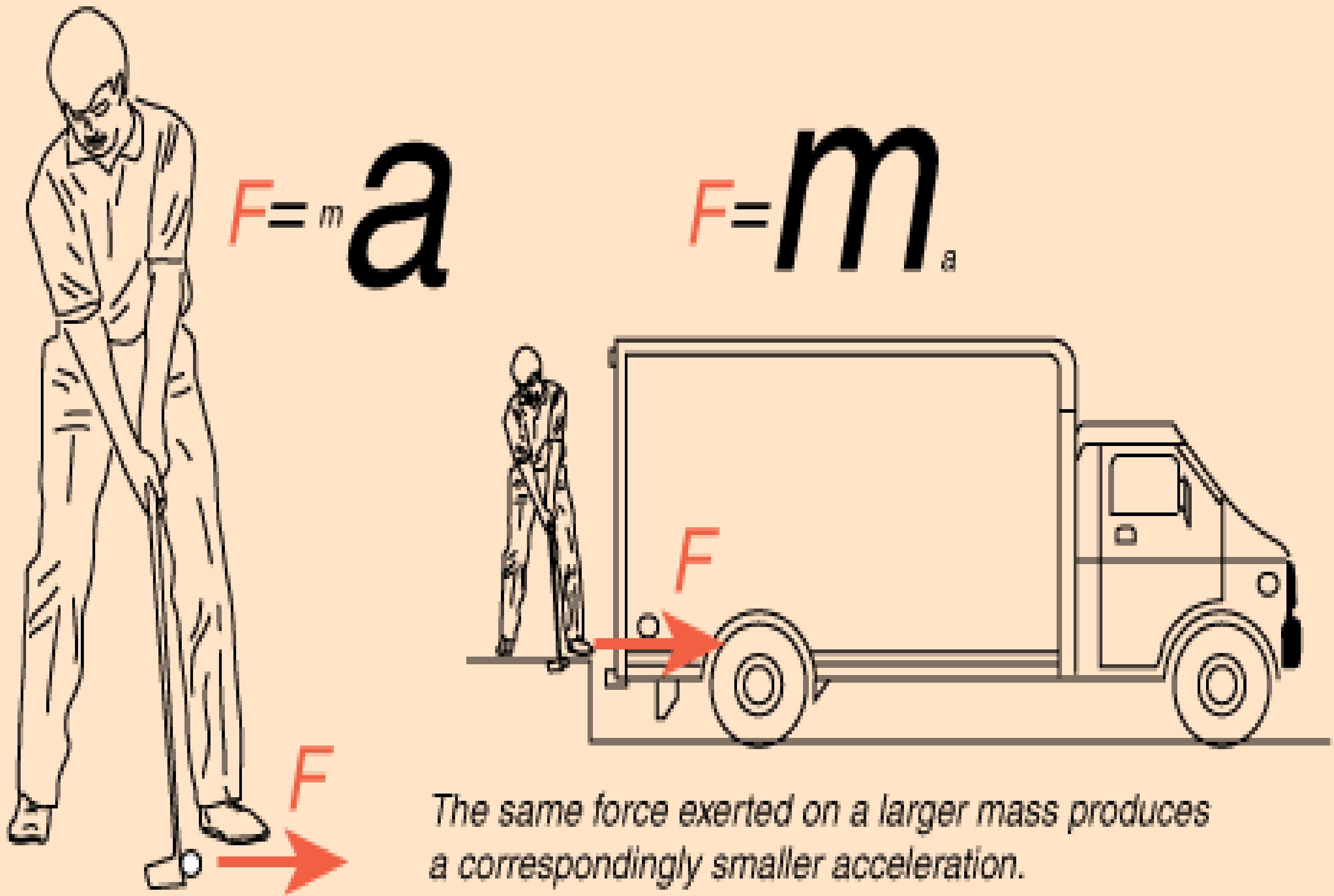
Newton's Second Law as stated below applies to a wide range of physical phenomena, but it is not a fundamental principle like the Conservation Laws. It is applicable only if the force is the net external force. It does not apply directly to situations where the mass is changing, either from loss or gain of material, or because the object is traveling close to the speed of light where relativistic effects must be included. It does not apply directly on the very small scale of the atom where quantum mechanics must be used.

$$F_{\text{net external}} = ma$$

Net force on object = mass of object x acceleration

Newton's Second Law Illustration

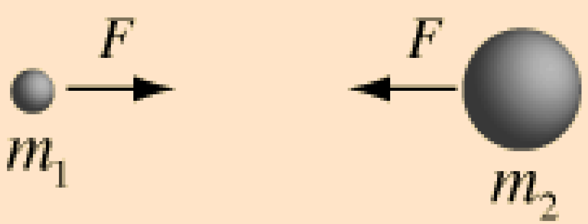
Newton's 2nd Law enables us to compare the results of the same force exerted on objects of different mass.



The same force exerted on a larger mass produces
a correspondingly smaller acceleration.

Newton's Third Law

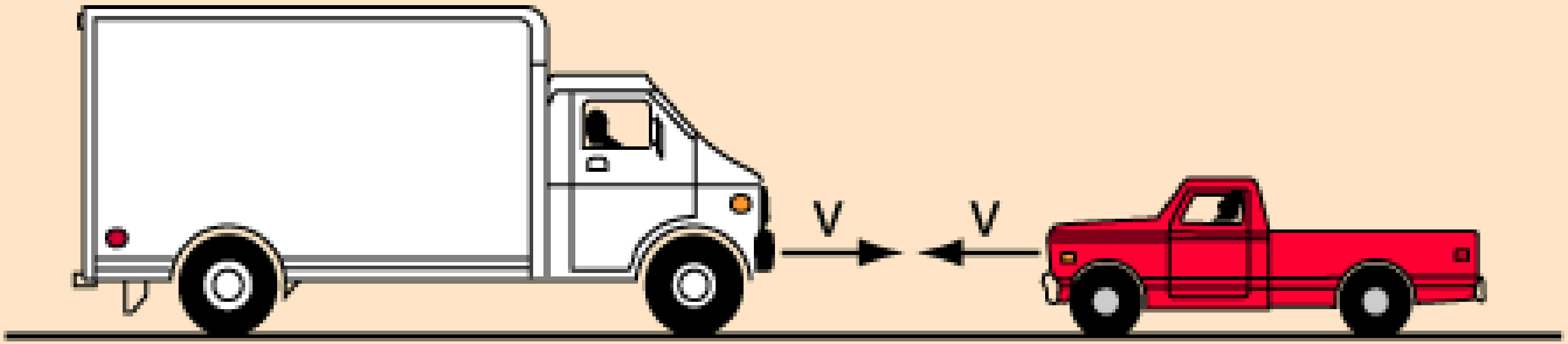
Newton's third law: All forces in the universe occur in equal but oppositely directed pairs. There are no isolated forces; for every external force that acts on an object there is a force of equal magnitude but opposite direction which acts back on the object which exerted that external force. In the case of internal forces, a force on one part of a system will be countered by a reaction force on another part of the system so that an isolated system cannot by any means exert a net force on the system as a whole. A system cannot "bootstrap" itself into motion with purely internal forces - to achieve a net force and an acceleration, it must interact with an object external to itself.



Without specifying the nature or origin of the forces on the two masses, Newton's 3rd law states that if they arise from the two masses themselves, they must be equal in magnitude but opposite in direction so that no net force arises from purely internal forces.

Newton's third law is one of the fundamental symmetry principles of the universe. Since we have no examples of it being violated in nature, it is a useful tool for analyzing situations which are somewhat counter-intuitive. For example, when a small truck collides head-on with a large truck, your intuition might tell you that the force on the small truck is larger. Not so!

Truck Collision



In a head-on collision:

Which truck will experience the greatest force?

Which truck will experience the greatest impulse?

Which truck will experience the greatest change in momentum?

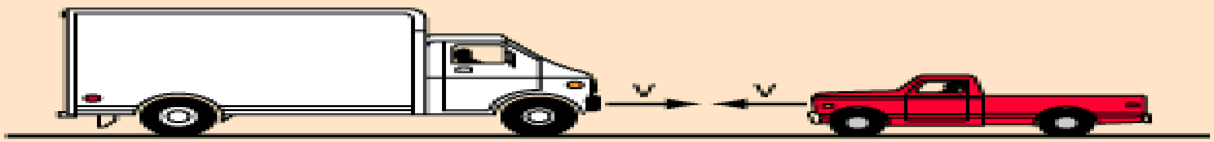
Which truck will experience the greatest change in velocity?

Which truck will experience the greatest acceleration?

Which truck would you rather be in during the collision?

Truck Collision

Comparison of the collision variables for the two trucks:



Force	F	$=$	F
Impulse	F_t	$=$	F_t
Change in momentum	$m_{\Delta v}$	$=$	$_m\Delta v$
Acceleration	m_a	$=$	$_m a$

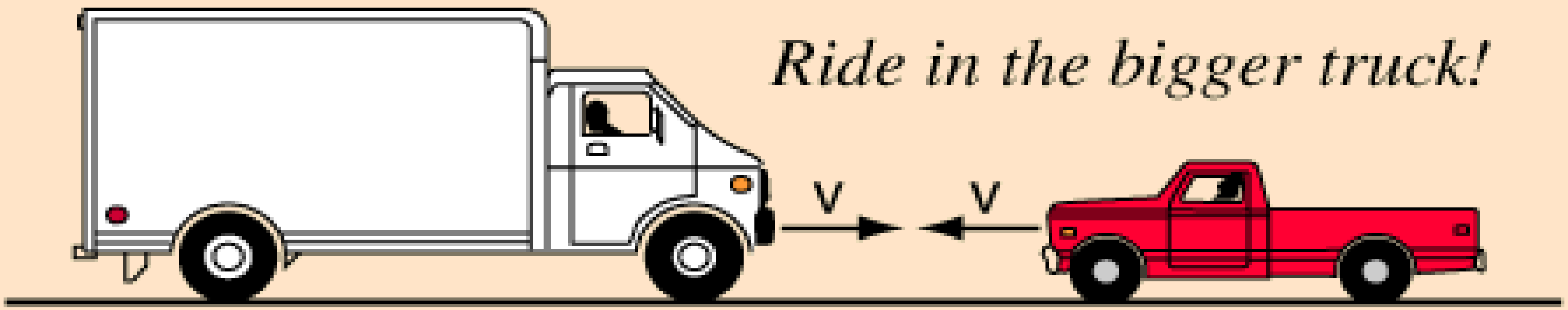
Ride in the bigger truck! There are good physical reasons!

In a head-on collision:

Newton's third law dictates that the forces on the trucks are equal but opposite in direction.

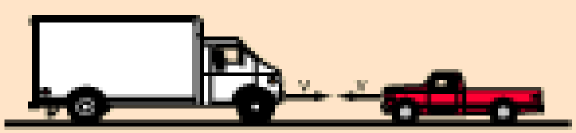
Impulse is force multiplied by time, and time of contact is the same for both, so the impulse is the same in magnitude for the two trucks. Change in momentum is equal to impulse, so changes in momenta are equal. With equal change in momentum and smaller mass, the change in velocity is larger for the smaller truck. Since acceleration is change in velocity over change in time, the acceleration is greater for the smaller truck.

Truck Collision



In a head-on collision the forces on the two vehicles are constrained to be the same by Newton's third law. But from both Newton's second law and the work-energy principle it becomes evident that it is safer to be in the bigger truck.

$$m\,\Delta v_{\text{big truck}} = m\,\Delta v_{\text{little truck}}$$



$$F_{\text{big truck}} = F_{\text{little truck}}$$

$$m\,a_{\text{big truck}} = m\,a_{\text{little truck}}$$

The change in velocity of the driver will be the same as the truck in which he/she is riding. A greater change in velocity implies a greater change in kinetic energy and therefore more work done on the driver.

Newton's Second Law

$$F_{net\ external} = ma$$

Net force on object = mass of object x acceleration

Limitations on Newton's 2nd Law

One of the best known relationships in physics is Newton's 2nd Law

$$F = ma$$

but, though extremely useful, it is not a fundamental principle like the conservation laws. F must be the net external force, and even then a more fundamental relationship is

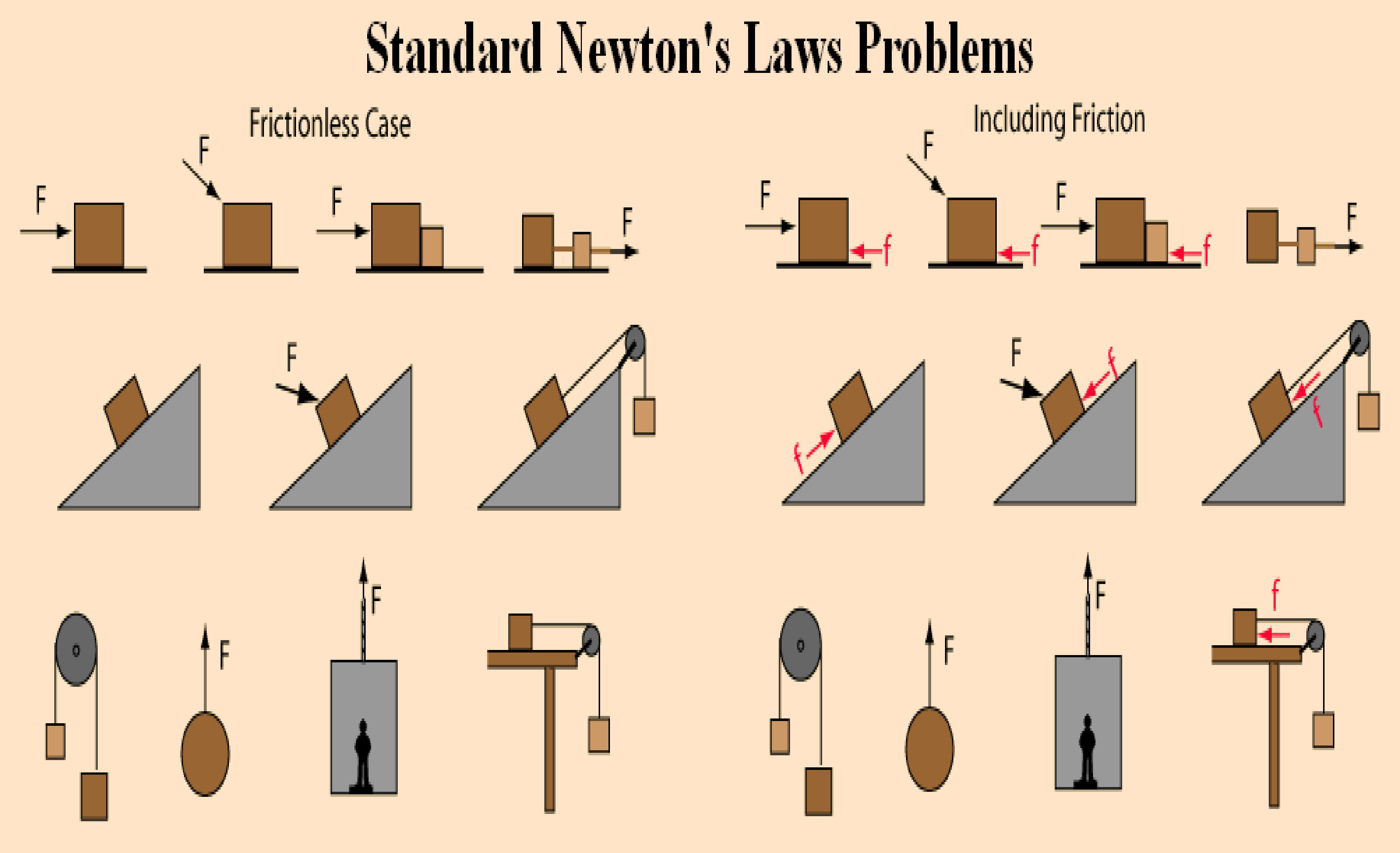
$$F_{\text{average net external}} = \frac{\Delta(mv)}{\Delta t}$$

The net force should be defined as the rate of change of momentum; this becomes

$$F_{\text{net external}} = ma \qquad F_{net\ \ external} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

only if the mass is constant. Since the mass changes as the speed approaches the speed of light, F=ma is seen to be strictly a non-relativistic relationship which applies to the acceleration of constant mass objects. Despite these limitations, it is extremely useful for the prediction of motion under these constraints.

Some Cases for Newton's Laws



Friction

Frictional resistance to the relative motion of two solid objects is usually proportional to the force which presses the surfaces together as well as the roughness of the surfaces. Since it is the force perpendicular or "normal" to the surfaces which affects the frictional resistance, this force is typically called the "normal force" and designated by N. The frictional resistance force may then be written:

$$f_{friction} = \mu N$$

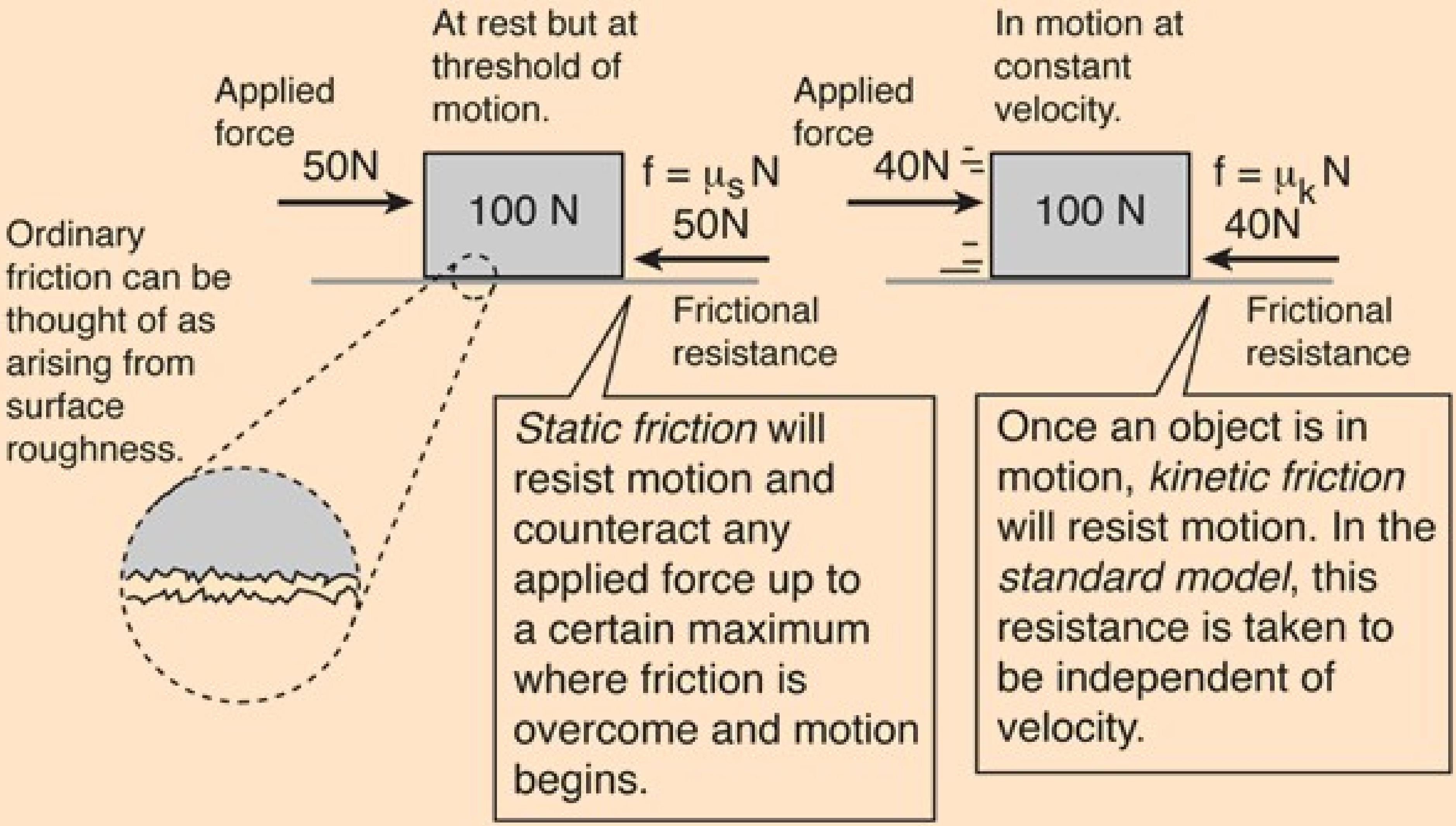
μ = coefficient of friction

μ_k = coefficient of kinetic friction

μ_s = coefficient of static friction

Standard model of friction

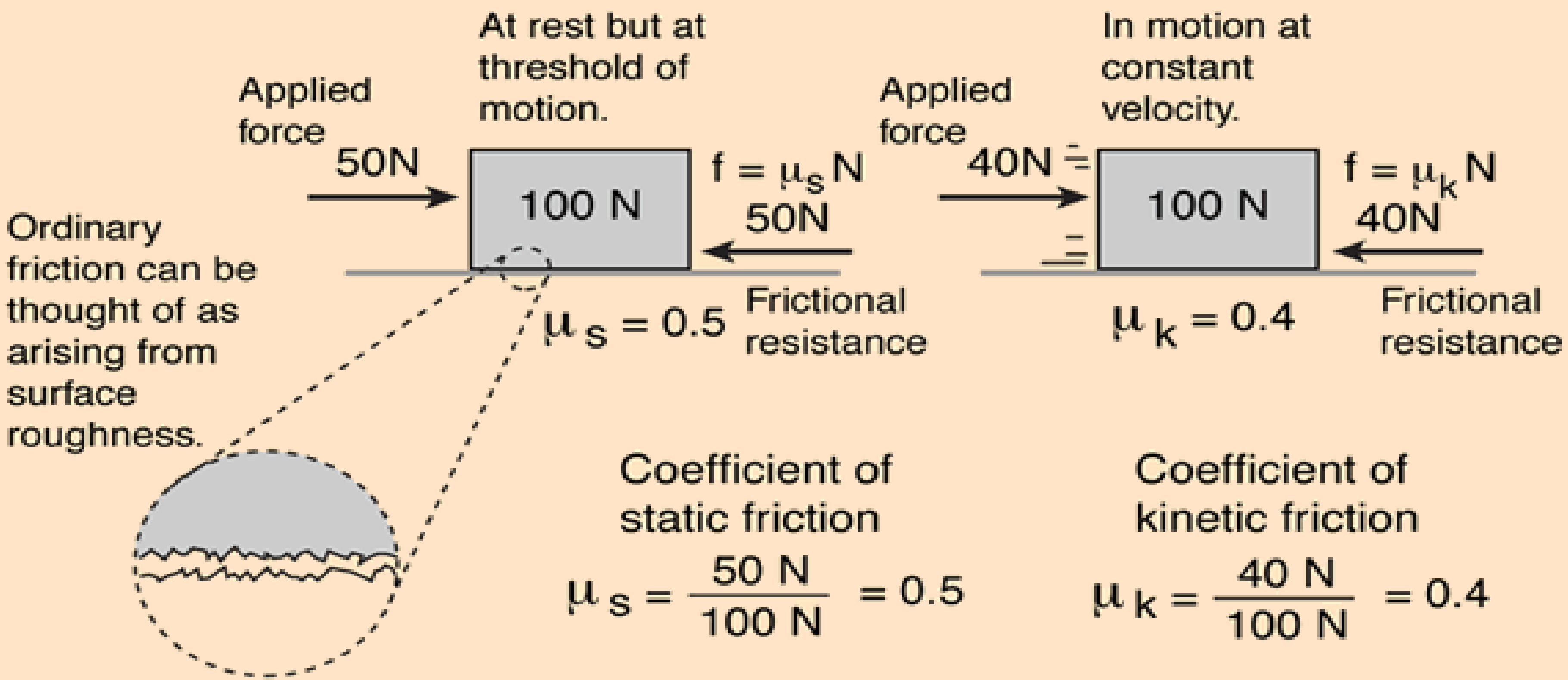
Friction and Surface Roughness



In general, the coefficients of friction for static and kinetic friction are different.

Coefficients of Friction

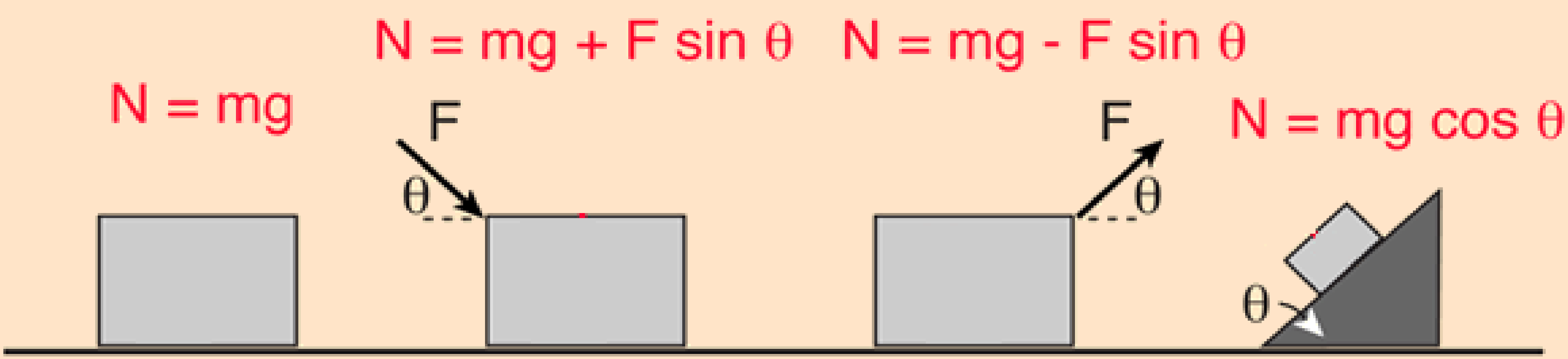
Friction is typically characterized by a coefficient of friction which is the ratio of the frictional resistance force to the normal force which presses the surfaces together. In this case the normal force is the weight of the block. Typically there is a significant difference between the coefficients of static friction and kinetic friction.



Note that the static friction coefficient does not characterize static friction in general, but represents the conditions at the threshold of motion only.

Normal Force

Frictional resistance forces are typically proportional to the force which presses the surfaces together. This force which will affect frictional resistance is the component of applied force which acts perpendicular or "normal" to the surfaces which are in contact and is typically referred to as the normal force. In many common situations, the normal force is just the weight of the object which is sitting on some surface, but if an object is on an incline or has components of applied force perpendicular to the surface, then it is not equal to the weight.



For an object sitting on a flat surface, the normal force is just its weight.

If a force acts downward on the object, the normal force is greater than the weight.

If a force pulls upward on the object, the normal force is less than the weight.

For an object sitting on an incline, the normal force is less than the weight.

Mass and Weight

The mass of an object is a fundamental property of the object; a numerical measure of its inertia; a fundamental measure of the amount of matter in the object. Definitions of mass often seem circular because it is such a fundamental quantity that it is hard to define in terms of something else. All mechanical quantities can be defined in terms of mass, length, and time. The usual symbol for mass is m and its SI unit is the kilogram. While the mass is normally considered to be an unchanging property of an object, at speeds approaching the speed of light one must consider the increase in the relativistic mass.

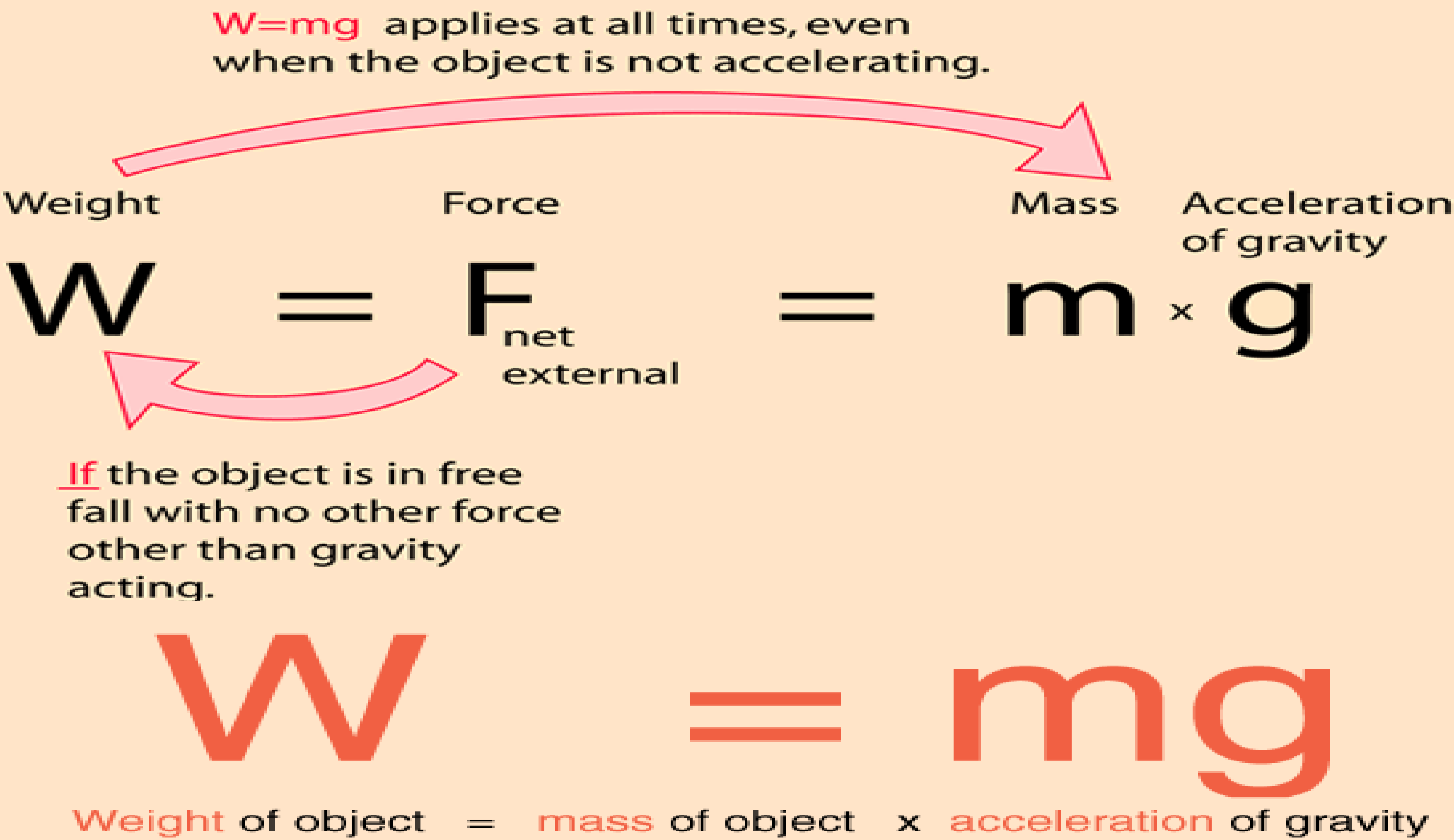
The weight of an object is the force of gravity on the object and may be defined as the mass times the acceleration of gravity, $w = mg$. Since the weight is a force, its SI unit is the newton. Density is mass/volume.

If an object has a mass of 1 kg on the earth, it would have a mass of 1 kg on the moon, even though it would weigh only one-sixth as much.

Weight

The weight of an object is defined as the force of gravity on the object and may be calculated as the mass times the acceleration of gravity, w = mg. Since the weight is a force, its SI unit is the newton.

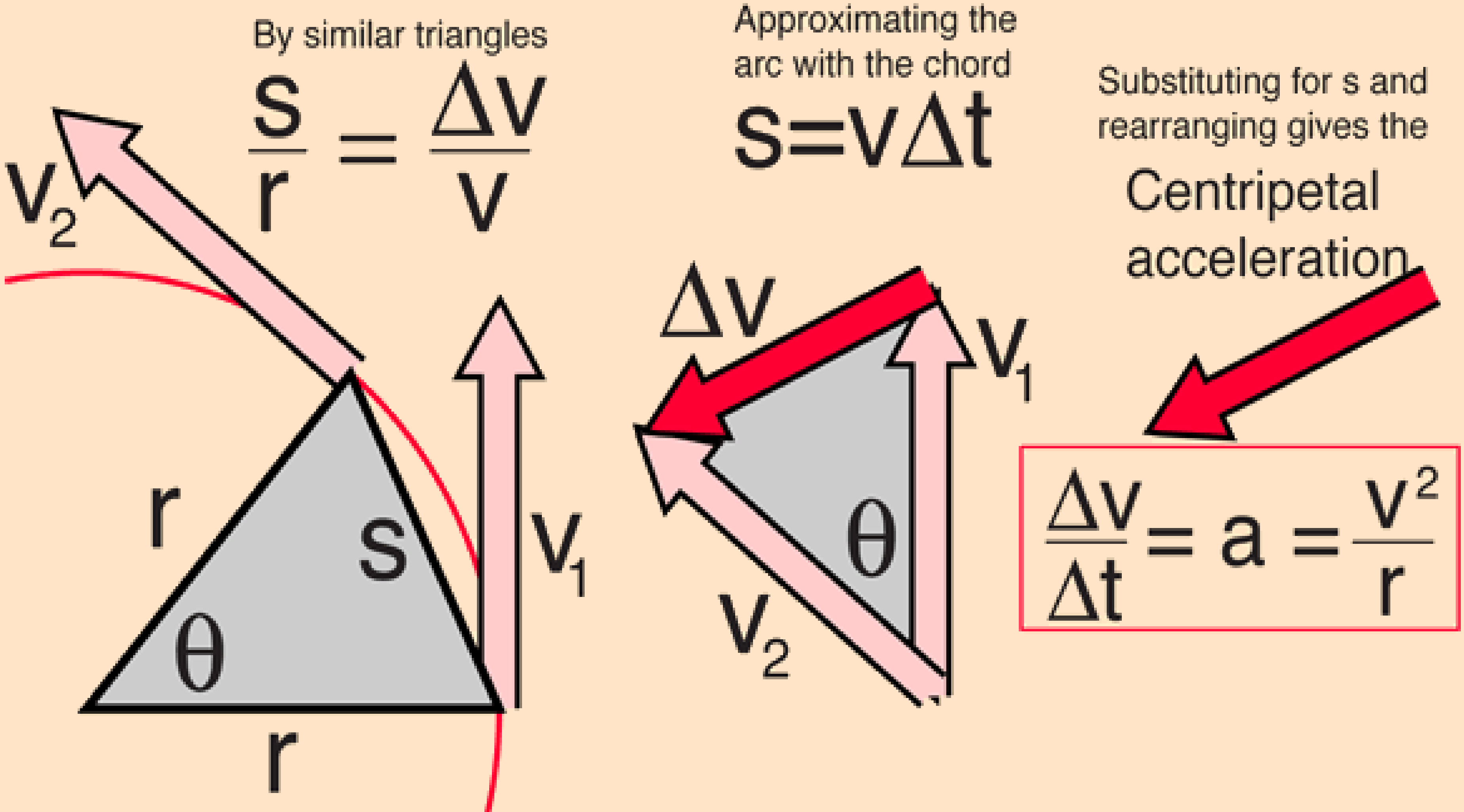
For an object in free fall, so that gravity is the only force acting on it, then the expression for weight follows from Newton's second law.



At the Earth's surface, where $g=9.8\ m/s^2$:

Centripetal Acceleration

The centripetal acceleration expression is obtained from analysis of constant speed circular motion by the use of similar triangles. From the ratio of the sides of the triangles:



Centripetal Force

Any motion in a curved path represents accelerated motion, and requires a force directed toward the center of curvature of the path. This force is called the centripetal force which means "center seeking" force. The force has the magnitude

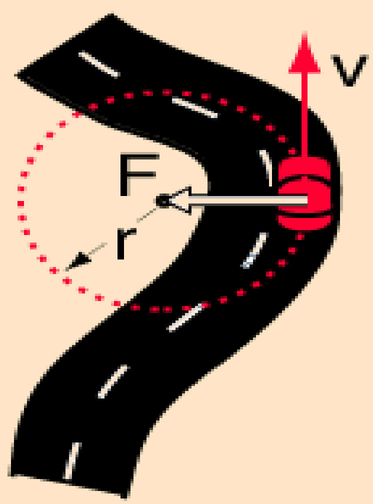
$$F_{centripetal} = m\frac{v^2}{r}$$

Swinging a mass on a string requires string tension, and the mass will travel off in a tangential straight line if the string breaks.

The centripetal acceleration can be derived for the case of circular motion since the curved path at any point can be extended to a circle.

$$F_{centripetal} = m\frac{v^2}{r}$$

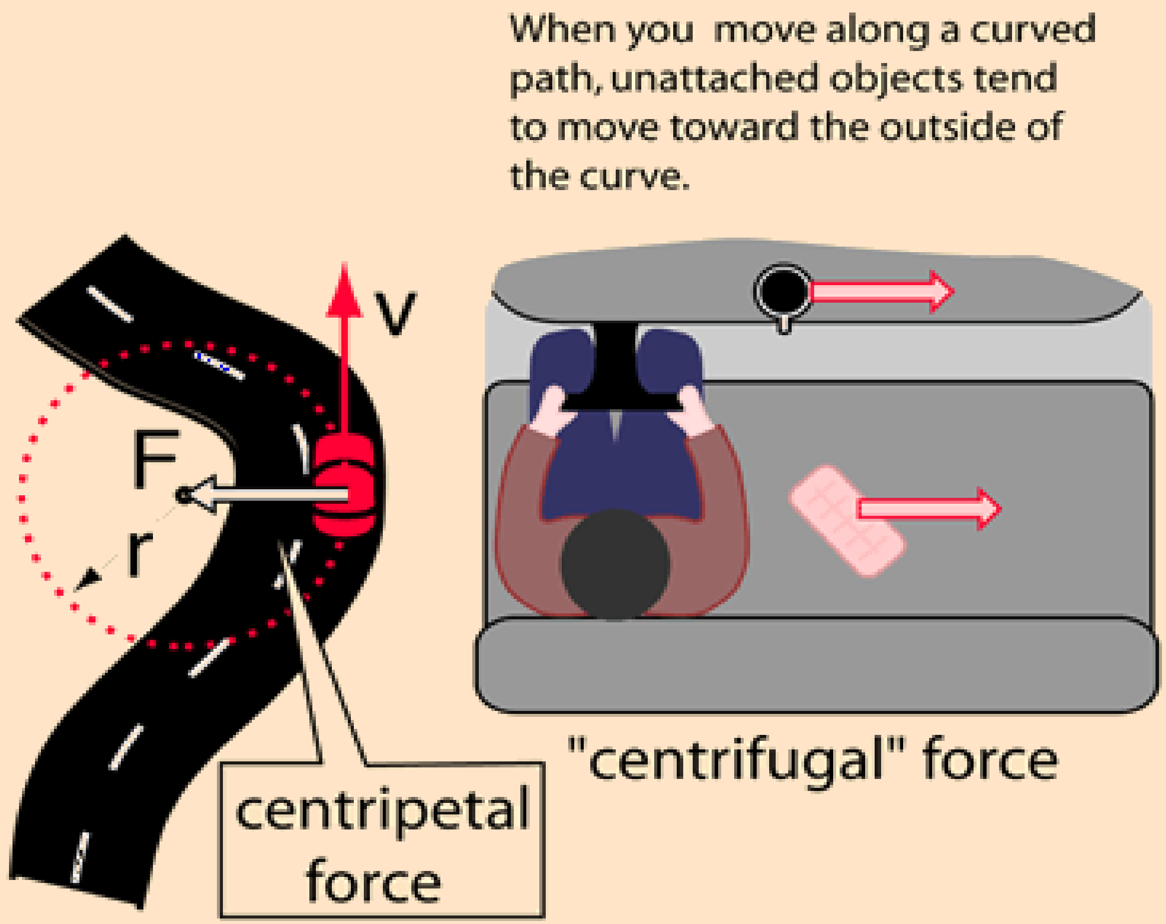
$\frac{v^2}{r}$ is the centripetal acceleration



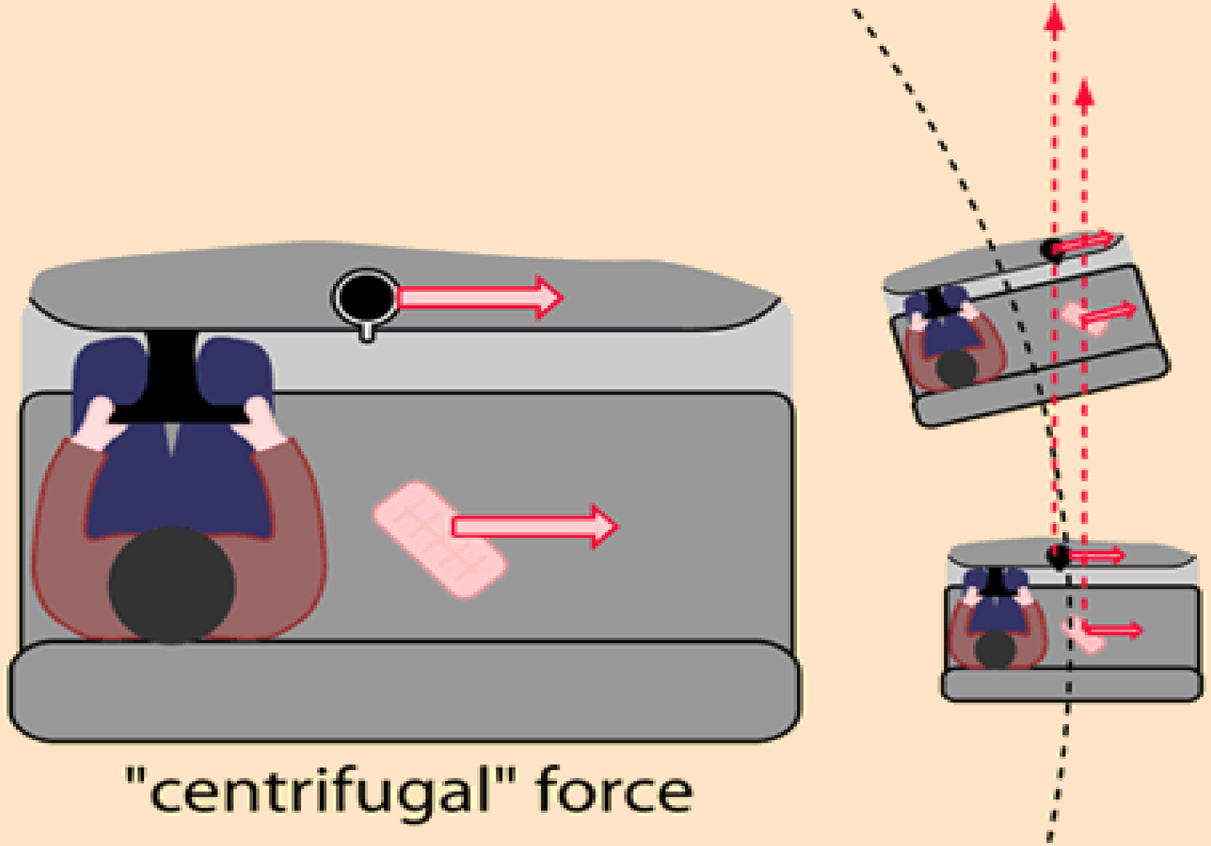
Note that the centripetal force is proportional to the square of the velocity, implying that a doubling of speed will require **four times** the centripetal force to keep the motion in a circle. If the centripetal force must be provided by friction alone on a curve, an increase in speed could lead to an unexpected skid if friction is insufficient.

Centrifugal Force

Whereas the centripetal force is seen as a force which must be applied by an external agent to force an object to move in a curved path, the centrifugal and coriolis forces are "effective forces" which are invoked to explain the behavior of objects from a frame of reference which is rotating.



The driver of a car on a curve is in a rotating reference frame and he could invoke a "centrifugal" force to explain why his coffee cup and the carton of eggs he has on the seat beside him tend to slide sideways. The friction of the seat or dashboard may not be sufficient to accelerate these objects in the curved path.

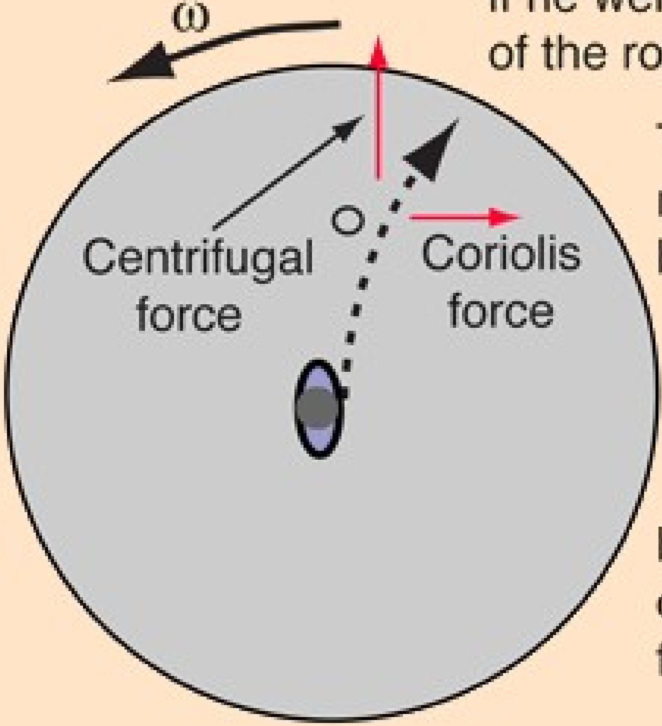


A person in a hovering helicopter above the car could describe the movement of the cup and the egg carton as just going straight while the car travels in a curved path. It is similar to the "broken string" example.

The centrifugal force is a useful concept when the most convenient reference frame is one which is moving in a curved path, and therefore experiencing a centripetal acceleration. Since the car above will be experiencing a centripetal acceleration v^2/r, then an object of mass m on the seat will require a force mv^2/r toward the center of the circle to stay at the same spot on the seat. From the reference frame of a person in the car, there seems to be an outward centrifugal force mv^2/r acting to move the mass radially outward. In practical descriptive terms, you would say that your carton of eggs is more likely to slide outward if you have a higher speed around the curve (the velocity squared factor) and more likely to slide outward if you go around a sharper curve (the inverse dependence upon r).

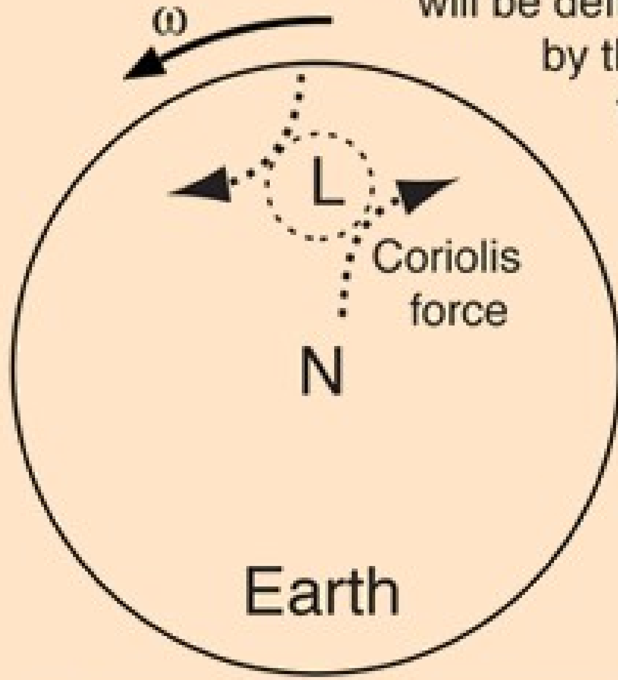
Coriolis Force

A golfer who is putting the ball from the center of a rotating platform would tend to miss to the right and overshoot the hole if he were unaware of the rotation.



The rightward miss he could blame on the coriolis force and the overshoot he could blame on centrifugal force.

Air which moves from the North Polar region toward a low pressure area in the northern hemsiphere will be deflected west by the coriolis force.



Air approaching the low pressure area from near the equator will be deflected eastward by the coriolis force. This gives the cyclonic counterclockwise rotation of air around lows in the northern hemisphere.

Conditions for Equilibrium

An object at equilibrium has no net influences to cause it to move, either in translation (linear motion) or rotation. The basic conditions for equilibrium are:

1. Net force = 0

x and y components of force may be separately set = 0.

$$\sum_i F_i = 0$$

Forces left = forces right
Forces up = forces down.

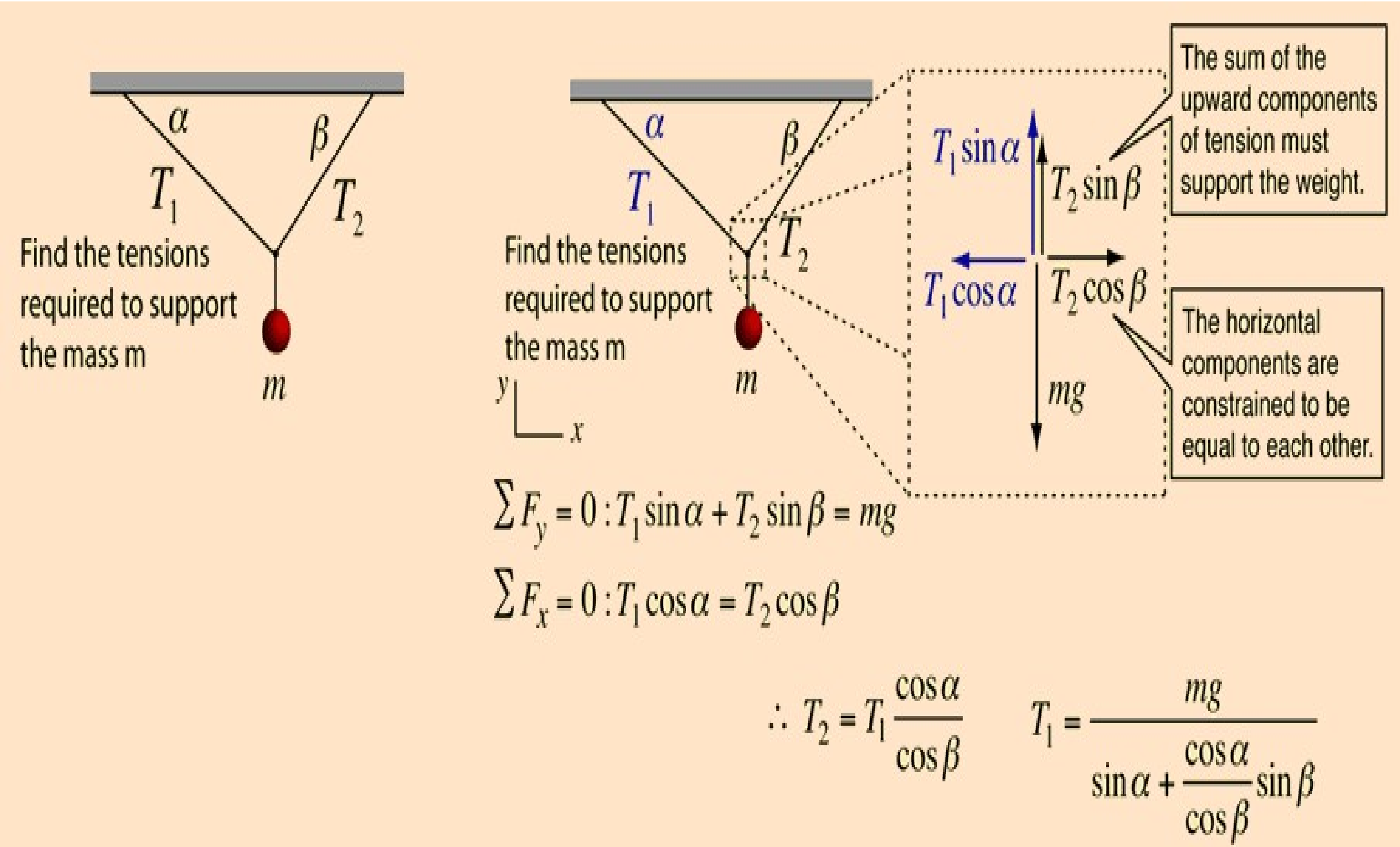
2. Net torque = 0

The axis may be chosen for advantage to eliminate some unknown forces..

$$\sum_i \tau_i = 0$$

The sum of the clockwise torques is equal to the sum of the counterclockwise torques.

Force Equilibrium Example



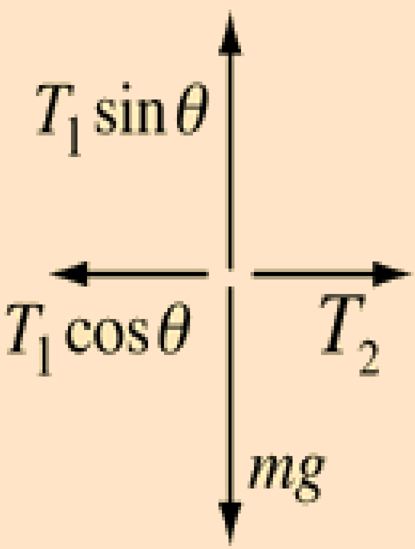
Force Equilibrium Example

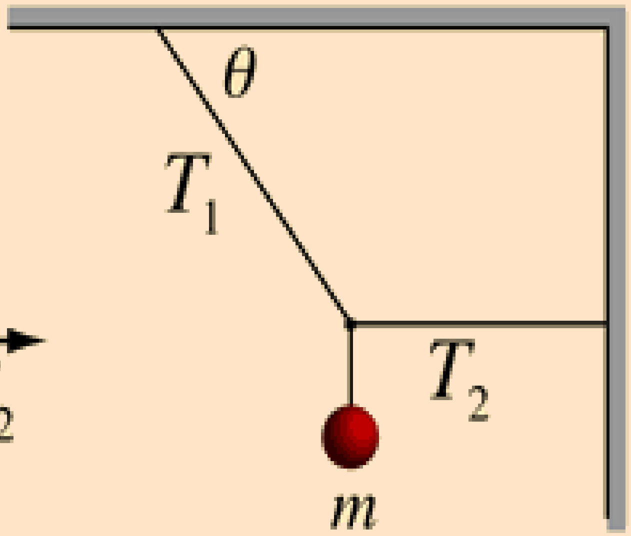
Force equilibrium problems like this can be analyzed by drawing a free body diagram of the point of attachment of the mass m, since it must be at equilibrium. The tensions should be resolved into horizontal and vertical components to apply the force equilibrium condition.

$$\sum F_y = 0 : T_1 \sin\theta = mg$$

$$\sum F_x = 0 : T_1 \cos\theta = T_2$$

$$T_1 = \frac{mg}{\sin\theta}$$





Find the required tensions if one cable is horizontal.

Causes of Motion

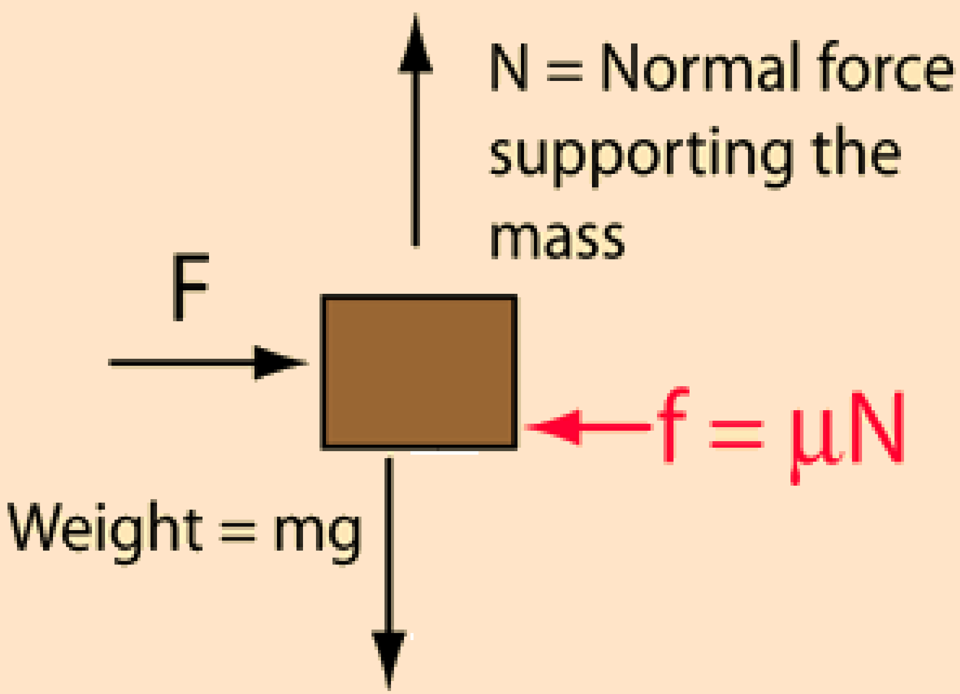
The influences which cause changes in the motion of objects are forces and torques. The effects of forces on objects are described by Newton's Laws. A force may be defined as any influence which tends to change the motion of an object. The relationship between force, mass, and acceleration is given by Newton's Second Law:

$$F_{\text{net external}} = ma$$

Newton's First Law states that an object will continue at rest or in motion in a straight line at constant velocity unless acted upon by an external force. Newton's Third Law states that all forces in nature occur in pairs of forces which are equal in magnitude and opposite in direction.

Free-Body Diagram

A free-body diagram is a sketch of an object of interest with all the surrounding objects stripped away and all of the forces acting on the body shown. The drawing of a free-body diagram is an important step in the solving of mechanics problems since it helps to visualize all the forces acting on a single object. The net external force acting on the object must be obtained in order to apply Newton's Second Law to the motion of the object.

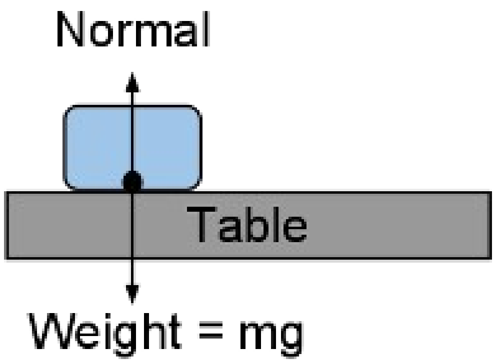


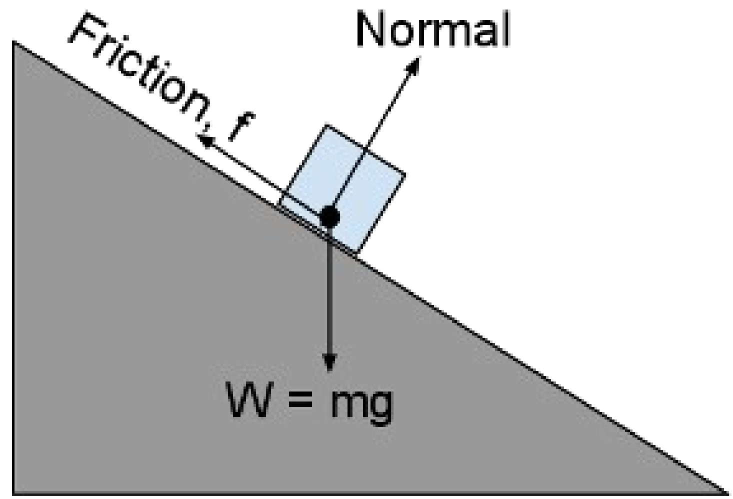
Free Body Diagram

e-body diagram or isolated-body am is useful in problems involving equilibrium of forces.

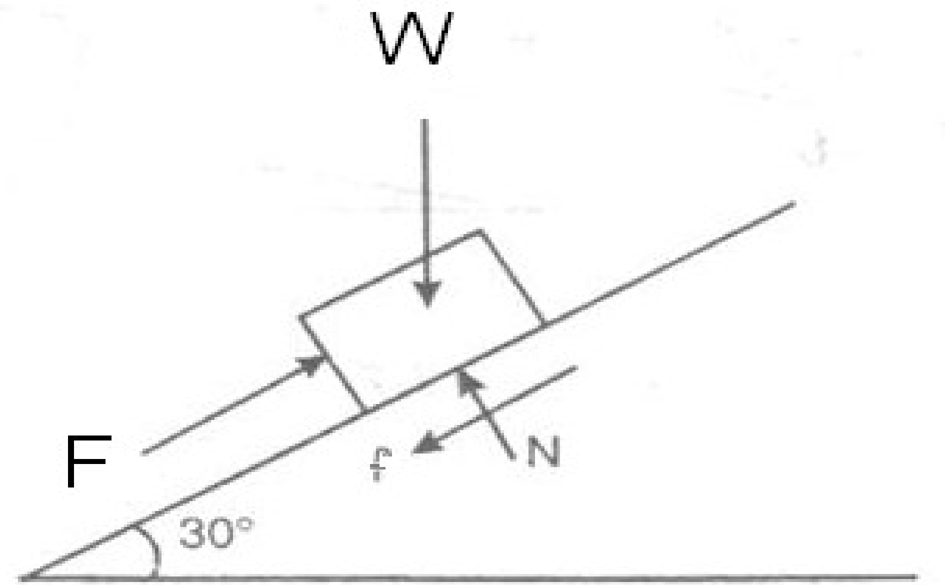
Free-body diagrams are useful for setting up standard mechanics problems.

Free Body Diagrams

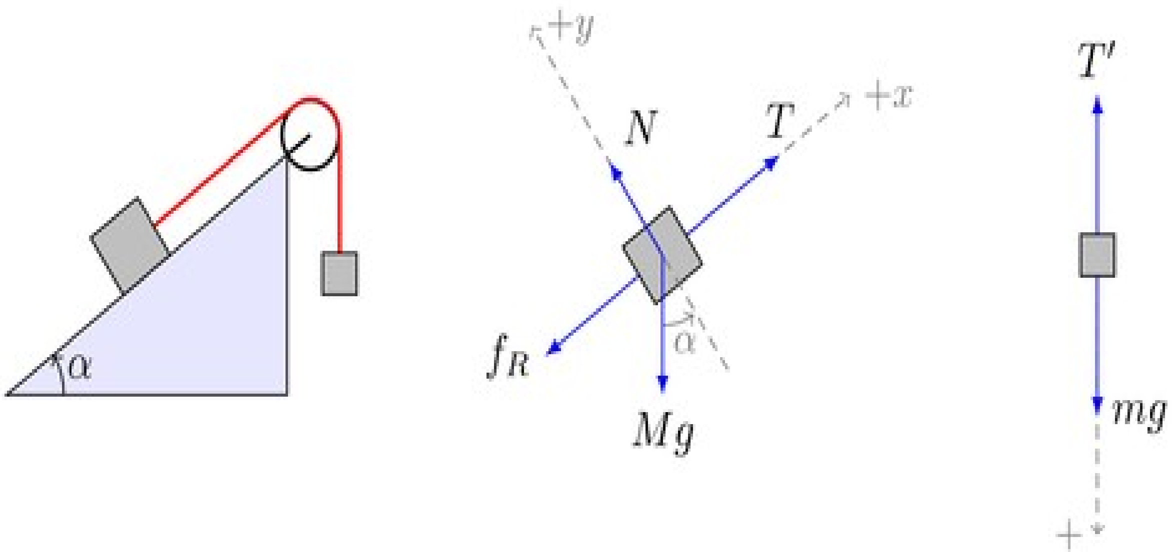




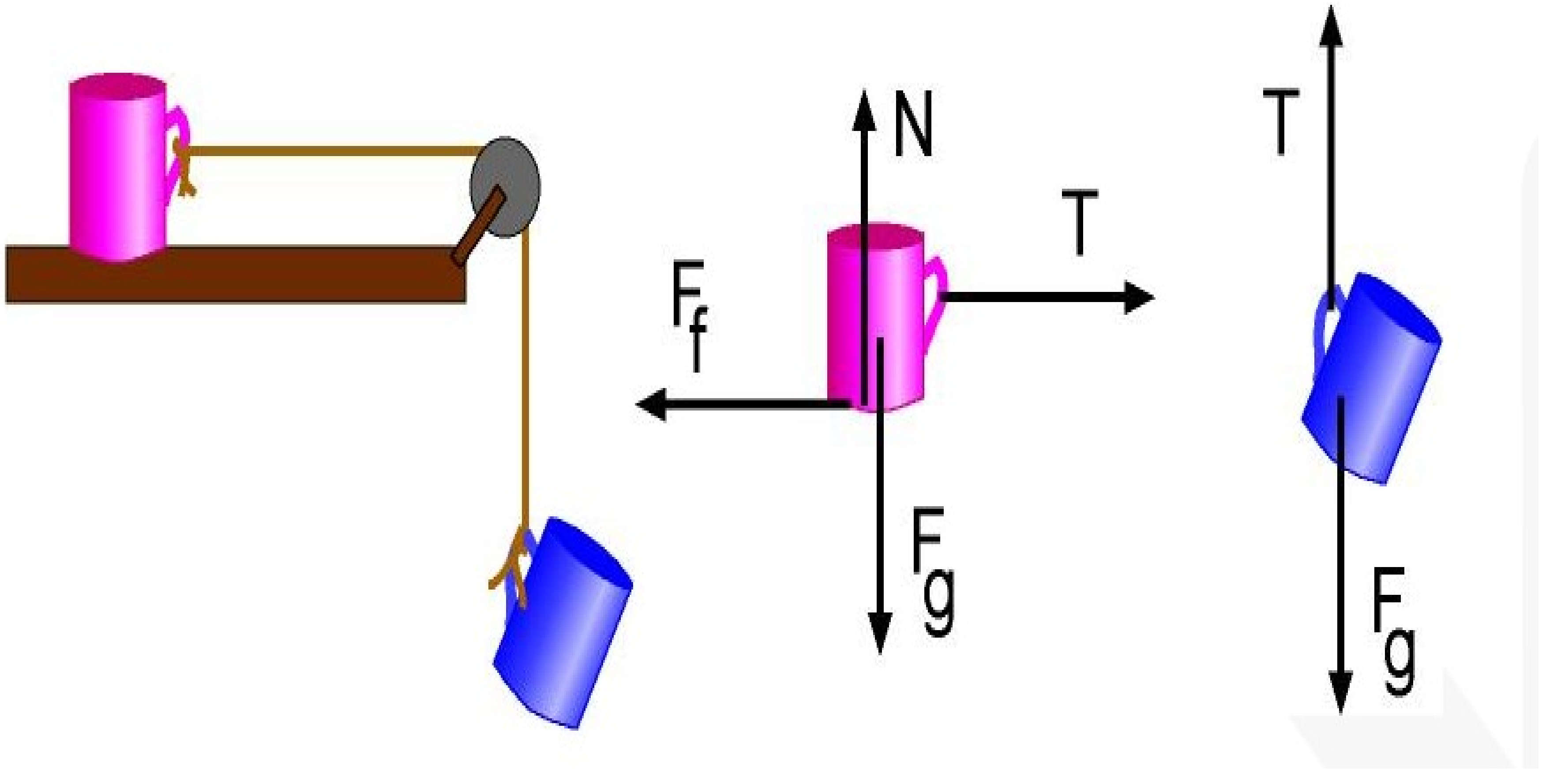
Free body Diagrams



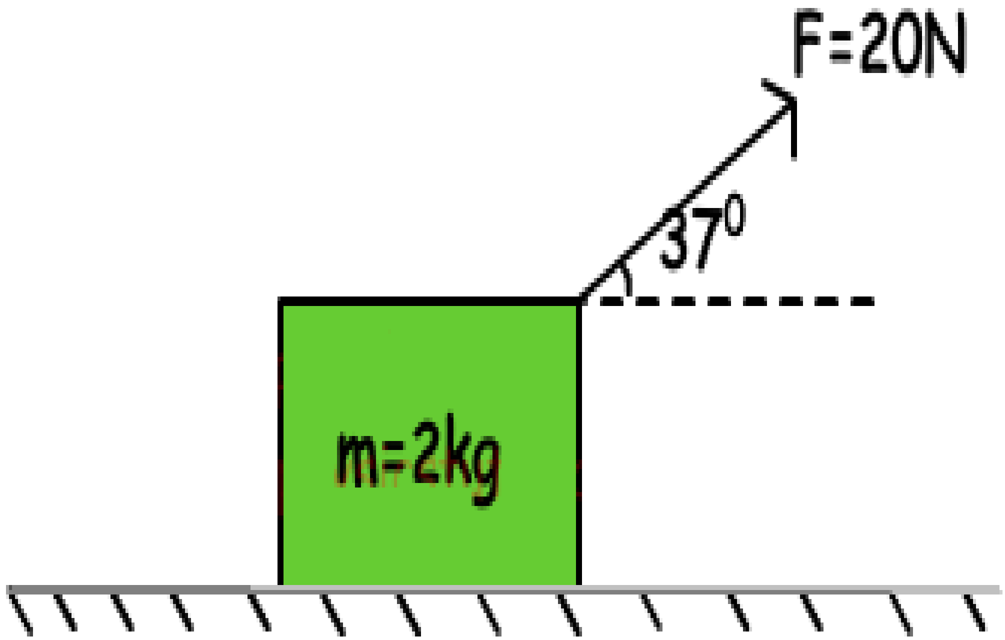
Example: Free body diagrams



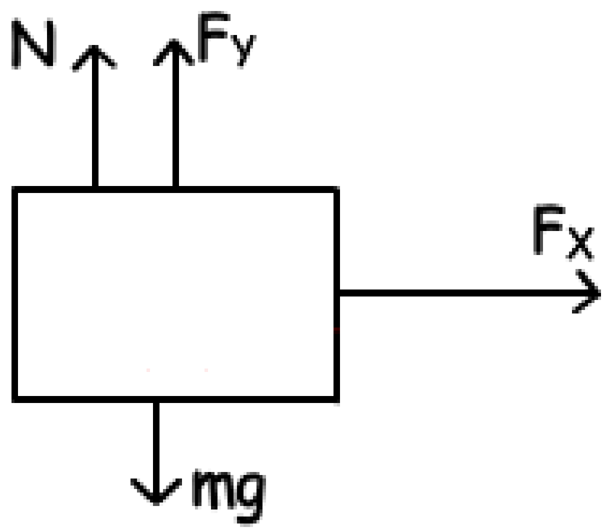
Free Body Diagrams



A box is pulled with 20N force. Mass of the box is 2kg and surface is frictionless. Find the acceleration of the box.



We show the forces acting on the box with following free body diagram.



X component of force gives acceleration to the box.

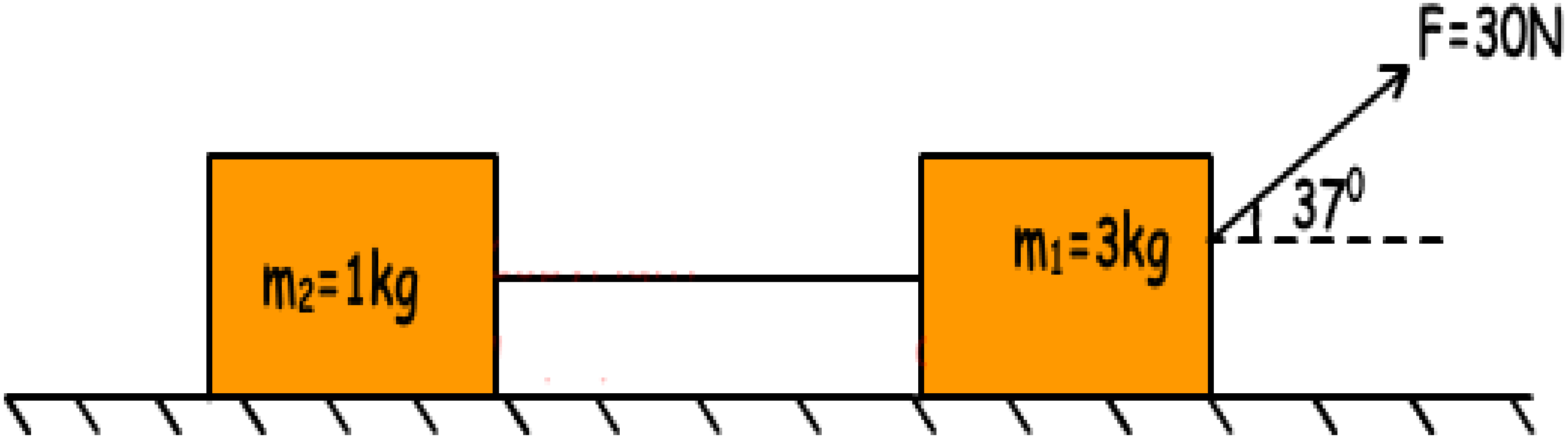
$F_X = F.\cos 37^0 = 20.0,8 = 16N$

$F_X = m.a$

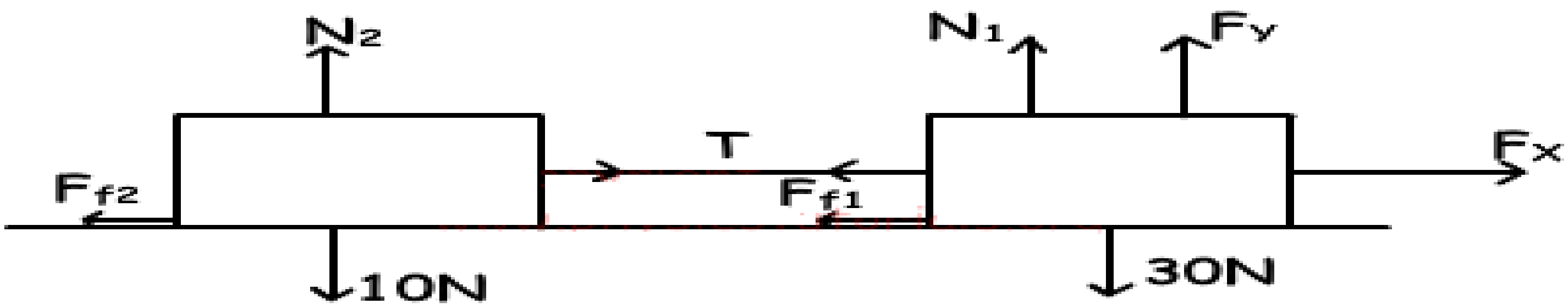
$16N = 2kg.a$

$a = 8m/s$

Picture given below shows the motion of two boxes under the effect of applied force. Friction constant between the surfaces is k=0,4. Find the acceleration of the boxes and tension on the rope. ($g=10 m/s^2$, $\sin 37^0=0,6$, $\cos 37^0=0,8$)



Free body diagram of these boxes given below.



Components of force,

$F_X = F.\cos 37^o = 30.0,8 = 24N$

$F_Y = F.\sin 37^o = 30.0,6 = 18N$

$N_1 = m_1.g - F_y = 30 - 18 = 12N$

$N_2 = 10N$

F_{f1} and F_{f2} are the friction forces acting on boxes.

$F_{f1} = k.N_1 = 0,4.12 = 4,8N$ and $F_{f2} = k.N2 = 0,4.10 = 4N$

We apply Newton's second law on two boxes.

m_1: $F_{net} = m.a$

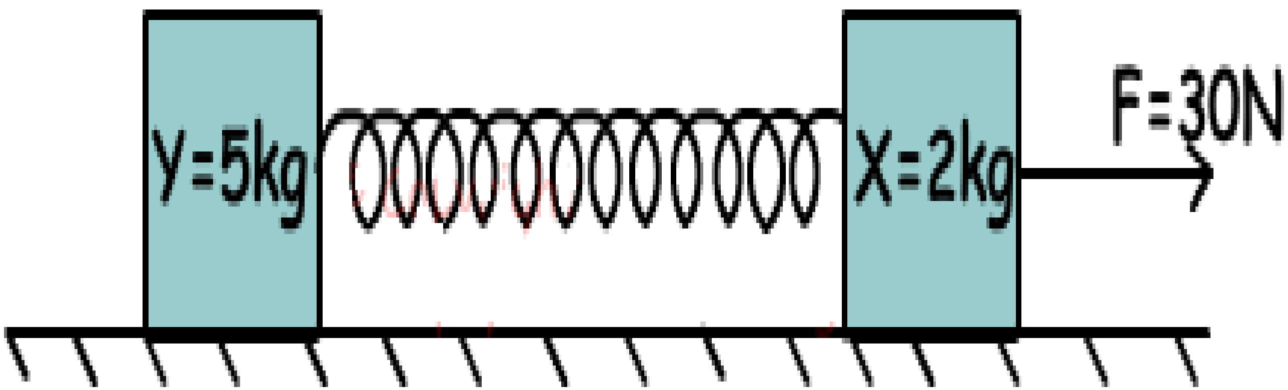
$20 - T - F_{f1} = 3.a$ $\quad 20 - T - 4,8 = 3.a$

m_2: $T - F_{f2} = 1.a$ $\quad T - 4 = a$

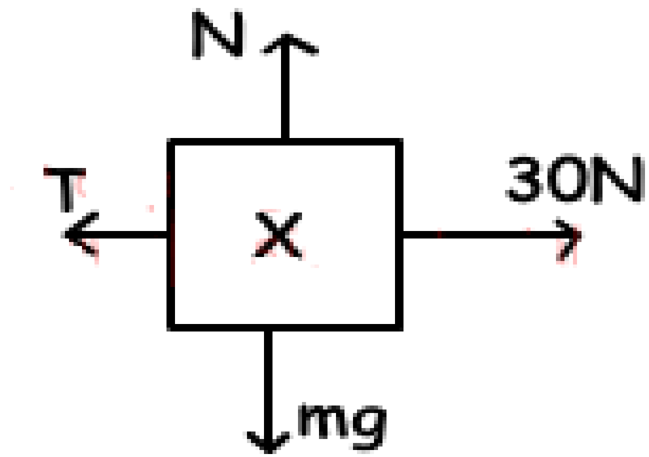
$a = 2,8 m/s^2$

$T = 6,8N$

As you can see in the picture given below, two boxes are placed on a frictionless surface. If the acceleration of the box X is $5 m/s^2$, find the acceleration of the box Y.



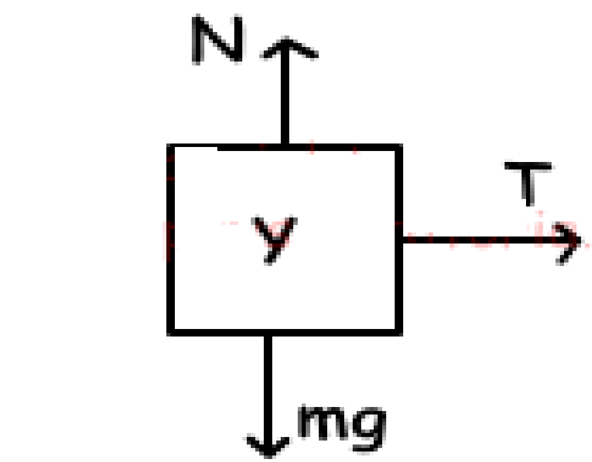
Free body diagrams of boxes are given below;



$F_{net}=m.a$

$(30-T)=2.5$

$T=20N$

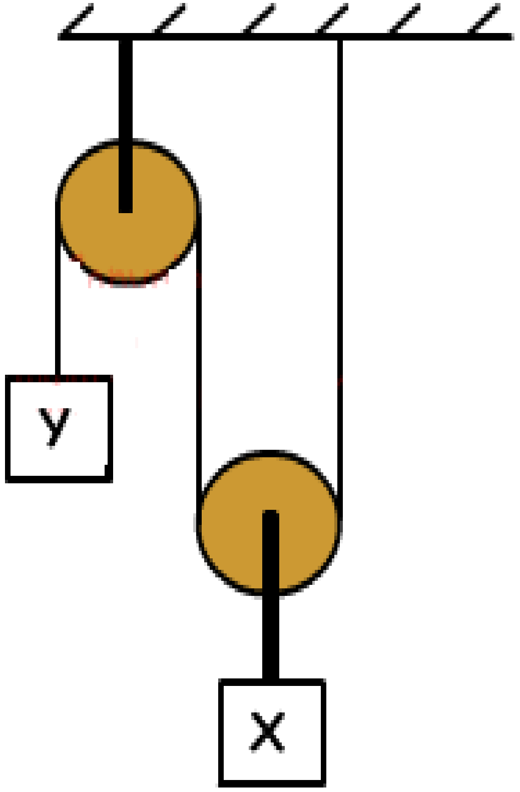


$F_{net}=m.a$

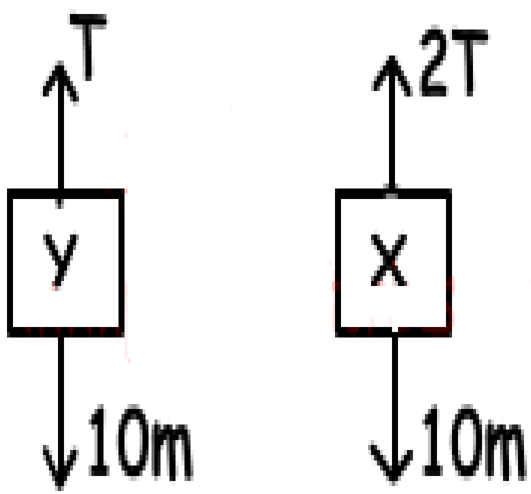
$T=5.a$

$20=5.a \qquad a=4m/s^2$

In the system given below ignore the friction and masses of the pulleys. If masses of X and Y are equal find the acceleration of the X?($g=10m/s^2$)



Free body diagrams of boxes are given below;



Since force acting on X is double of force acting on Y,

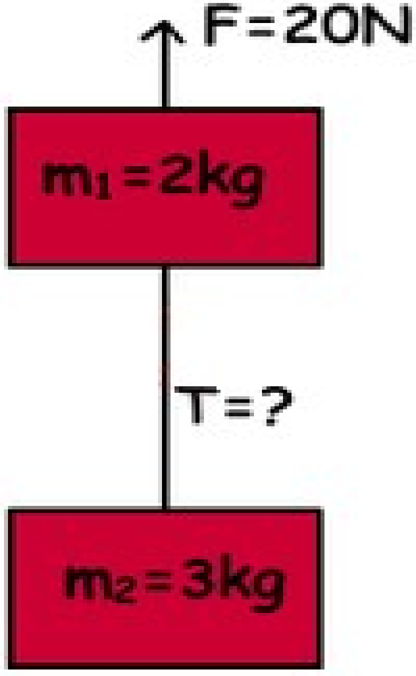
$a_X=2a_Y$

For X: 2T-10m=m.a

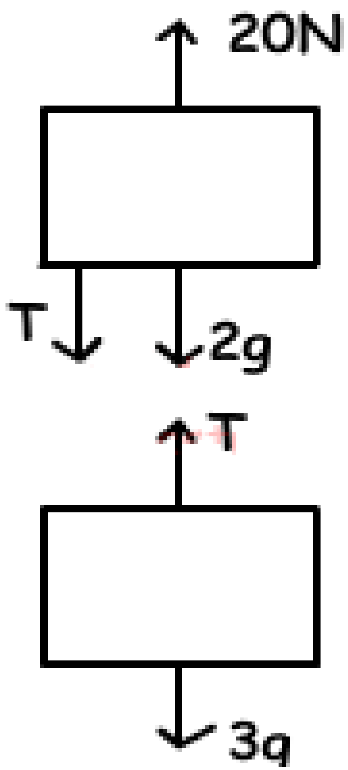
For Y: T-10m=m.2a

$a=2m/s^2$

When system is in motion, find the tension on the rope.



Free body diagrams of boxes are given below.



m_1: T+2g-20=2.a

m_2: 3g-T=3.a

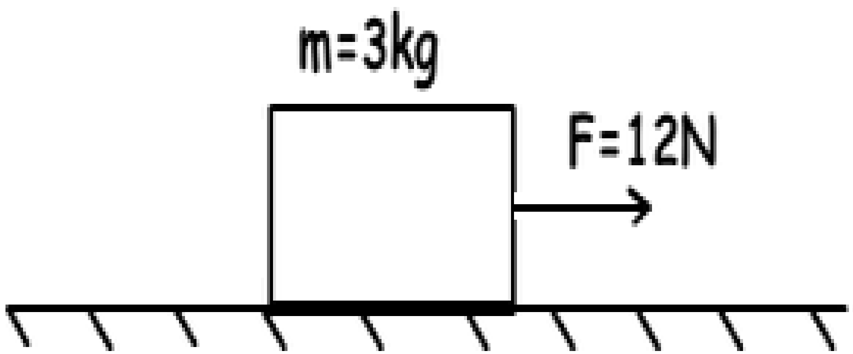
5g-20=5.a

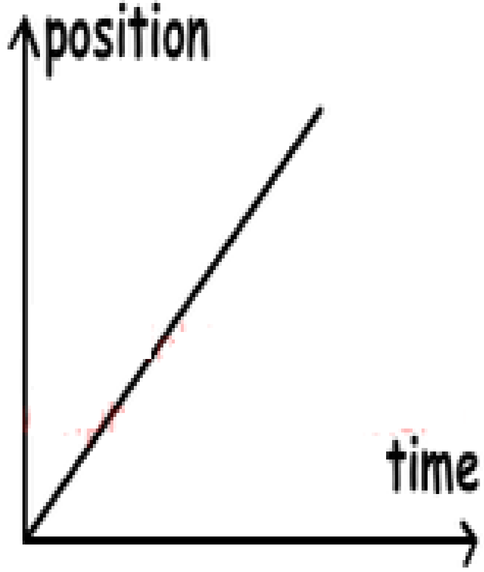
a=g-4 putting it into m_1 equation;

T+2g-20=2(g-4)

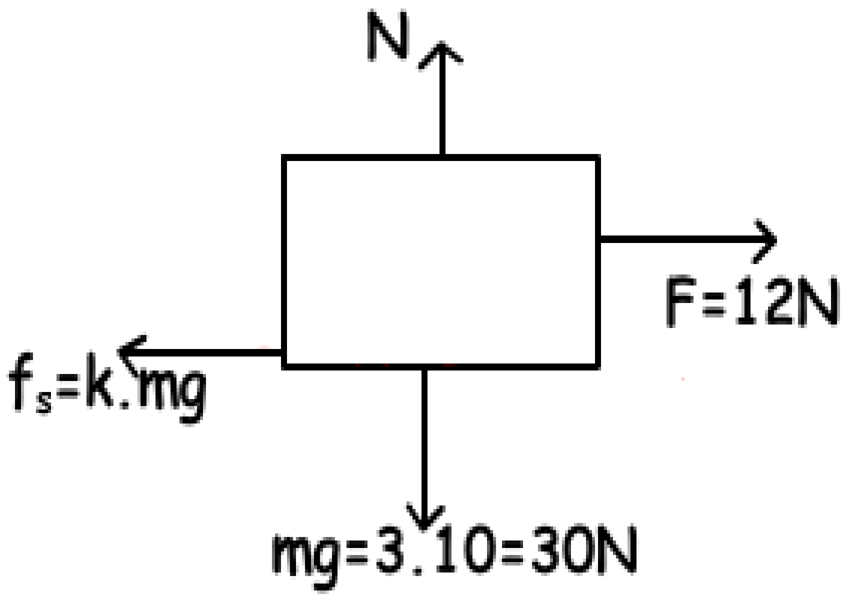
T=12N

Position time graph of the box is given below. Find the friction constant between box and surface? ($g=10m/s^2$)





Slope of the graph gives us velocity of the box. Since the slope of the position time graph is constant, velocity of the box is also constant. As a result, acceleration of the box becomes zero.



$F_{net}=F-f_s=m.a=0$

$F_{net}=f_s$

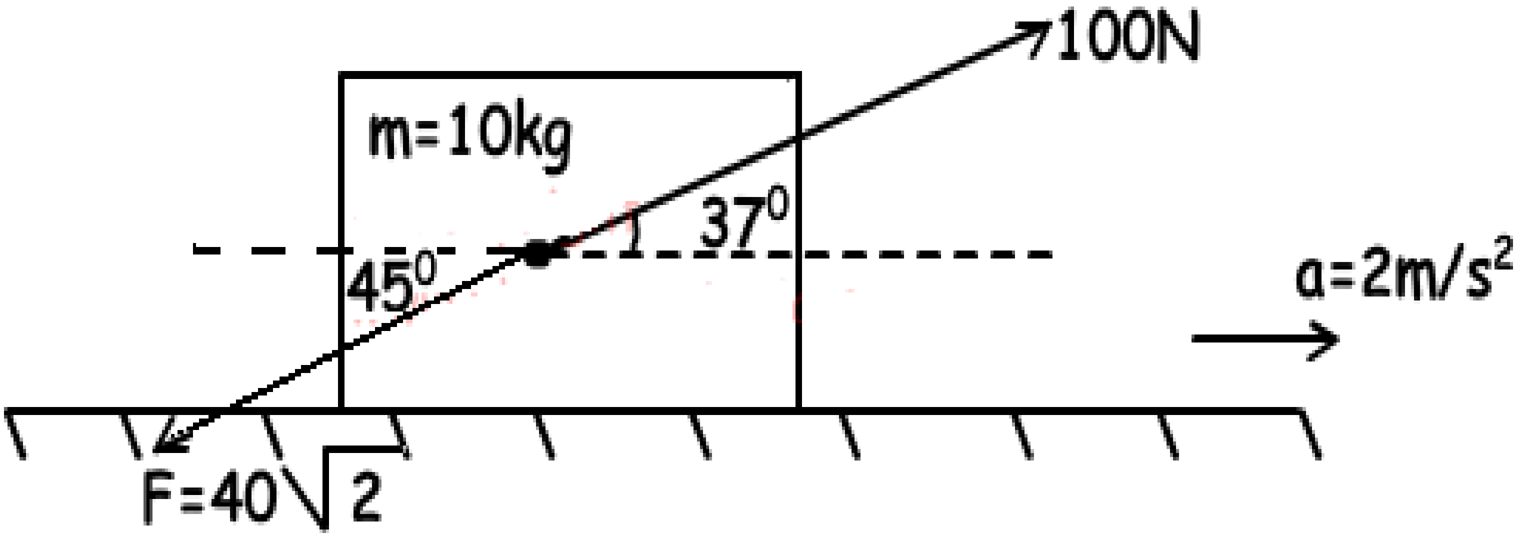
$f_s=12$

$k.mg=12$

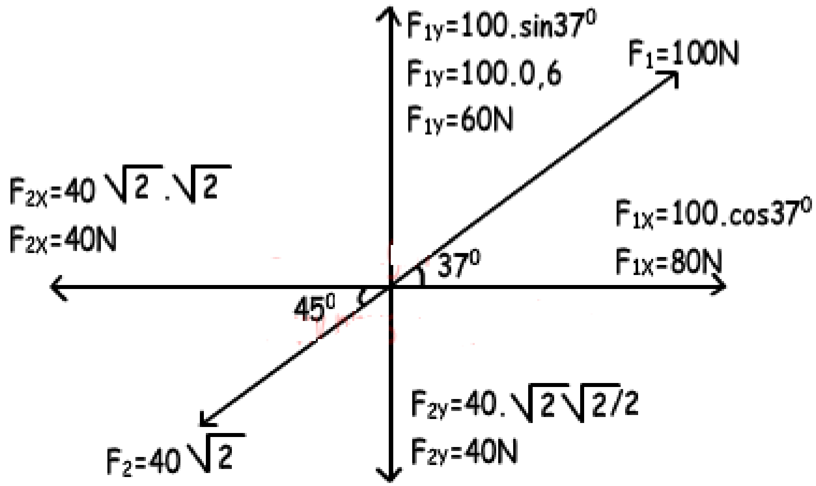
$k.3.10=12$

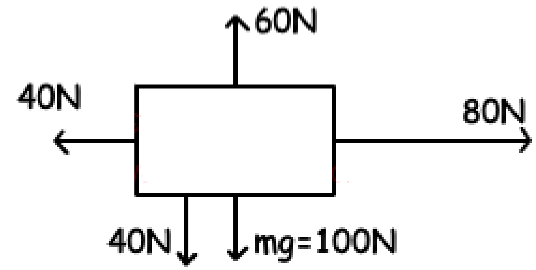
$k=0,4$

If the acceleration of the system given below is 3m/s2, find the friction constant between box and surface. (sin370=0,6, cos370=0,8, sin450=cos450=√2/2)



Free body diagrams of the system are given below.





Acceleration of the 10 kg box is $2 m/s^2$. Thus, net force acting on this box is;

$F_{net}=m.a$

$F_{net}=10.2=20N$

Normal force of the box is;

$N=100+40-60=80N$

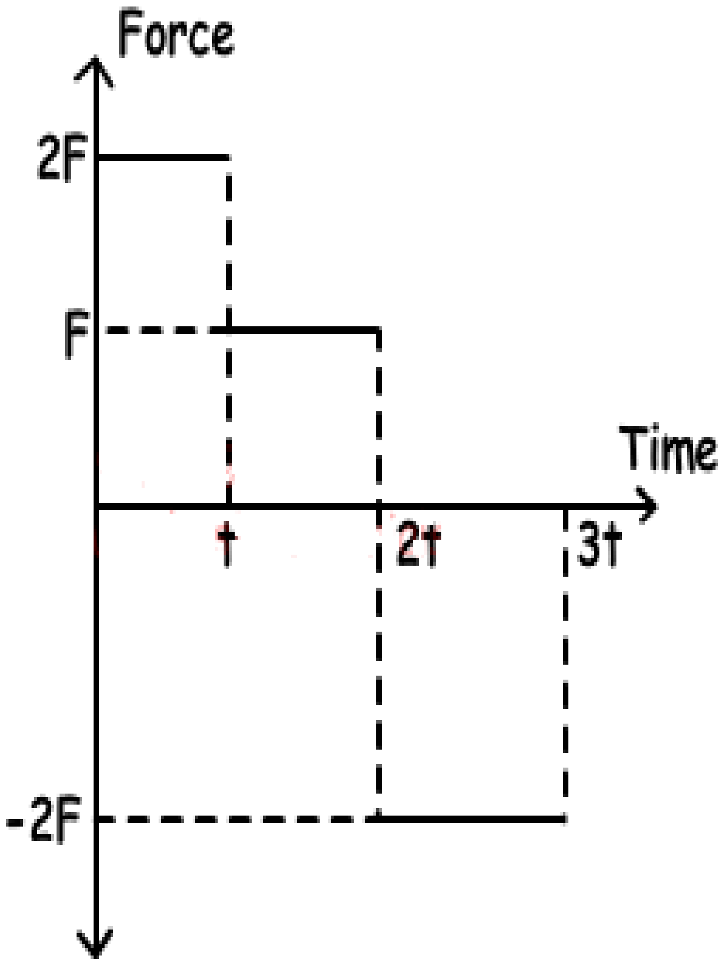
$F_{net}=80-40-F_{friction}$

$20=80-40-k.80$

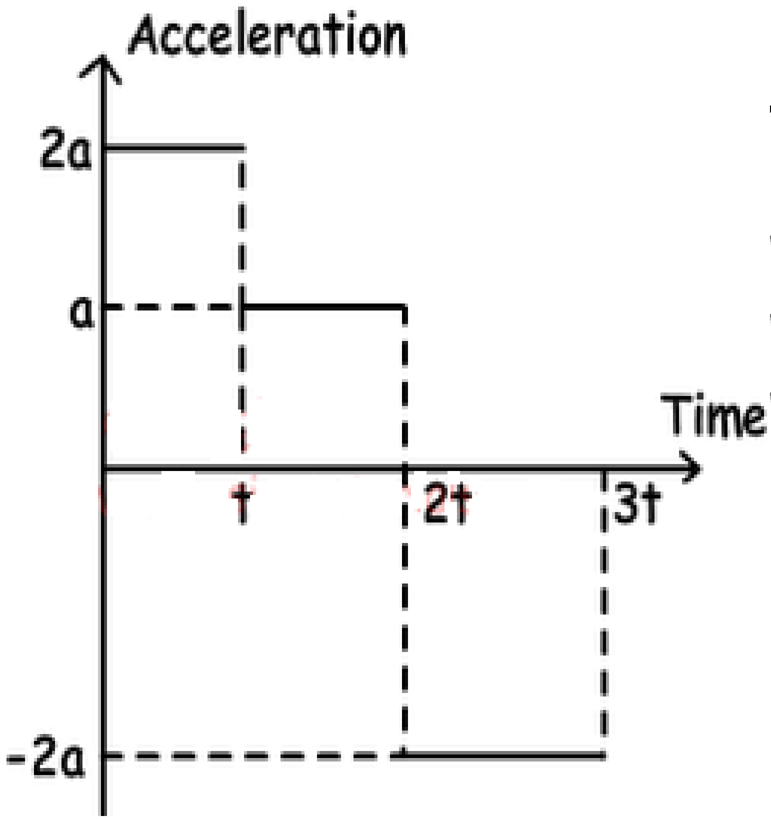
$k.80=20$

$k=1/4$

Net force vs. time graph of object is given below. If displacement of this object between t-2t is 75m, find the displacement of the object between 0-3t.



We draw acceleration vs. time graph using force vs time graph of the object.



Area under the graph gives velocity.

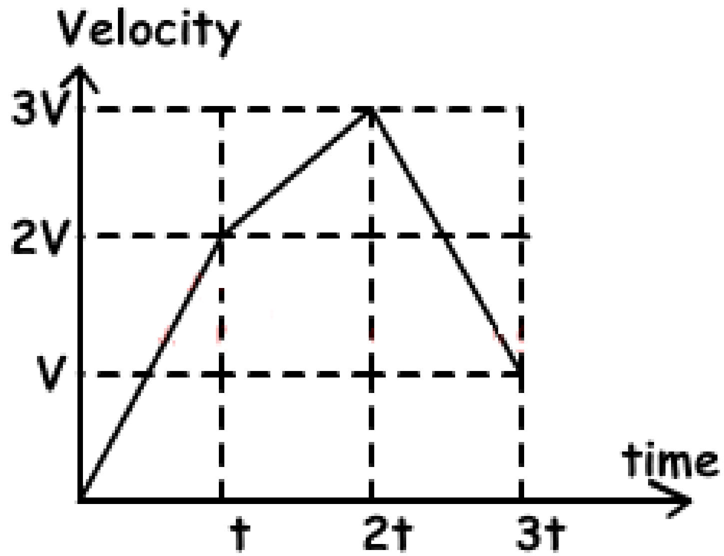
If we say at=V then,

Vt=2V

V2t=3V

V3t=V

We draw velocity vs. time graph now.



Area under the velocity vs. time graph gives us displacement of the object.

0-t: $\Delta X_1=2Vt/2=Vt$

t-2t: $\Delta X_2=5/2.Vt$

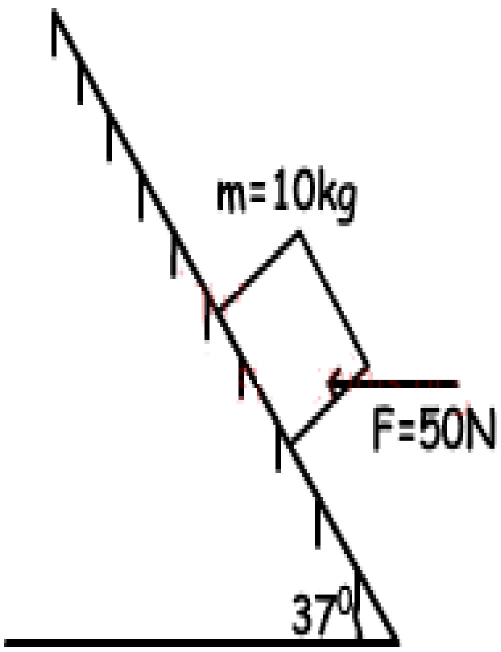
2t-3t: $\Delta X_3= 2.Vt$

We know $\Delta X_2=5/2.Vt=75m$, $Vt=30m$

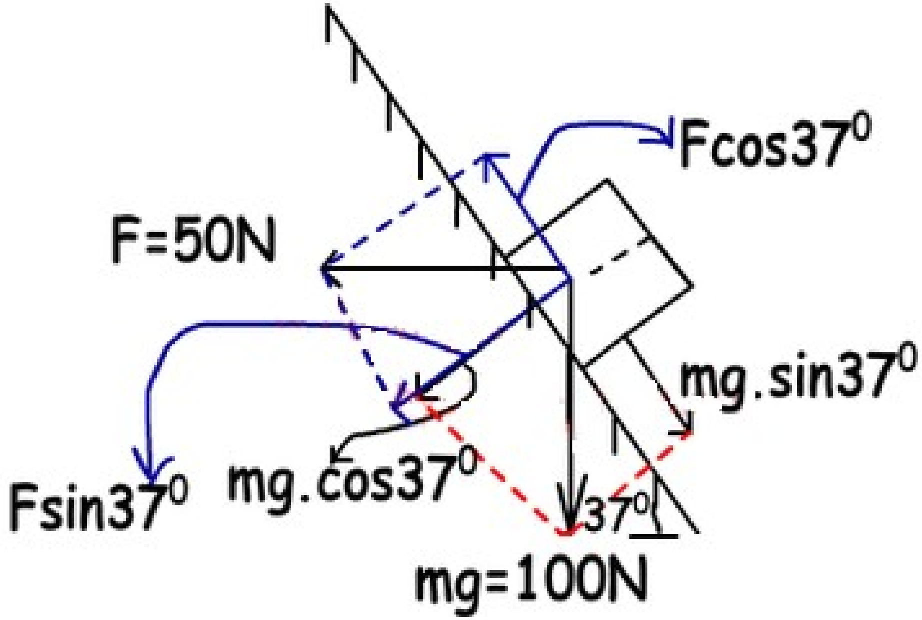
Total displacement=$\Delta X_1+\Delta X_2+\Delta X_3=Vt+5/2.Vt+2Vt$

Total displacement=30+75+2.30=165m

System in the given picture below, box moves under the effect of applied force and gravity with $1 m/s^2$ acceleration. Find the friction constant between the box and surface.



Free body diagram of the system is given below;



Forces acting on the box perpendicularly;

30+80=110N

Box moves downward with 1m/s^2 acceleration.

F_{net}=m.a

60-40-$F_{friction}$=10.1

20-k.110=10

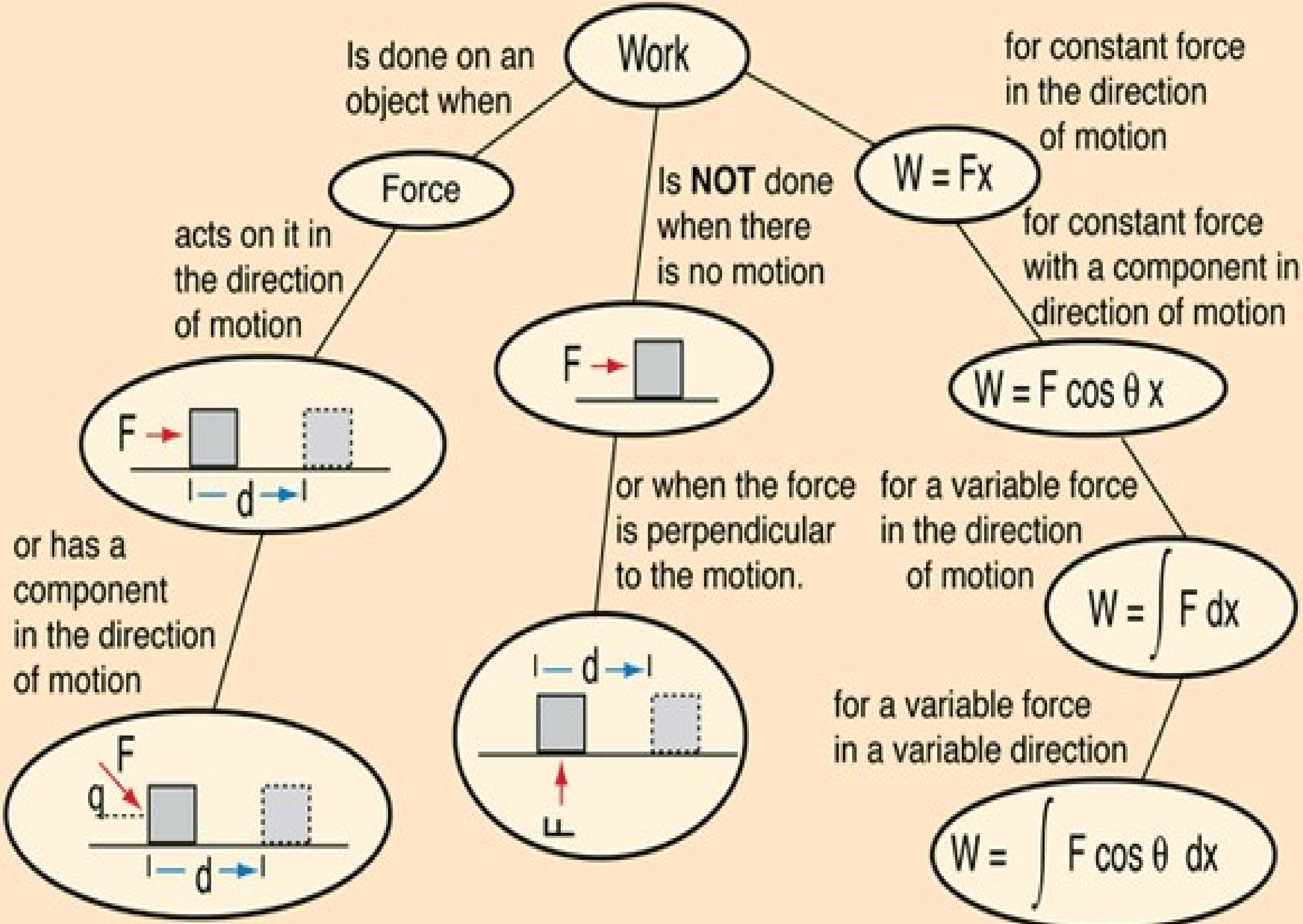
10=110k

k=1/11

WORK, ENERGY AND POWER

Work

refers to an activity involving a force and movement in the directon of the force. A force of 20 newtons pushing an object 5 meters in the direction of the force does 100 joules of work.


Work
Is done on an object when
Force
acts on it in the direction of motion
F
d
or has a component in the direction of motion
F
q
d
Is NOT done when there is no motion
F
or when the force is perpendicular to the motion.
d
F
W = Fx
for constant force in the direction of motion
for constant force with a component in direction of motion
W = F cos θ x
for a variable force in the direction of motion
W = ∫ F dx
for a variable force in a variable direction
W = ∫ F cos θ dx

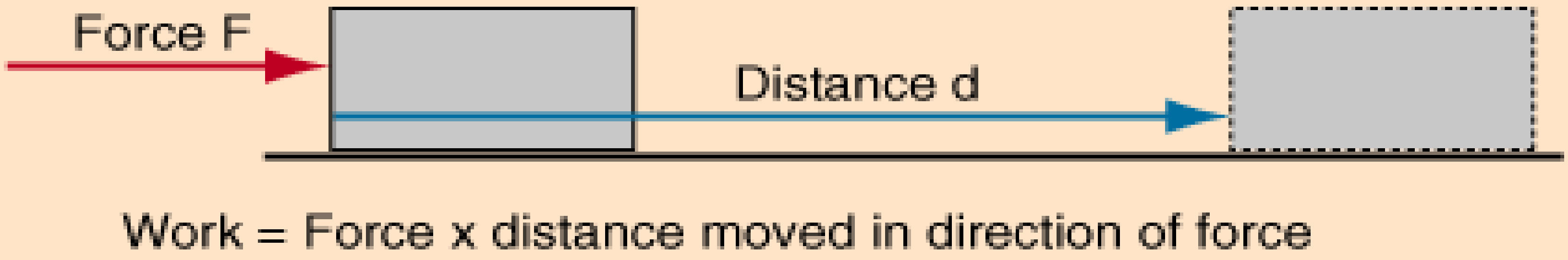
Newton's Second Law

$$F_{\text{net external}} = ma$$

Net force on object = mass of object x acceleration

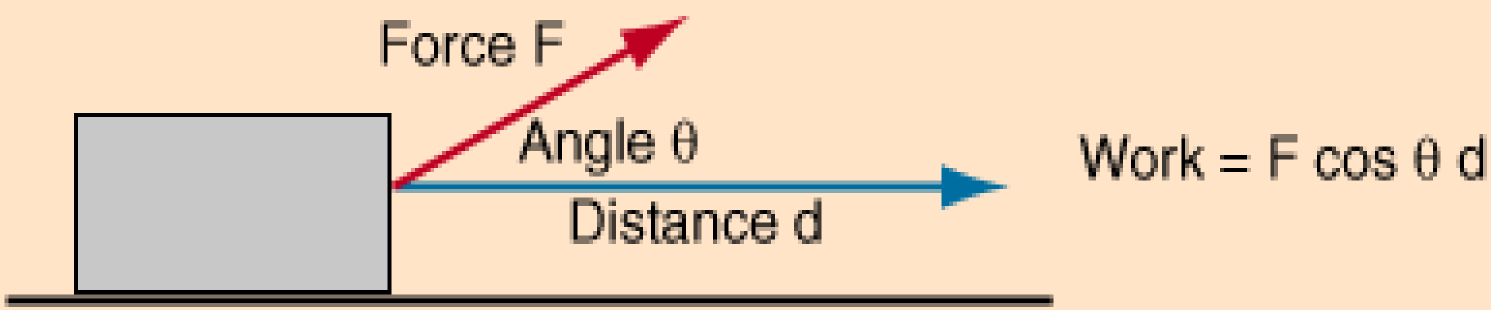
Work Example

In order to accomplish work on an object there must be a force exerted on the object and it must move in the direction of the force.

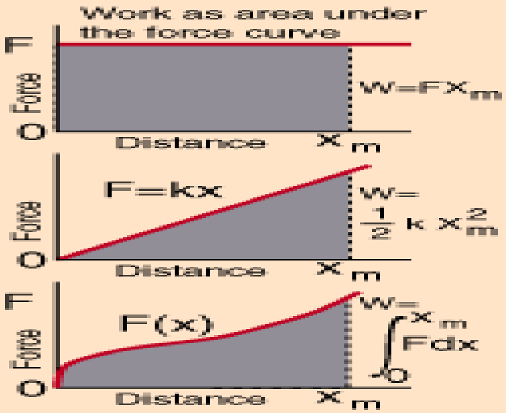


Work Example

In order for a force to do work on an object there must be motion which has a component in the direction of the force.



Work done by a variable force



The basic work relationship W=Fx is a special case which applies only to constant force along a straight line. That relationship gives the area of the rectangle shown, where the force F is ploted as a function of distance. In the more general case of a force which changes with distance, the work may still be calculated as the area under the curve. For example, for the work done to stretch a spring, the area under the curve can be readily determined as the area of the triangle. The power of calculus can also be applied since the integral of the force over the distance range is equal to the area under the force curve:

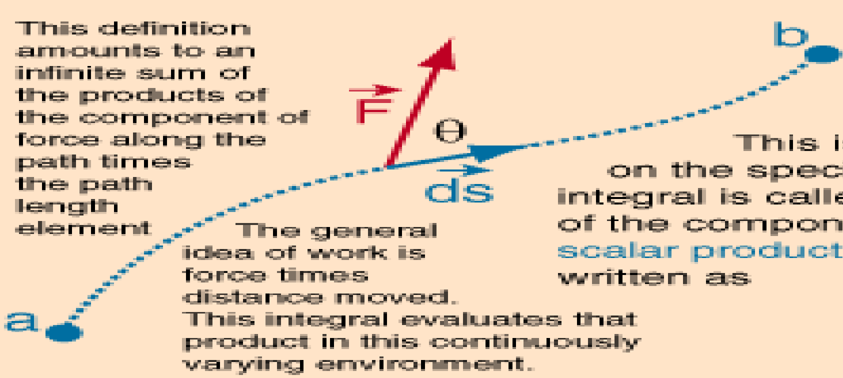
$$\text{Work} = \int_0^{x_m} F(x)\,dx = \int_0^{x_m} kx\,dx = \tfrac{1}{2}kx_m^2$$

For any function of x, the work may be calculated as the area under the curve by performing the integral

$$\text{Work} = \int_{x_1}^{x_2} F(x)\,dx$$

Work:General Definition

The general definition of work done by a force must take into account the fact that the force may vary in both magnitude and direction, and that the path followed may also change in direction. All these things can be taken into account by defining work as an integral.



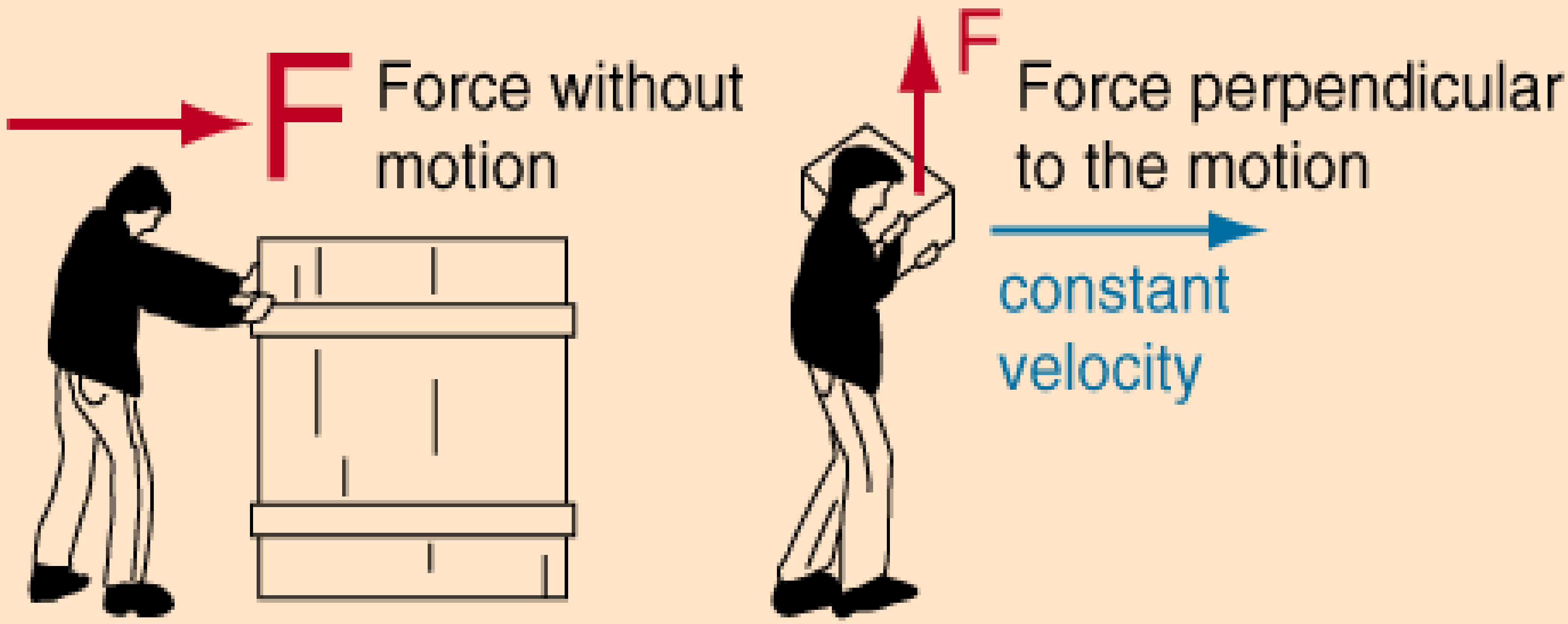
$$W_{ab} = \int_a^b \vec{F}\cdot\vec{ds}$$

This is taken to represent the work done by the force on the specified path between points a and b. This type of integral is called a line integral because it evaluates the integral of the component of F along a line. The integrand is a scalar product of the vector elements and it can also be written as

$$W_{ab} = \int_a^b F\cos\theta\, ds$$

When a force does no work

A force with no motion or a force perpendicular to the motion does no work



When a force is exerted on an object which does not move, no work is done on the object.

When an object is carried at constant velocity by a force which acts at right angles to the motion, no work is done on the object.

Energy

is the capacity for doing work. You must have energy to accomplish work - it is like the "currency" for performing work. To do 100 joules of work, you must expend 100 joules of energy.

Energy Forms

Energy can be defined as the capacity for doing work, but that capacity can reside in many different forms. Broad forms such as mechanical, electrical, chemical, nuclear, solar can be envisioned.

Large amounts of energy exist in the form of internal energy within objects at normal temperatures. As several of the energy examples point out, the processes of heating and cooling are much more energy intensive than purely mechanical processes.

Almost any process in nature can be viewed as some kind of energy transfer process. While it is not practical to try to categorize all the kinds of energy transfer processes, we can state that none of them involve any net gain or loss of energy. The principal of conservation of energy constrains the kind of processes which can occur in nature.

Energy Unit Comparison

One of the difficulties of reading articles about energy resources is the plethora of units used. This is an attempt to give a comparison of some commonly quoted energy units. You may enter a number in any of the boxes to calculate the comparisons in other units.

Joules	10	x 10^	1
.....ft-lb	0.73746312€	x 10^	2
..........calorie	2.38891543‹	x 10^	1
...............BTU	0.94781698‹	x 10^	-1
....................kWh	2.77777777‹	x 10^	-5
.........................gallons of gasoline	0.76923076‹	x 10^	-6
..............................1000 cu ft of natural gas	0.90909090‹	x 10^	-7
...................................barrel of oil	1.69491525‹	x 10^	-8
....ton of coal	0.38461538‹	x 10^	-8

The energy values for the common fuels are nominal values since the energy content of these fuels varies with the source.

Kinetic Energy

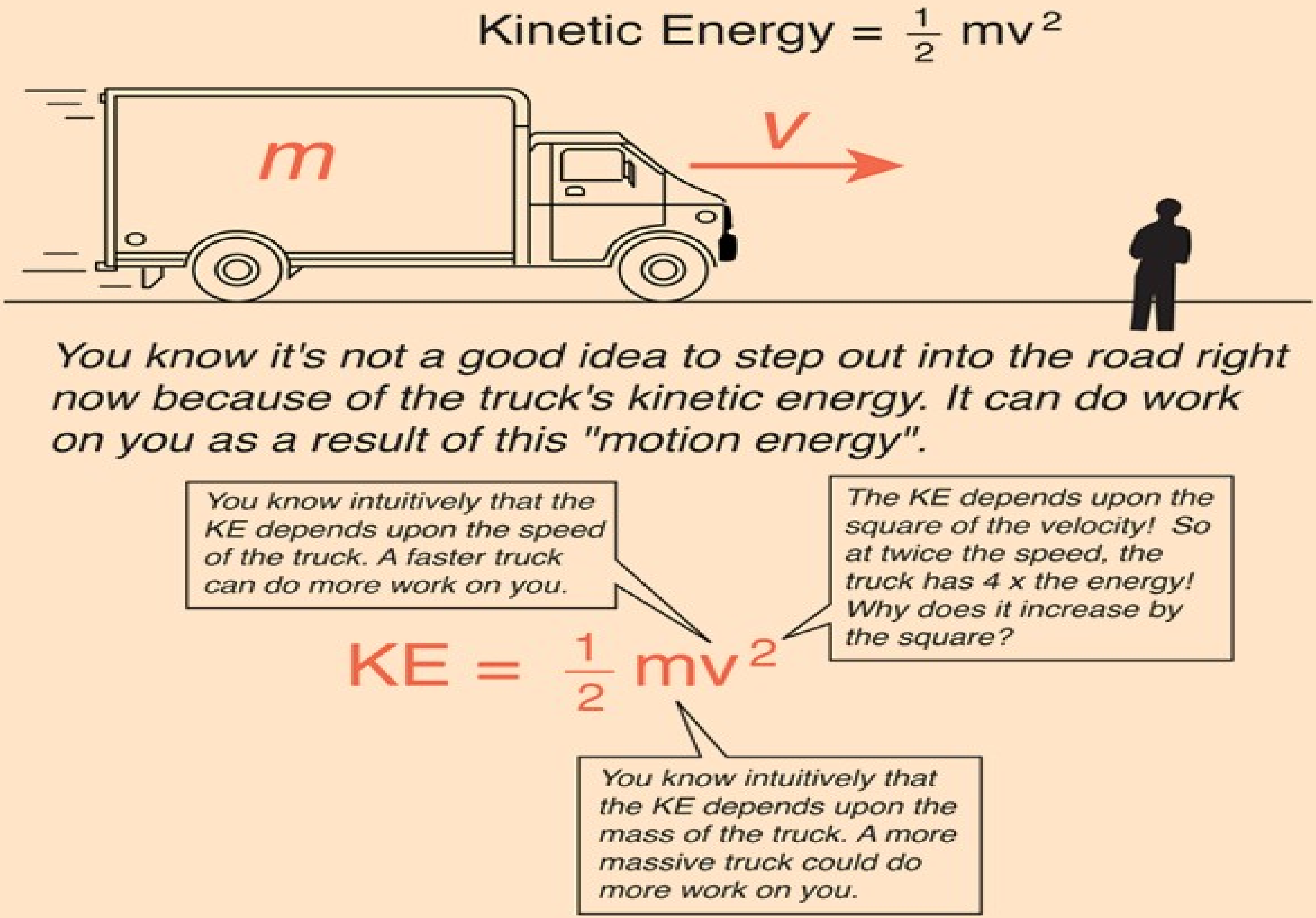
Kinetic energy is energy of motion. The kinetic energy of an object is the energy it possesses because of its motion. The kinetic energy* of a point mass m is given by

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

Kinetic energy is an expression of the fact that a moving object can do work on anything it hits; it quantifies the amount of work the object could do as a result of its motion. The total mechanical energy of an object is the sum of its kinetic energy and potential energy. The total energy of an isolated system is subject to the conservation of energy principle.

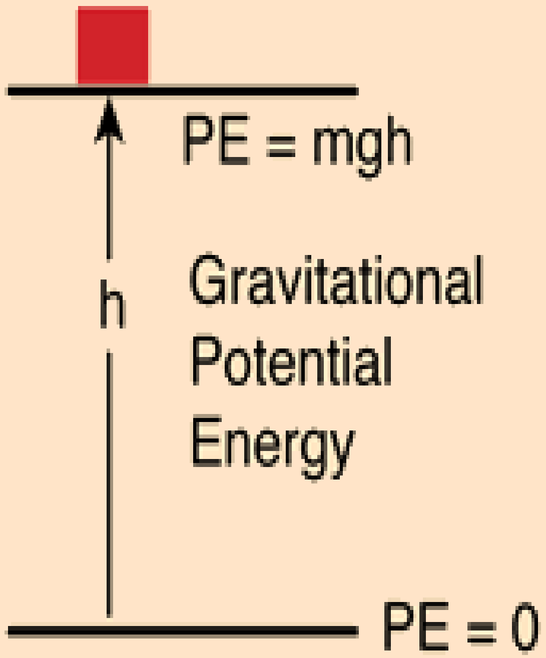
For an object of finite size, this kinetic energy is called the translational kinetic energy of the mass to distinguish it from any rotational kinetic energy it might possess - the total kinetic energy of a mass can be expressed as the sum of the translational kinetic energy of its center of mass plus the kinetic energy of rotation about its center of mass.

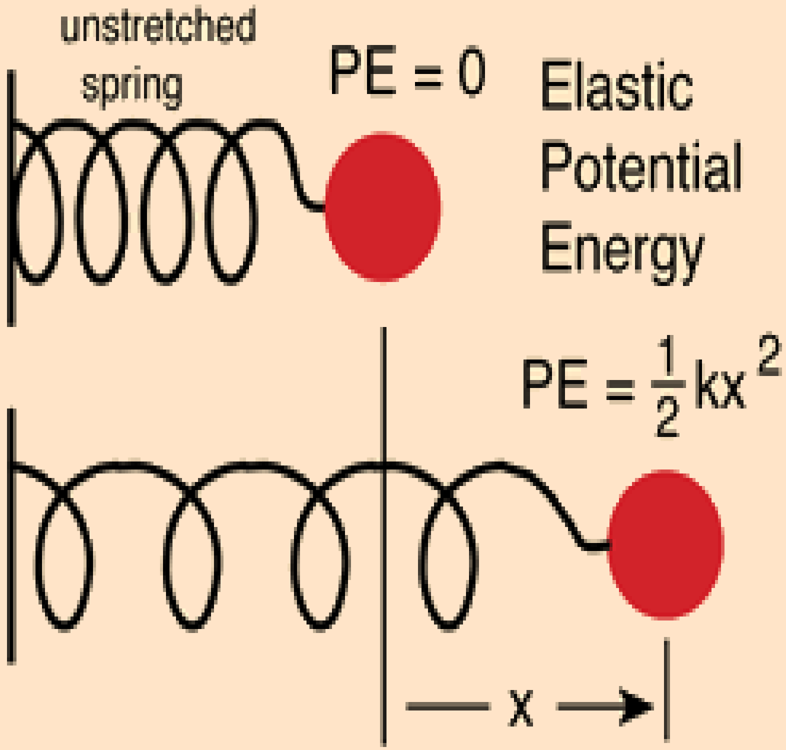
Kinetic energy is energy of motion. The kinetic energy of an object is the energy it possesses because of its motion. The kinetic energy of a point mass m is given by



Potential Energy

Potential energy is energy which results from position or configuration. An object may have the capacity for doing work as a result of its position in a gravitational field (gravitational potential energy), an electric field (electric potential energy), or a magnetic field (magnetic potential energy). It may have elastic potential energy as a result of a stretched spring or other elastic deformation.





Gravitational Potential Energy

Gravitational potential energy is energy an object possesses because of its position in a gravitational field. The most common use of gravitational potential energy is for an object near the surface of the Earth where the gravitational acceleration can be assumed to be constant at about 9.8 m/s^2. Since the zero of gravitational potential energy can be chosen at any point (like the choice of the zero of a coordinate system), the potential energy at a height h above that point is equal to the work which would be required to lift the object to that height with no net change in kinetic energy. Since the force required to lift it is equal to its weight, it follows that the gravitational potential energy is equal to its weight times the height to which it is lifted.

$$PE_{gravitational} = \text{weight} \times \text{height} = mgh$$

Elastic Potential Energy

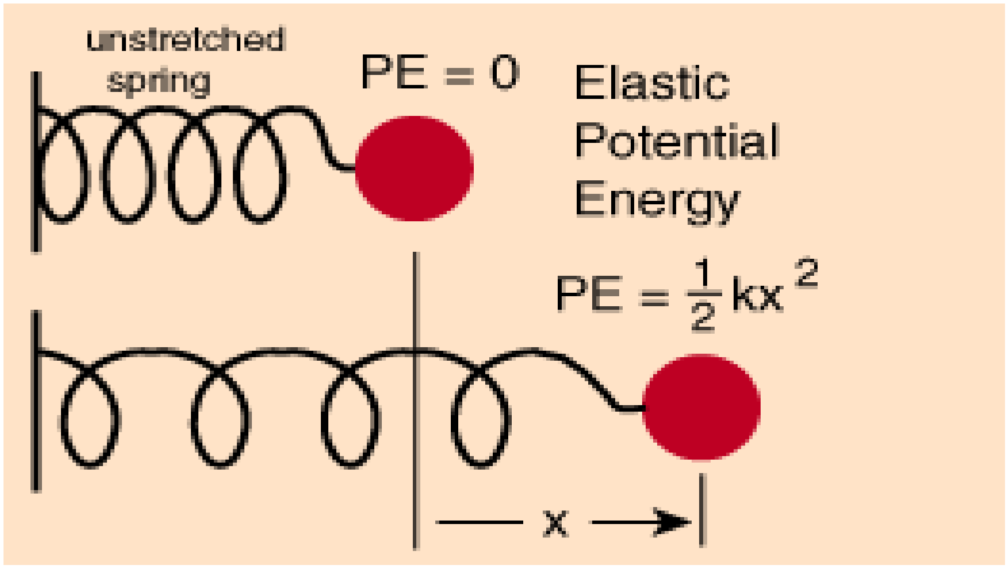
Elastic potential energy is Potential energy stored as a result of deformation of an elastic object, such as the stretching of a spring. It is equal to the work done to stretch the spring, which depends upon the spring constant k as well as the distance stretched. According to Hooke's law, the force required to stretch the spring will be directly proportional to the amount of stretch.

Since the force has the form

$$F = -kx$$

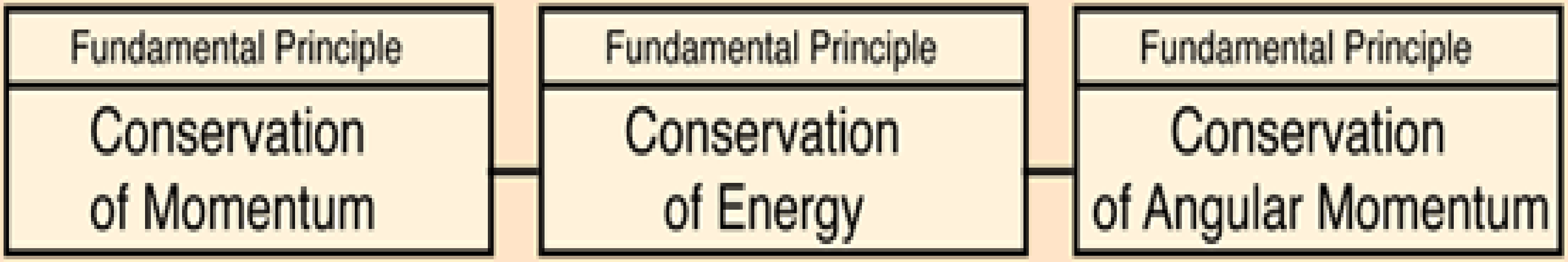
then the work done to stretch the spring a distance x is

$$\text{Work} = \mathbf{PE} = \frac{1}{2}\mathbf{kx}^2$$



Conservation Laws

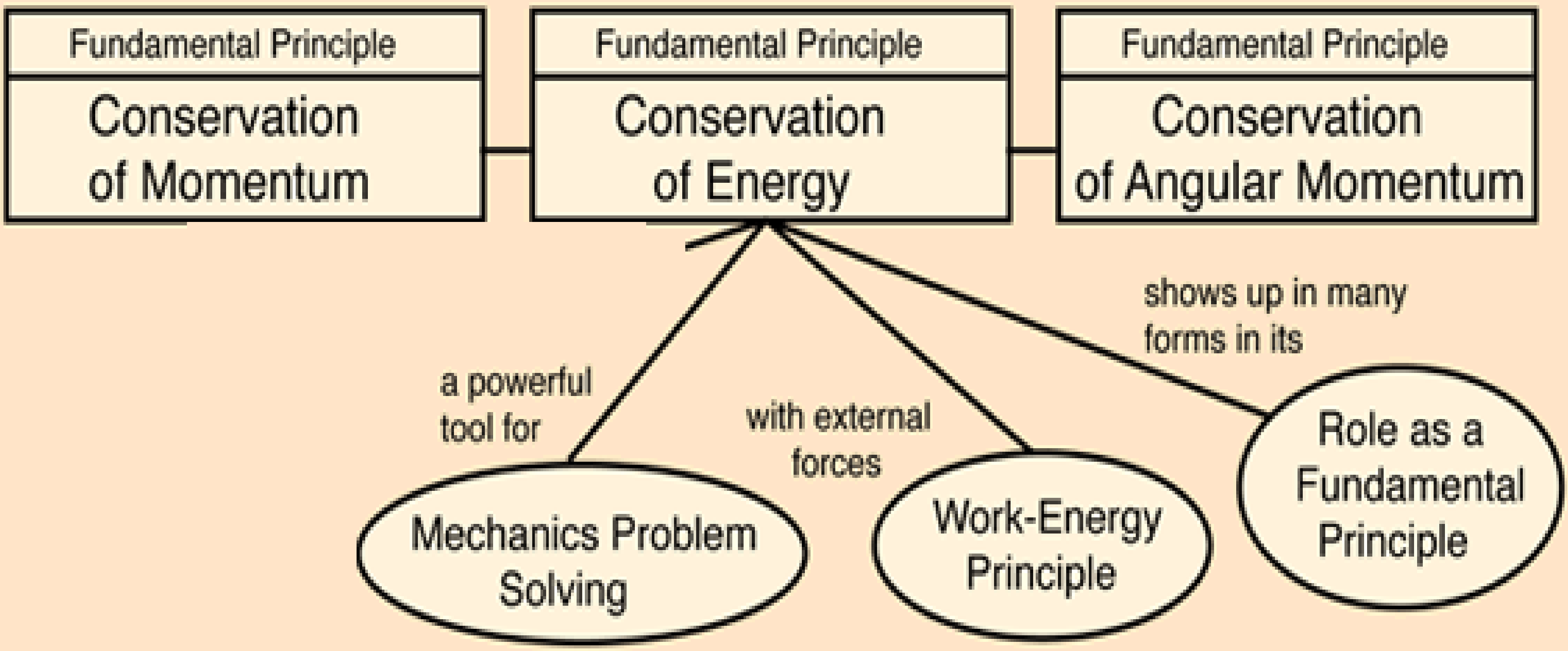
If a system does not interact with its environment in any way, then certain mechanical properties of the system cannot change. They are sometimes called "constants of the motion". These quantities are said to be "conserved" and the conservation laws which result can be considered to be the most fundamental principles of mechanics. In mechanics, examples of conserved quantities are energy, momentum, and angular momentum. The conservation laws are exact for an isolated system.



Stated here as principles of mechanics, these conservation laws have far-reaching implications as symmetries of nature which we do not see violated. They serve as a strong constraint on any theory in any branch of science.

Conservation of Energy

Energy can be defined as the capacity for doing work. It may exist in a variety of forms and may be transformed from one type of energy to another. However, these energy transformations are constrained by a fundamental principle, the Conservation of Energy principle. One way to state this principle is "Energy can neither be created nor destroyed". Another approach is to say that the total energy of an isolated system remains constant.



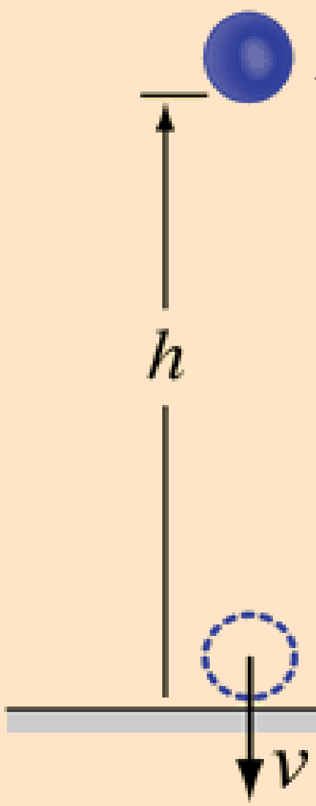
Energy as a tool for mechanics problem solving

The application of the conservation of energy principle provides a powerful tool for problem solving. Newton's laws are used for the solution of many standard problems, but often there are methods using energy which are more straightforward. For example, the solution for the impact velocity of a falling object is much easier by energy methods. The basic reason for the advantage of the energy approach is that just the beginning and ending energies need be considered; intermediate processes do not need to be examined in detail since conservation of energy gaurantees that the final energy of the system is the same as the initial energy.

The work-energy principle is also a useful approach to the use of conservation of energy in mechanics problem solving. It is particularly useful in cases where an object is brought to rest as in a car crash or the normal stopping of an automobile.

Object Falling from Rest

As an object falls from rest, its gravitational potential energy is converted to kinetic energy. Conservation of energy as a tool permits the calculation of the velocity just before it hits the surface.



$PE = mgh$

$KE = 0$

By conservation of energy:

Energy before = Energy after

$$mgh = \frac{1}{2}mv^2$$

The beginning energy is all potential energy.

The m on both sides tells you that the final velocity doesn't depend upon the mass.

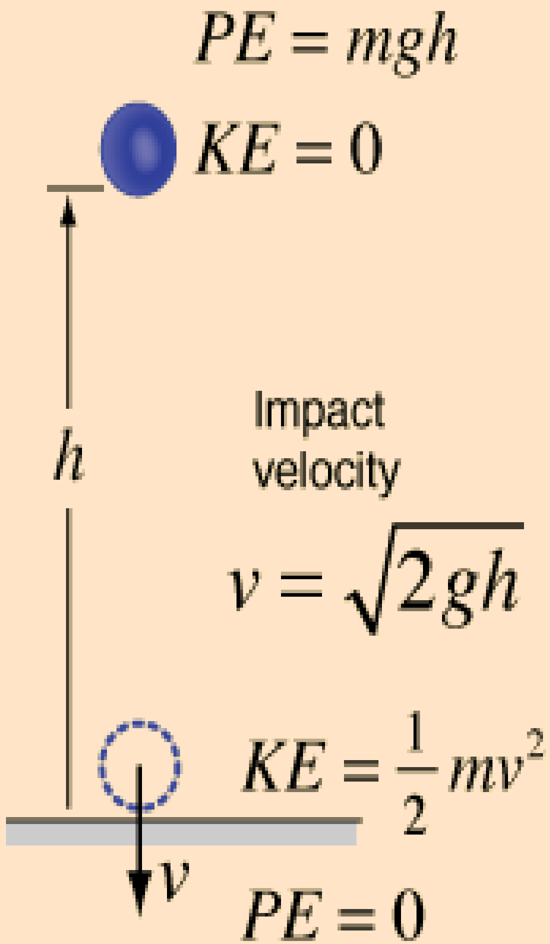
The final energy is all kinetic energy.

The velocity just before impact is $v = \sqrt{2gh}$

$KE = \frac{1}{2}mv^2$

$PE = 0$

Even though the application of conservation of energy to a falling object allows us to predict its impact velocity and kinetic energy, we cannot predict its impact force without knowing how far it travels after impact.



Work-Energy Principle

$$W_{net} = \frac{1}{2} mv^2_{final} - \frac{1}{2} mv^2_{initial}$$

The change in the kinetic energy of an object is equal to the net work done on the object.

This fact is referred to as the Work-Energy Principle and is often a very useful tool in mechanics problem solving. It is derivable from conservation of energy and the application of the relationships for work and energy, so it is not independent of the conservation laws. It is in fact a specific application of conservation of energy. However, there are so many mechanical problems which are solved efficiently by applying this principle that it merits separate attention as a working principle.

For a straight-line collision, the net work done is equal to the average force of impact times the distance traveled during the impact.

Average impact force x distance traveled = change in kinetic energy

Conservation of Energy as a Fundamental Principle

The conservation of energy principle is one of the foundation principles of all science disciplines. In varied areas of science there will be primary equations which can be seen to be just an appropriate reformulation of the principle of conservation of energy.

Fluids	**Bernoulli equation**
Electric circuits	**Voltage law**
Heat and thermodynamics	**First law of thermodyamics**

Power

is the rate of doing work or the rate of using energy, which are numerically the same. If you do 100 joules of work in one second (using 100 joules of energy), the power is 100 watts.

Power Calculation

Work = Force x distance moved in direction of force

$$P_{avg} = \frac{\text{Work}}{\text{time}} = \frac{\text{Force x distance}}{\text{time}}$$

Special case for constant force acting in the direction of motion.

Power

Power may be defined as the rate of doing work or the rate of using energy. These two definitions are equivalent since one unit of energy must be used to do one unit of work. Often it is convenient to calculate the average power.

$$P_{avg} = \frac{Work}{time} = \frac{F \cos \theta \, x}{t}$$

This can be rearranged in the form:

$$P_{avg} = F \cos \theta \, v_{avg}$$

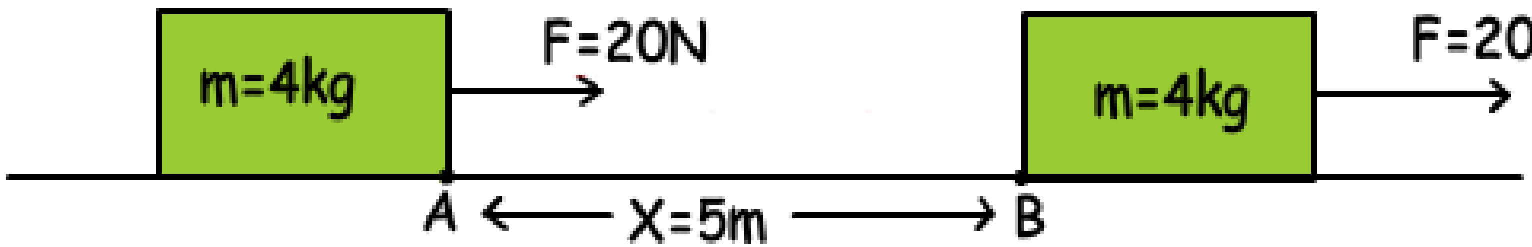
It turns out that this is a general form and that instantaneous power can be calculated from the expression:

$$P_{instantaneous} = F \cos \theta \, v$$

which in vector notation is the scalar product: $P = \vec{F} \cdot \vec{v}$

In the straightforward cases where a constant force moves an object at constant velocity, the power is just P = Fv. In a more general case where the velocity is not in the same direction as the force, then the scalar product of force and velocity must be used.

The standard unit for power is the watt (abbreviated W) which is a joule per second.



In the picture given above F pulls a box having 4kg mass from point A to B. If the friction constant between surface and box is 0,3; find the work done by F, work done by friction force and work done by resultant force.

Work done by F;

W_F=F.X=20.5=100 joule

Work done by friction force;

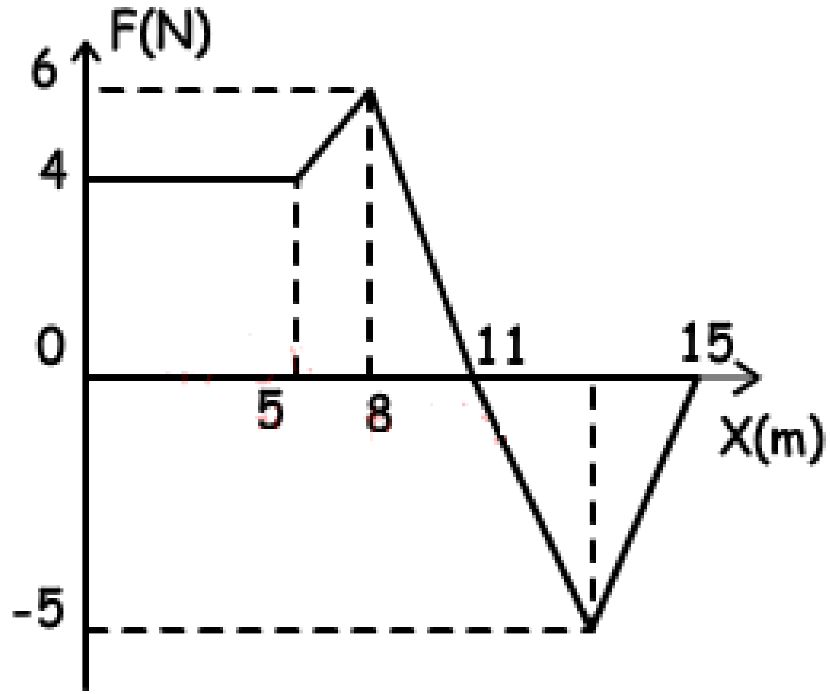
$W_{friction}$=-F_f.X=-k.mg.X=-0,3.4.10.5=-60 joule

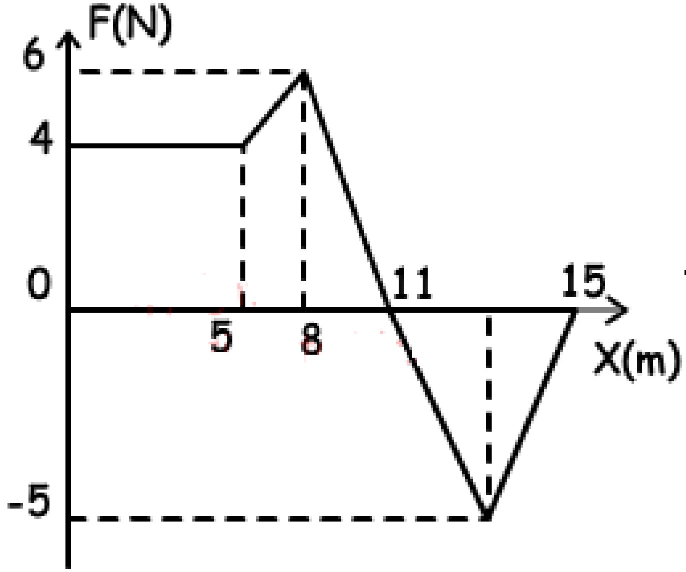
Work done by resultant force;

W_{net}=F_{net}.X=(F-F_f).X=(20-0,3.4.10)5

W_{net}=40 joule

Applied force vs. position graph of an object is given below. Find the work done by the forces on the object.





Area under the graph gives us work done by the force.

Work done between 0-5m:
$W_1=4.5=20$ joule

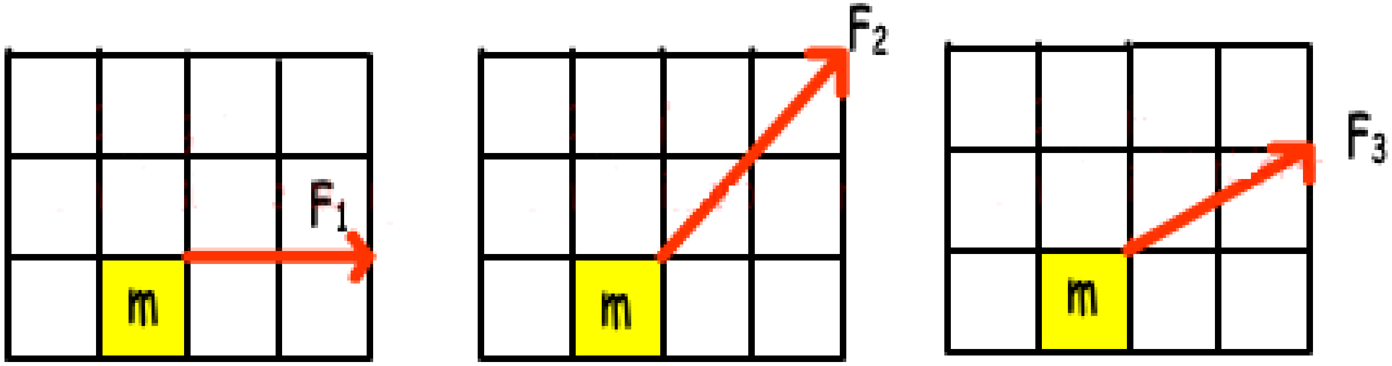
Work done between 5-8m:
$W_2=(6+4)/2.3=15$ joule

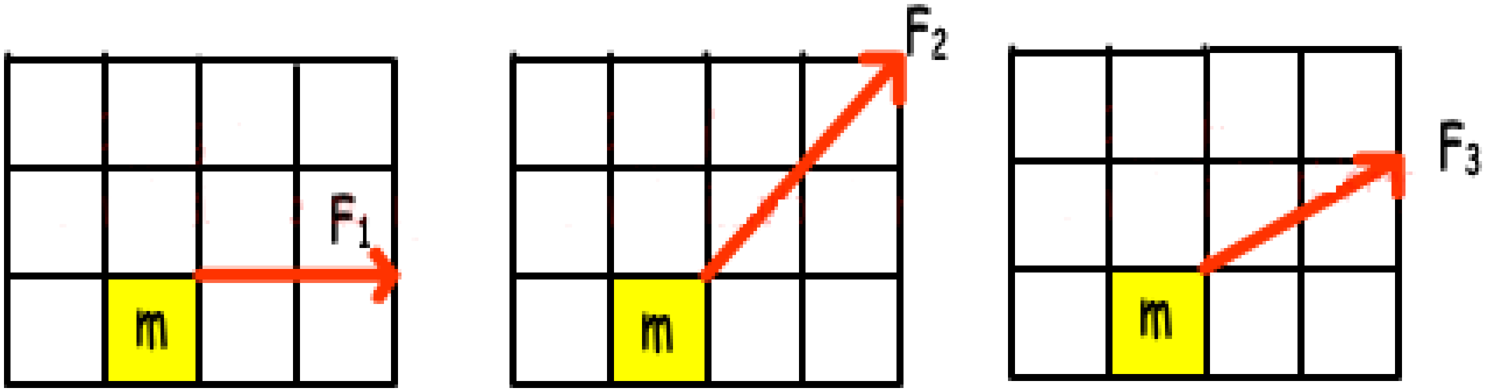
Work done between 8-11m:
$W_3= 6.3/2=9$ joule

Work done between 11-15m:
$W_4=-5.4/2=-10$ joule

$W_{net}=W_1+W_2+W_3+W_4=20+15+9+(-10)$
$W_{net}=34$ joule

In the picture given below, forces act on objects. Works done on objects during time t are W_1, W_2 and W_3. Find the relation of the works.





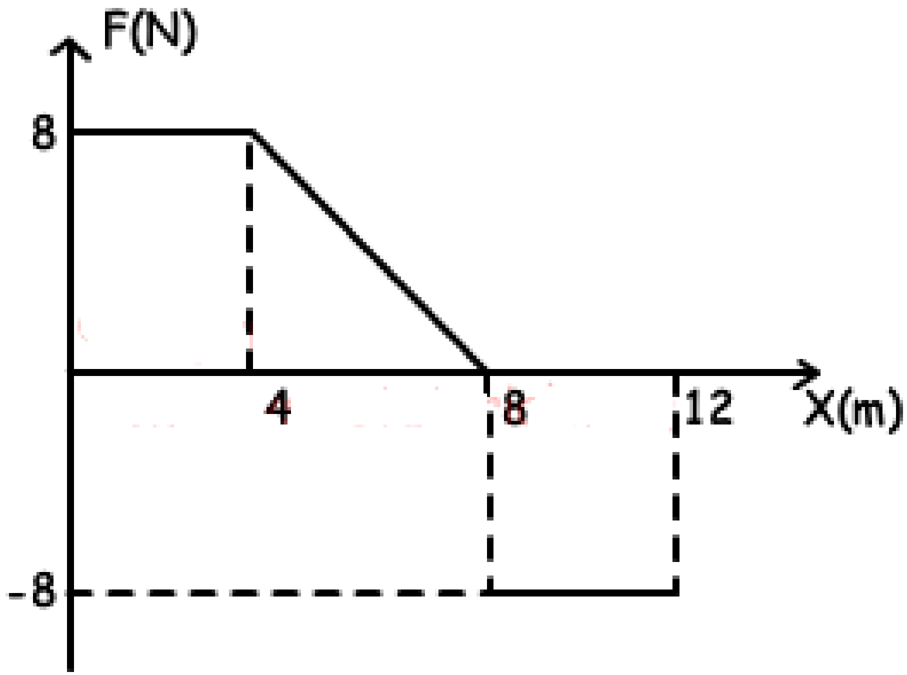
Horizontal components of the applied forces are equal to each other. Masses of the objects are also equal. Thus, acceleration of the objects and distances taken are also equal.

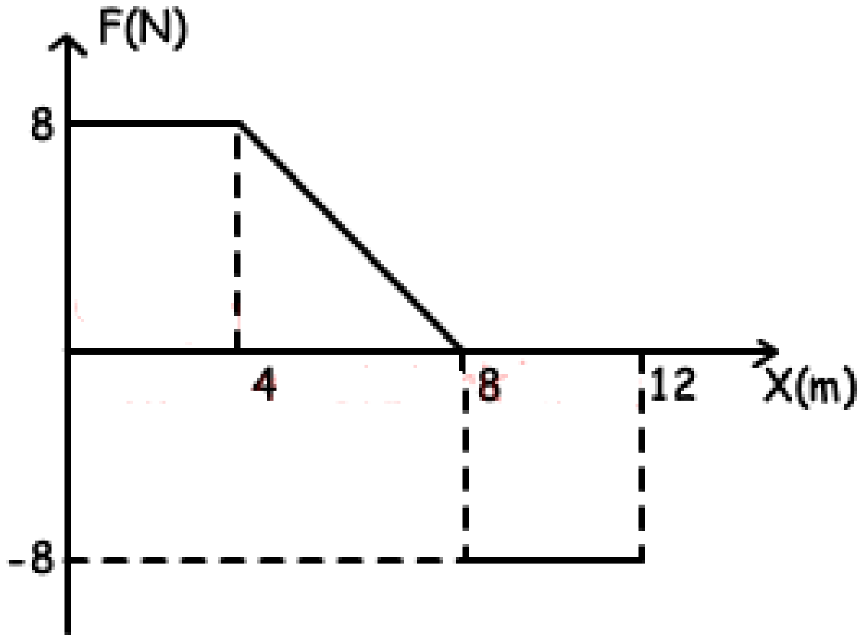
Work done:

$$W=F_X.X$$

$$W_1=W_2=W_3$$

Applied force vs. position graph of an object is given below. Find the kinetic energy gained by the object at distance 12m.



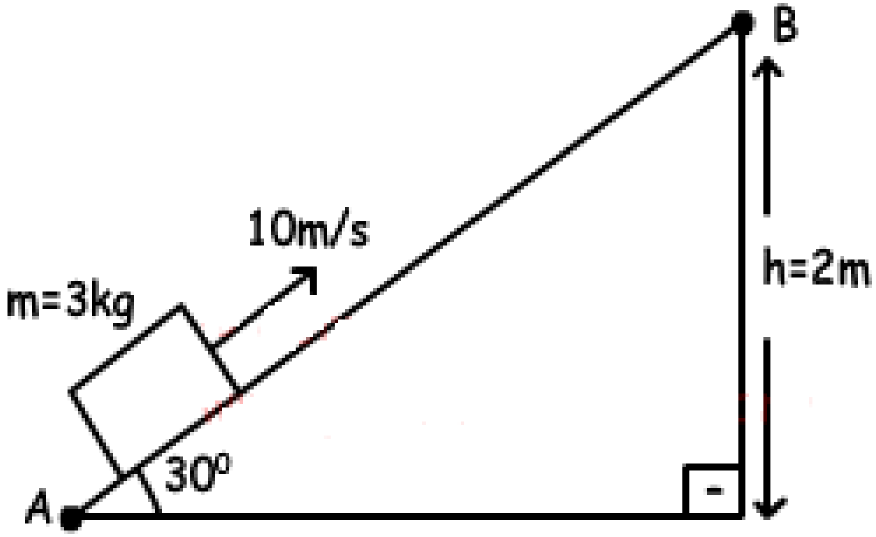


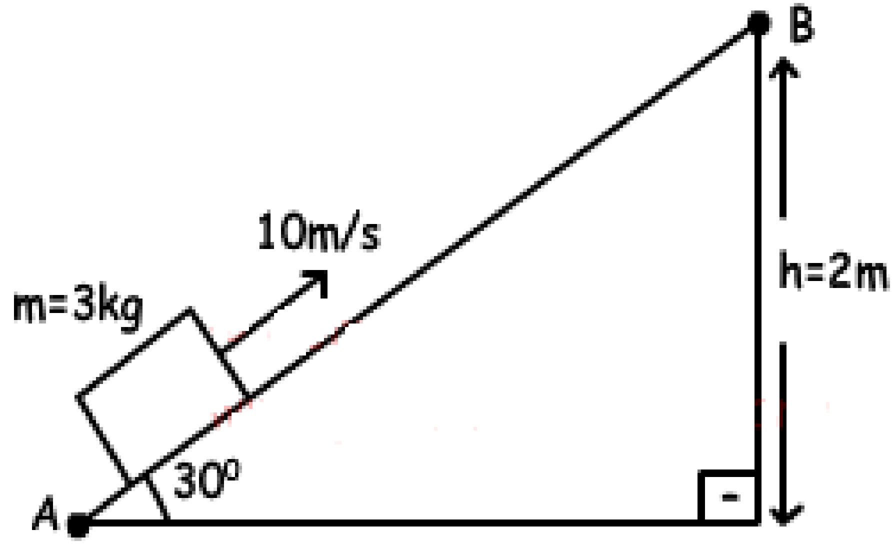
By using work and energy theorem we say that; area under the graph gives us work done by the force.

ΔEK=W=area under the graph=(8+4)/2.8-8(12-8)

ΔEK=12.4-8.4=16 joule

Box having mass 3kg thrown with an initial velocity 10 m/s on an inclined plane. If the box passes from the point B with 4m/s velocity, find the work done by friction force.





We use conservation of energy theorem.

$E_A = E_B + W_{friction}$

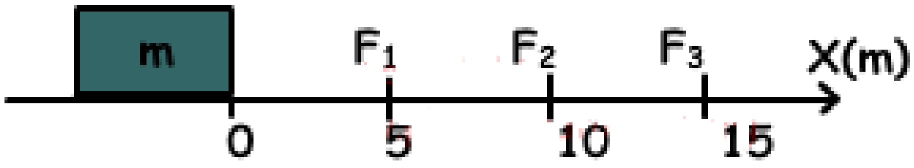
$$\overbrace{}^{E_A}\qquad\qquad\overbrace{}^{E_B}$$

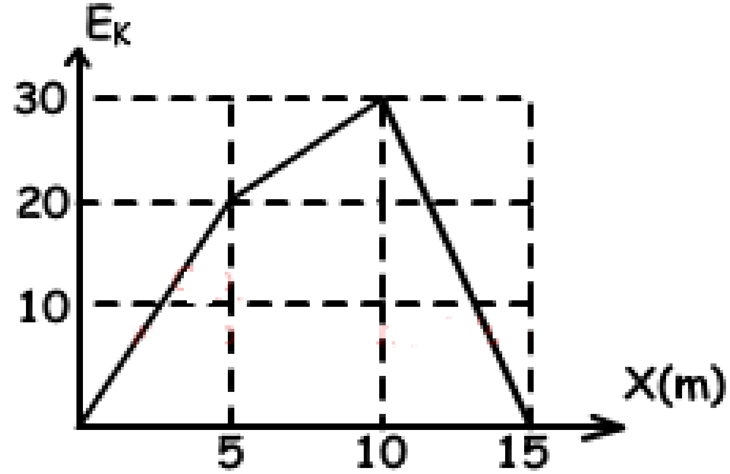
$W_{friction} = 1/2.m.V^2 - (mgh + 1/2mV_L^2)$

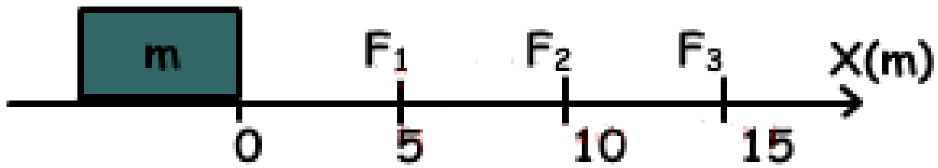
$W_{friction} = 1/2.3.10^2 - (3.10.2 + 1/2.3.4^2)$

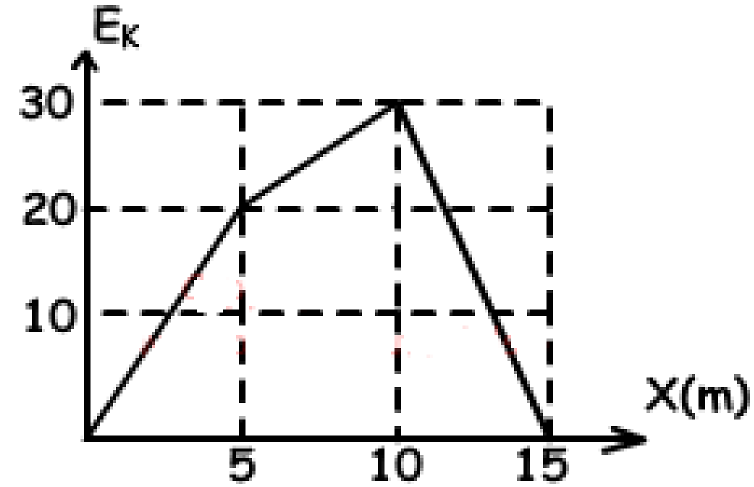
$W_{friction} = 66$ **joule**

Three different forces are applied to a box in different intervals. Graph, given below, shows kinetic energy gained by the box in three intervals. Find the relation between applied forces.









Slope of the E_K vs. position graph gives applied force

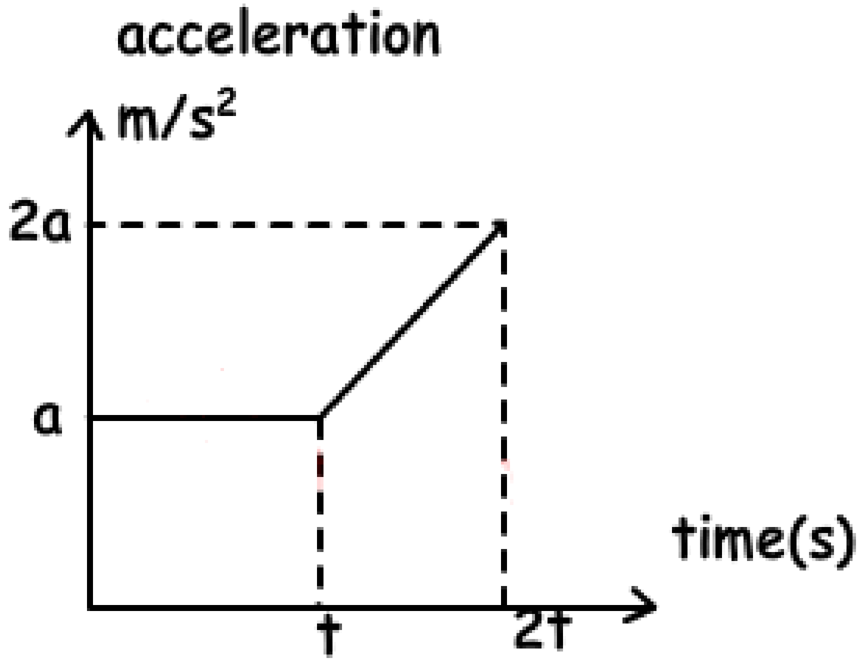
I. interval: $F_1=(20-0)/(5-0)=4N$

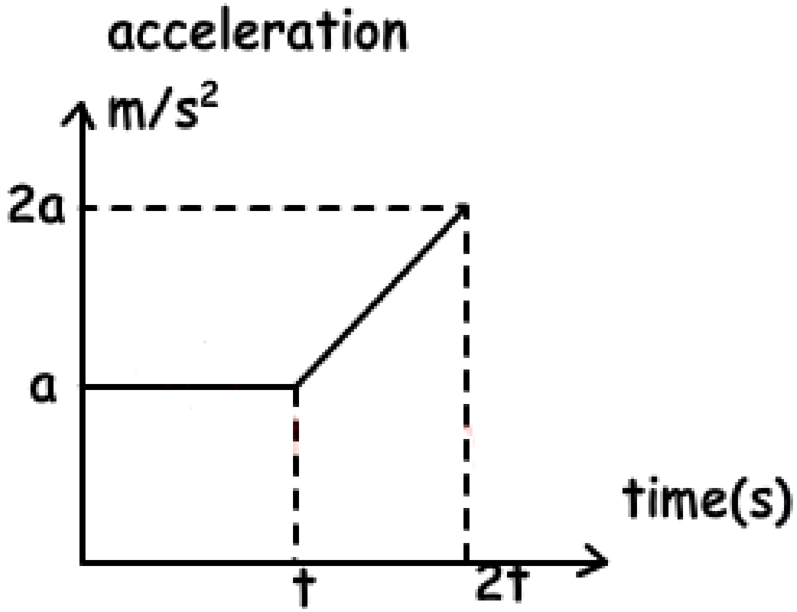
II. interval: $F_2=(30-20)/(10-5)=2N$

III. interval: $F_3= (0-30)/(15-10)=-6N$

$|F_{III}| > |F_I| > |F_{II}|$

A stationary object at t=0, has an acceleration vs. time graph given below. If object has kinetic energy E at t=t, find the kinetic energy of the object at t=2t in terms of E.





Area under the acceleration-time graph gives velocity

Object has velocity at $0<time<t$

$V_1=at$

Object has velocity at $0<time<2t$

$V_2=at+((2a+a)/2).t=at+3/2.at=5/2.at$

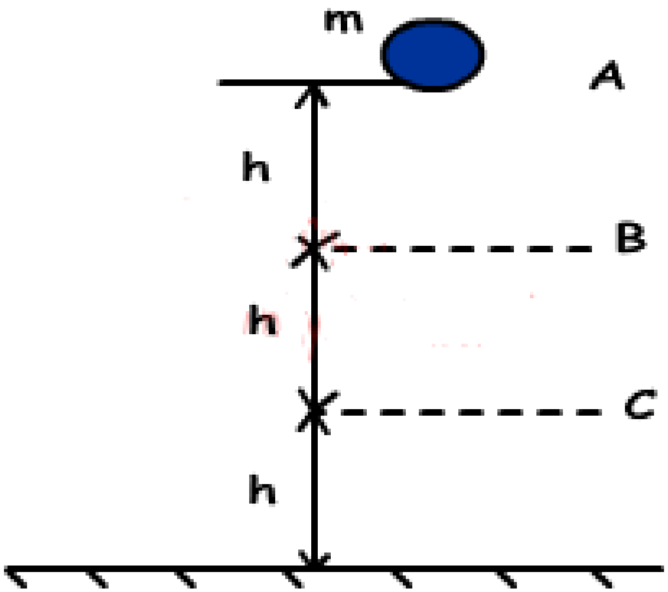
$V_2=5/2.V_1$

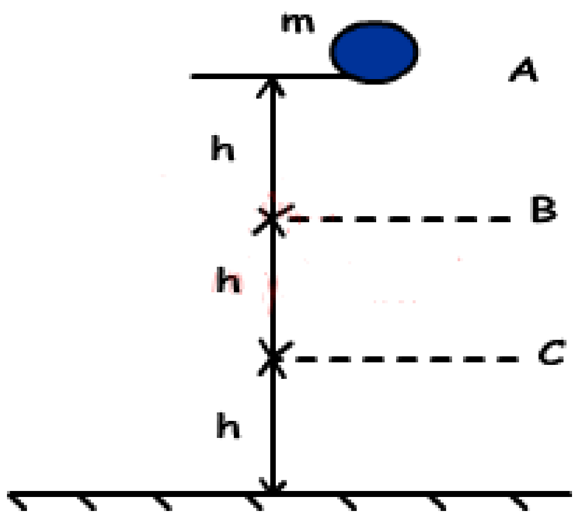
$E_2/E=(1/2.m.V_2^2)/(1/2.m.V_1^2)=(5/2.V_1)^2/V_1^2$

$E_2/E=25/4$

$E_2=25E/4$

An object does free fall. Picture given below shows this motion. Find the ratio of kinetic energy at point C to total mechanical energy of the object.





Object lost 2mgh potential energy from point A to C. According to conservation of energy theorem, this lost potential energy converted to the kinetic energy. Thus; we can say that kinetic energy of the object at point C is;

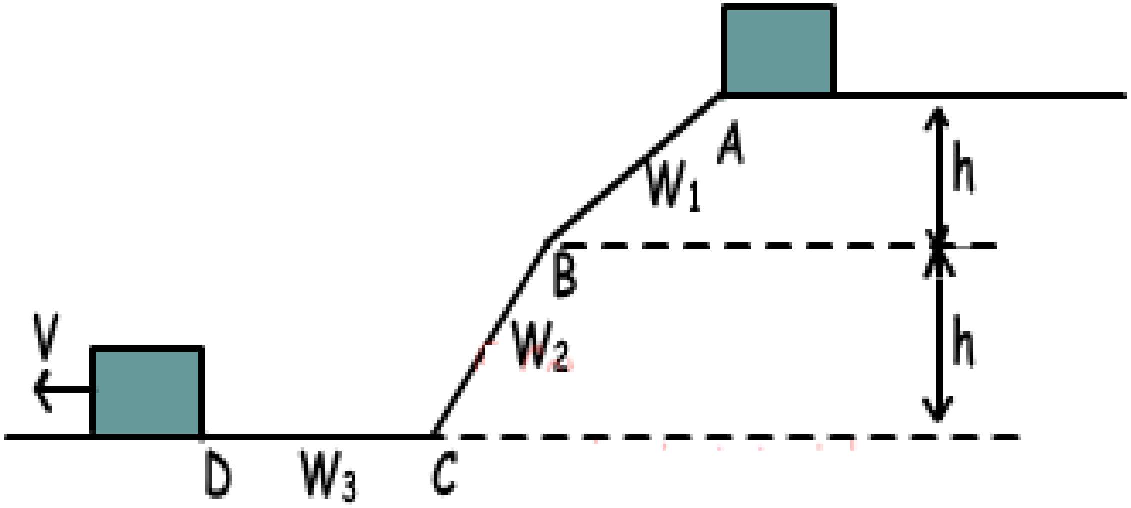
E_K=2mgh

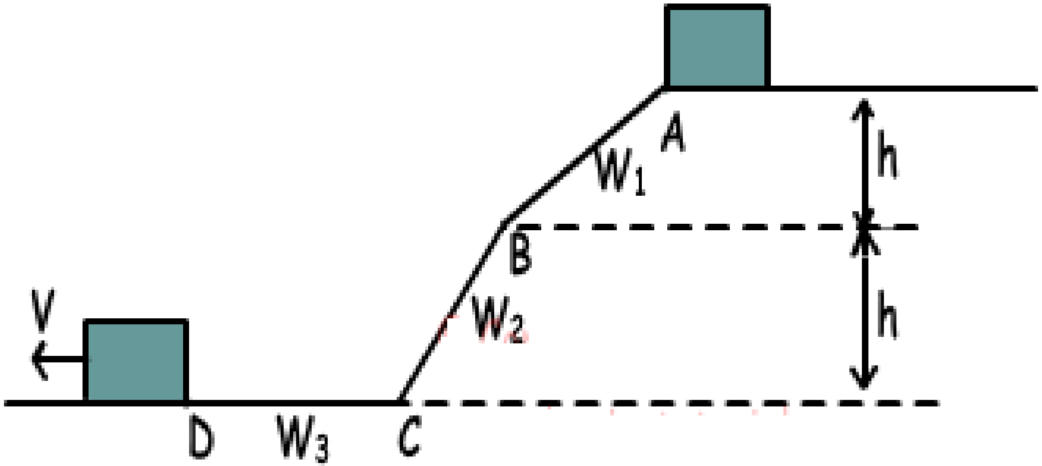
Total mechanical energy;

E_{total}=3mgh

E_K/E_{total}=2mgh/3mgh

A box is released from point A and it passes from point D with a velocity V. Works done by the gravity are W_1 between AB, W_2 between BC and W_3 between CD. Find the relation between them.





Work done by gravity is equal to change in potential energy of the object.

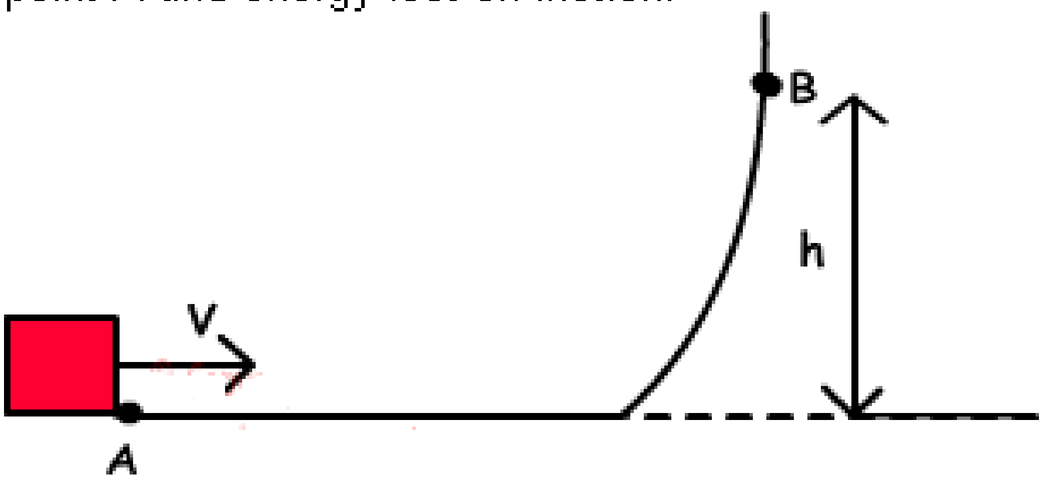
Interval AB: $W_1 = \Delta Ep = -mgh$

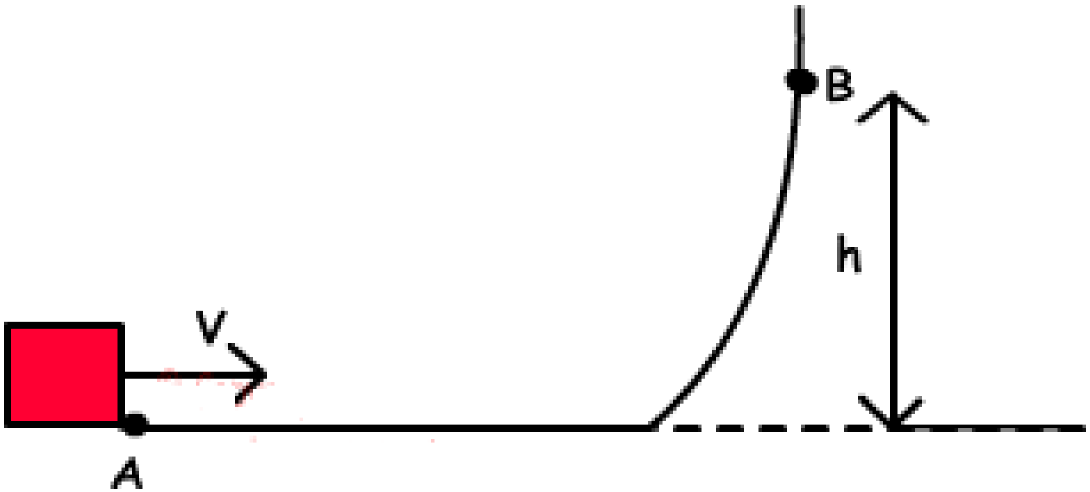
Interval BC: $W_2 = \Delta Ep = -mgh$

Interval CD: $W_3 = \Delta Ep = 0$

$W_1 = W_2 > W_3$

An object thrown with an initial velocity V from point A. It reaches point B and turns back to point A and stops. Find the relation between the kinetic energy object has at point A and energy lost on friction.





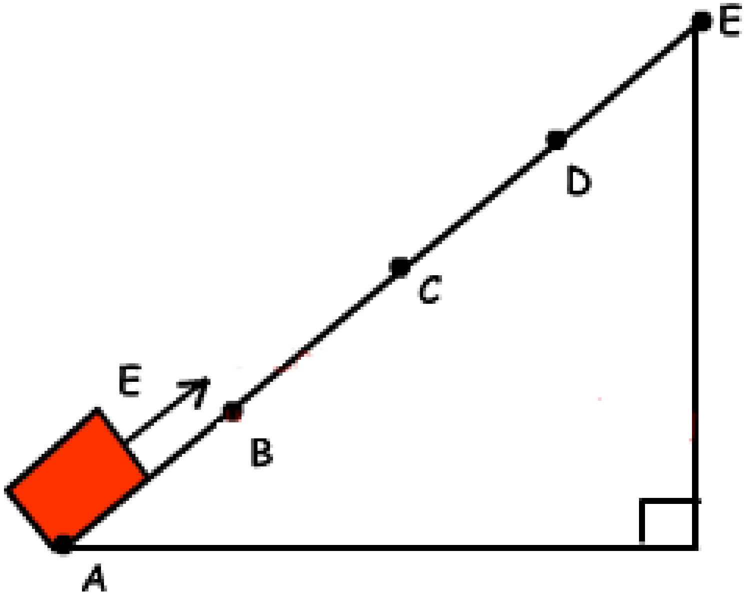
Object has kinetic energy at point A;

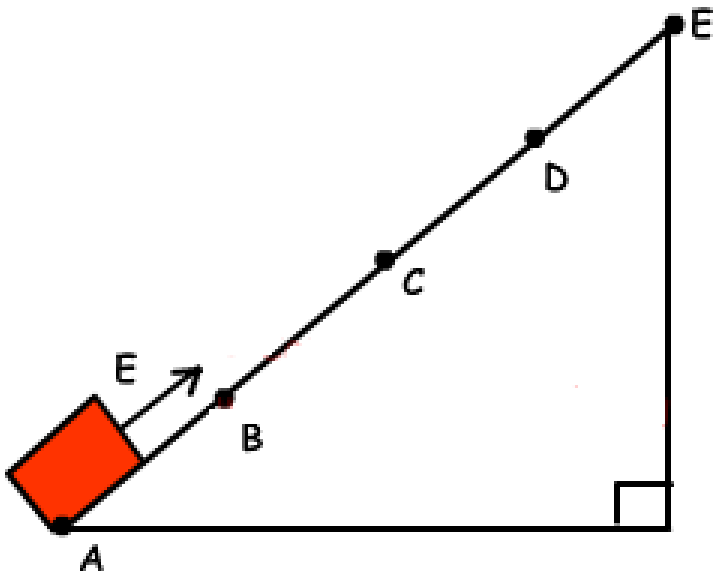
$E_K = 1/2.mV^2$

Object stops at point A, which means that all energy is lost on friction.

$E_K = E_{friction}$

We throw object from point A with an initial kinetic energy E, and it reaches point C. How much energy must be given to make object reach point D.



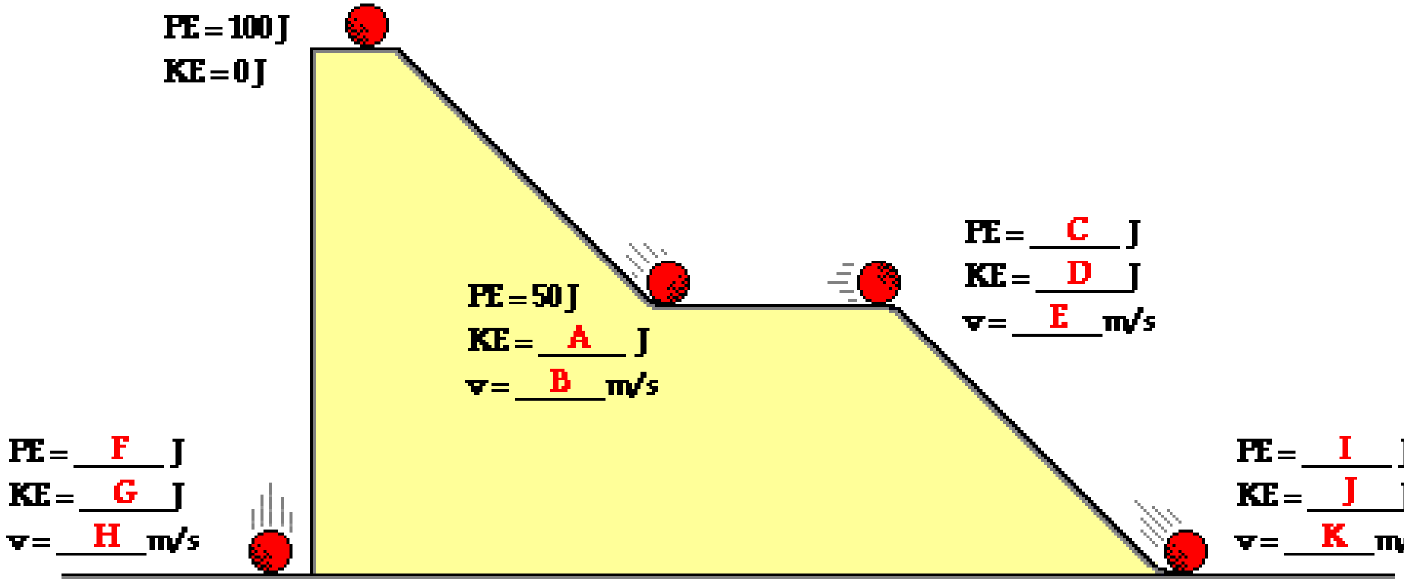


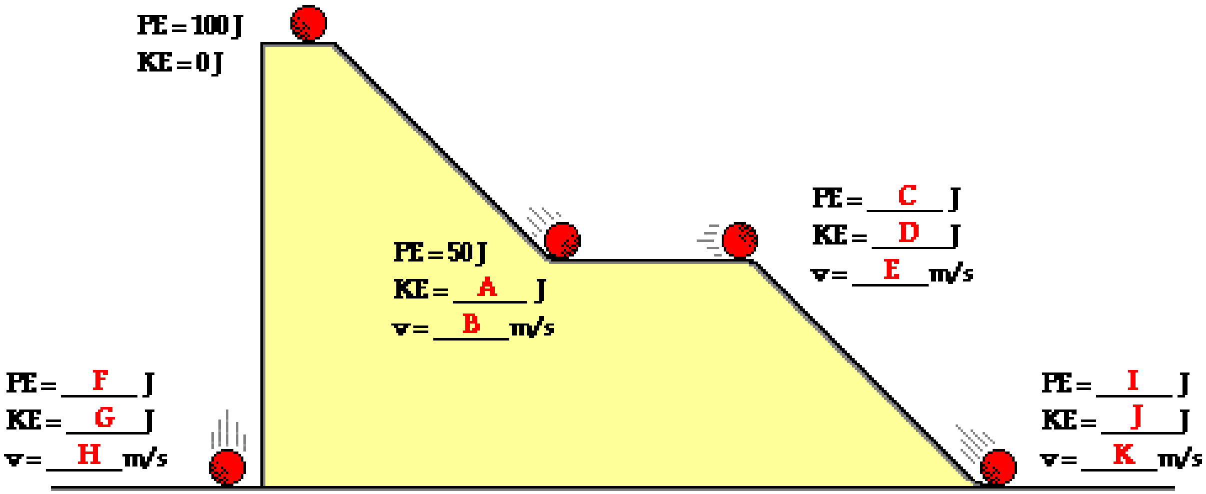
Using conservation of energy theorem;

$$E=2mgh+F_{friction}.2X$$
$$E=2(mgh+F_{friction}.X)$$
$$mgh+F_{friction}.X=E/2$$
$$E'=3mgh+Ffriction.3X$$
$$E'=3(mgh+Ffriction.X)$$
$$E'=3.E/2=1,5E$$

we must give 1,5 E energy to make object reach point D.

Consider the falling and rolling motion of the ball in the following two resistance-free situations. In one situation, the ball falls off the top of the platform to the floor. In the other situation, the ball rolls from the top of the platform along the staircase-like pathway to the floor. For each situation, indicate what types of forces are doing work upon the ball. Indicate whether the energy of the ball is conserved and explain why. Finally, fill in the blanks for the 2-kg ball.





The answers given here for the speed values are presuming that all the kinetic energy of the ball is in the form of translational kinetic energy.

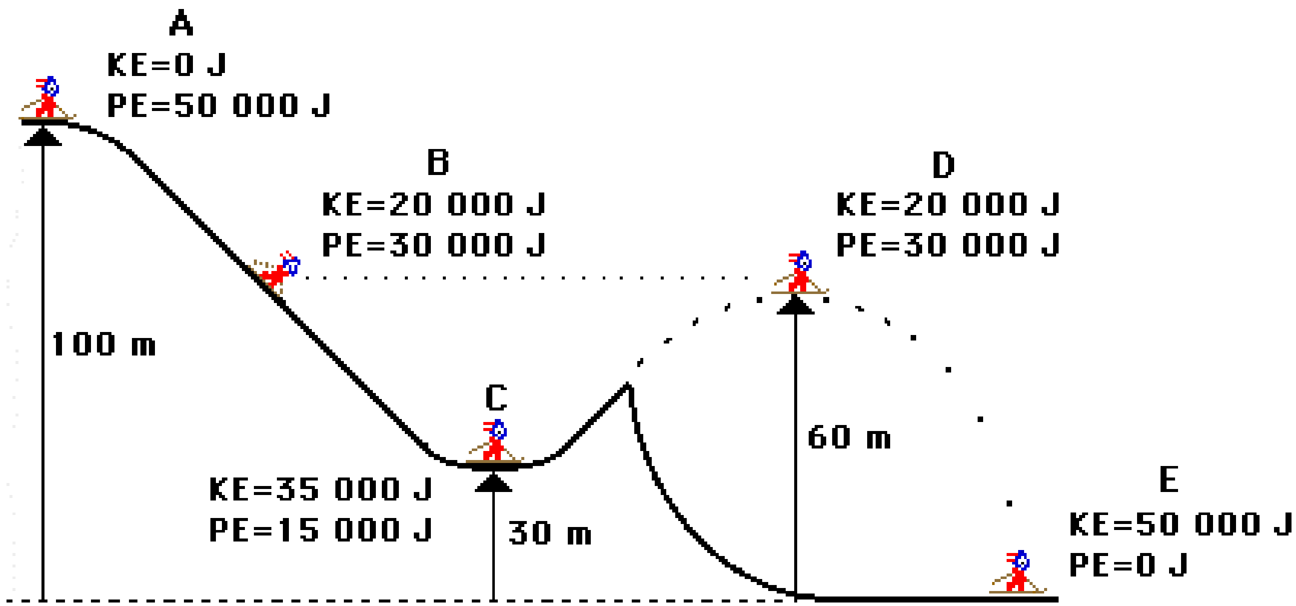
The only force doing work is gravity. Since it is an internal or conservative force, the total mechanical energy is conserved. Thus, the 100 J of original mechanical energy is present at each position. So the KE for A is 50 J.

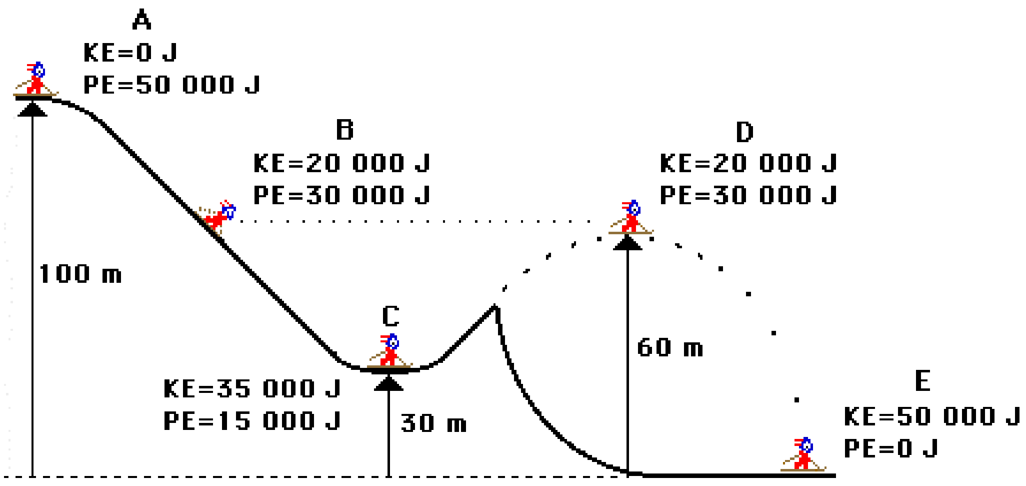
The PE at the same stairstep is 50 J (C) and thus the KE is also 50 J (D).

The PE at zero height is 0 J (F and I). And so the kinetic energy at the bottom of the hill is 100 J (G and J).

Using the equation $KE = 0.5*m*v^2$, the velocity can be determined to be 7.07 m/s for B and E and 10 m/s for H and K.

Determine Li Ping Phar's (a mass of approximately 50 kg) speed at locations B, C, D and E.





B: $KE = 0.5 \bullet m \bullet v^2$

$20\,000\ J = 0.5 \bullet (50\ kg) \bullet v^2$

v = 28.3 m/s

C: $KE = (0.5 \bullet m \bullet v^2$

$35\,000\ J = 0.5 \bullet (50\ kg) \bullet v^2$

v = 37.4 m/s

D: same as postition B

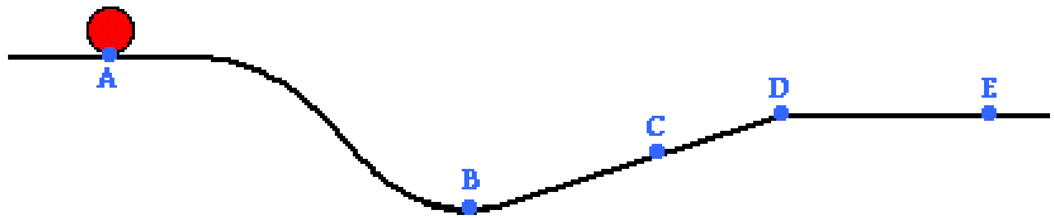
v = 28.3 m/s

E: $KE = 0.5 \bullet m \bullet v^2$

$50\,000\ J = 0.5 \bullet (50\ kg) \bullet v^2$

v = 44.7 m/s

Use the following diagram to answer question Neglect the effect of resistance forces.



As the object moves from point A to point D across the surface, the sum of its gravitational potential and kinetic energies ___.

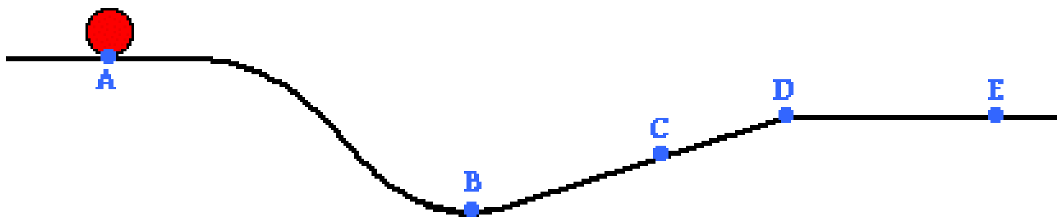
a. decreases, only

b. decreases and then increases

c. increases and then decreases

d. remains the same

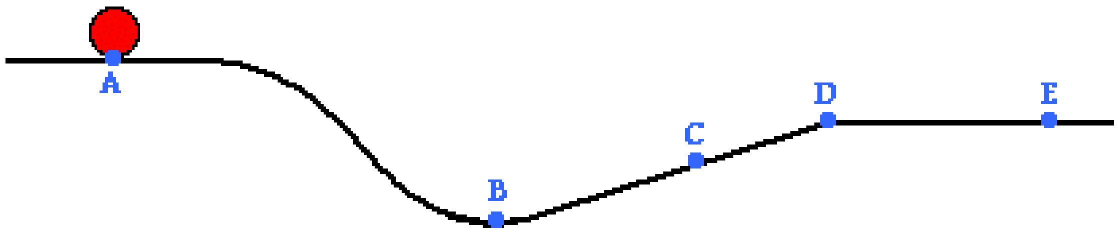
Use the following diagram to answer question Neglect the effect of resistance forces.



The object will have a minimum gravitational potential energy at point ____.

a. A b. B c. C d. D e. E

Use the following diagram to answer question Neglect the effect of resistance forces.



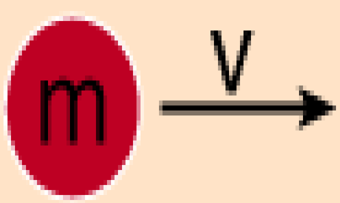
The object's kinetic energy at point C is less than its kinetic energy at point ___.

a. A only b. A, D, and E c. B only d. D and E

Momentum

The momentum of a particle is defined as the product of its mass times its velocity. It is a vector quantity. The momentum of a system is the vector sum of the momenta of the objects which make up the system. If the system is an isolated system, then the momentum of the system is a constant of the motion and subject to the principle of conservation of momentum.

The most common symbol for momentum is p. The SI unit for momentum is kg m/s.



$$\text{momentum} = \text{mass} \times \text{velocity}$$
$$p = m \times v$$

Conservation of Momentum

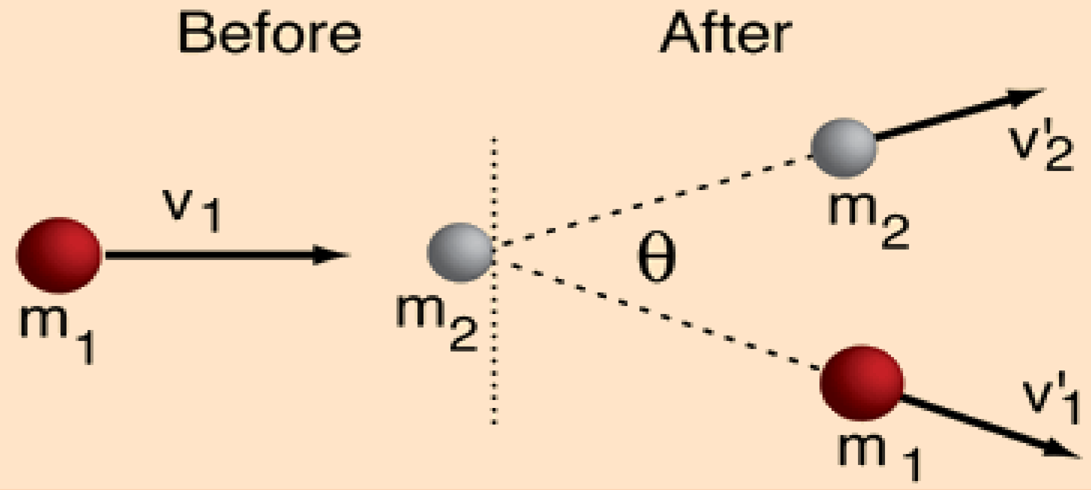
The momentum of an isolated system is a constant. The vector sum of the momenta mv of all the objects of a system cannot be changed by interactions within the system. This puts a strong constraint on the types of motions which can occur in an isolated system. If one part of the system is given a momentum in a given direction, then some other part or parts of the system must simultaneously be given exactly the same momentum in the opposite direction. As far as we can tell, conservation of momentum is an absolute symmetry of nature. That is, we do not know of anything in nature that violates it.

Elastic and Inelastic Collisions

A perfectly elastic collision is defined as one in which there is no loss of kinetic energy in the collision. An inelastic collision is one in which part of the kinetic energy is changed to some other form of energy in the collision.

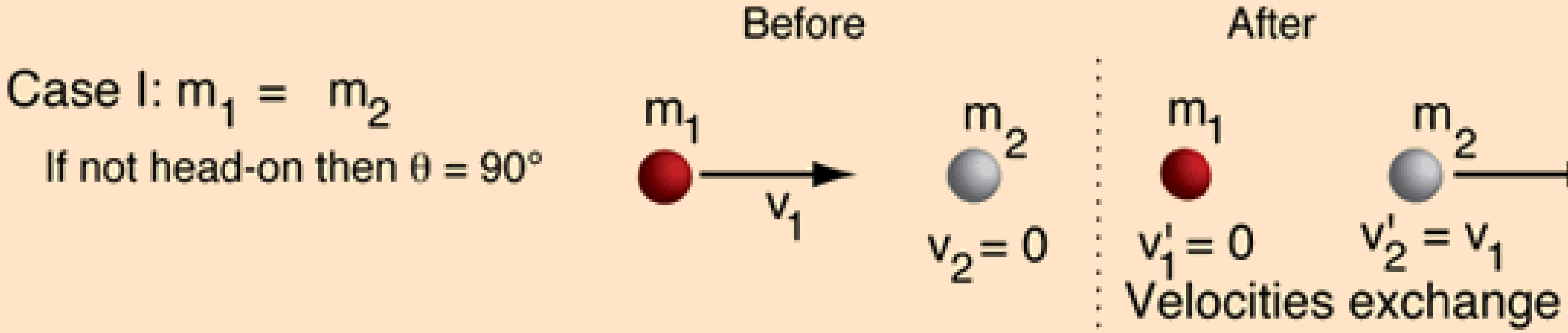
An elastic collision is defined as one in which both conservation of momentum and conservation of kinetic energy are observed. This implies that there is no dissipative force acting during the collision and that all of the kinetic energy of the objects before the collision is still in the form of kinetic energy afterward.

Collisions between hard spheres may be nearly elastic, so it is useful to calculate the limiting case of an elastic collision. The assumption of conservation of momentum as well as the conservation of kinetic energy makes possible the calculation of the final velocities in two-body collisions.

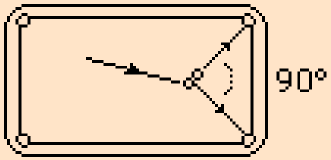


Elastic Collision, Equal Masses

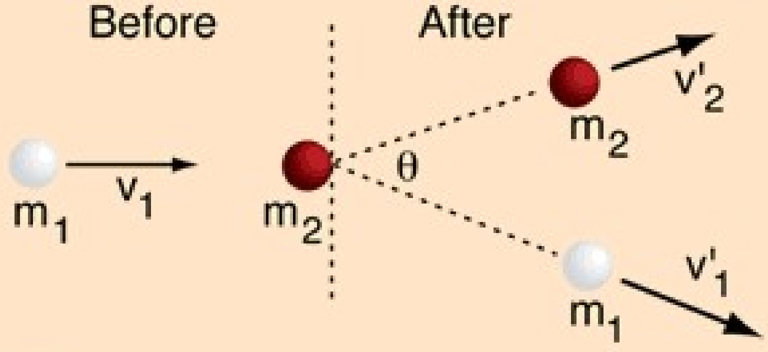
For a head-on collision with a stationary object of equal mass, the projectile will come to rest and the target will move off with equal velocity, like a head-on shot with the cue ball on a pool table. This may be generalized to say that for a head-on elastic collision of equal masses, the velocities will always exchange.



For a non-head-on elastic collision between equal masses, the angle between the velocities after the collision will always be 90 degrees. The spot on a pool table is placed so that a collision with a ball on the spot which sends it to a corner pocket will send the cue ball to the other corner pocket.



Elastic Collisions - Target Initially at Rest*



*This can actually apply to all elastic collisions since we can always choose a reference frame riding with m_2 before the collision.

General relationships:

a. Conservation of momentum: $m_1\vec{v}_1 = m_1\vec{v}'_1 + m_2\vec{v}'_2$

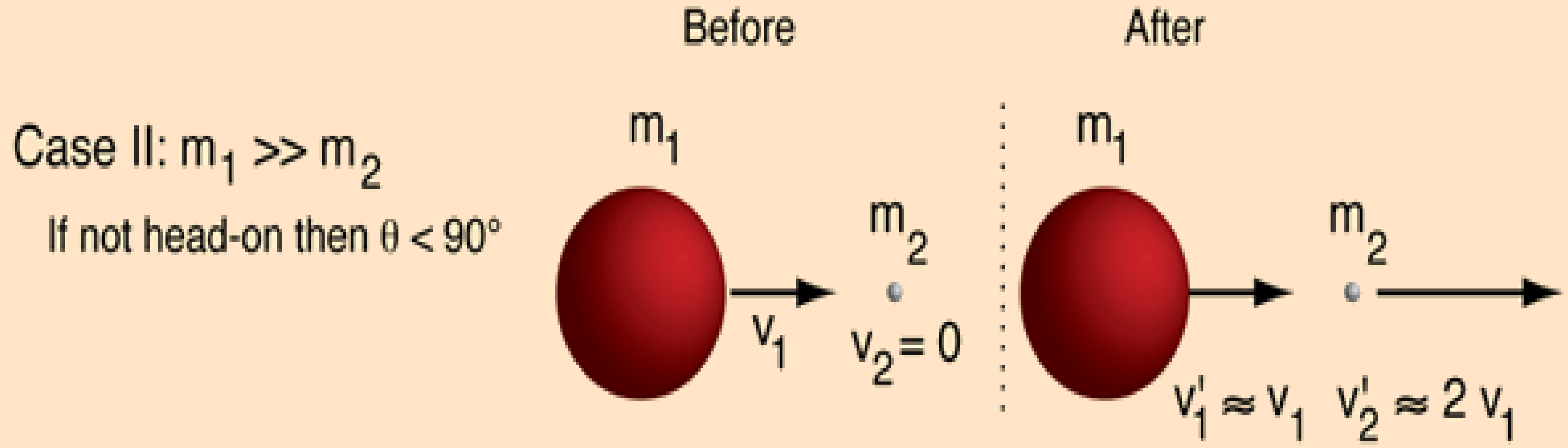
b. Conservation of kinetic energy: $\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1 v'^2_1 + \frac{1}{2}m_2 v'^2_2$
(elastic collision assumption)

c. For head-on collisions: $v'_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}v_1$; $v'_2 = \frac{2m_1}{(m_1 + m_2)}v_1$

d. For head-on collisions the velocity of approach is equal to the velocity of separation.

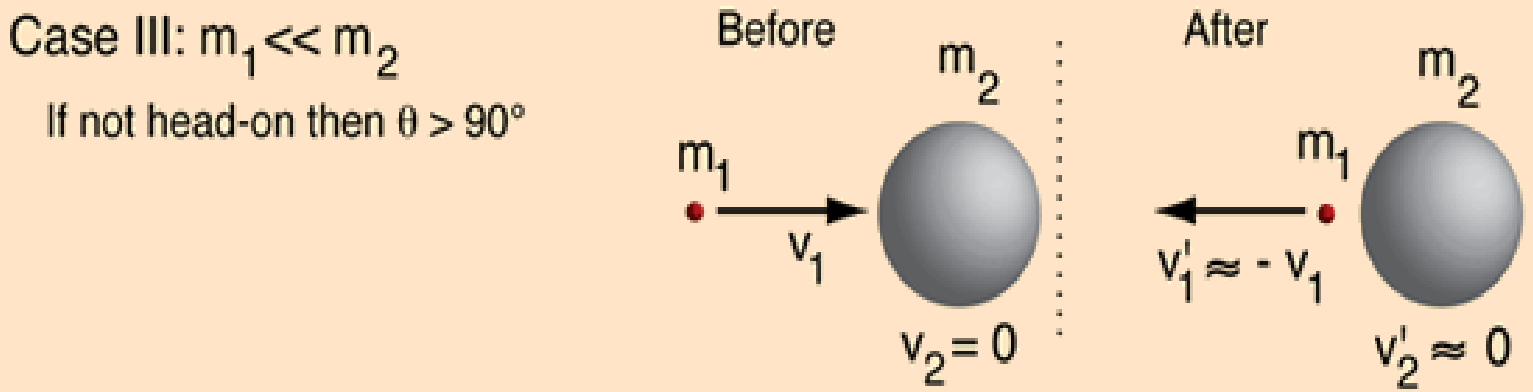
Elastic Collision, Massive Projectile

In a head-on elastic collision where the projectile is much more massive than the target, the velocity of the target particle after the collision will be about twice that of the projectile and the projectile velocity will be essentially unchanged.



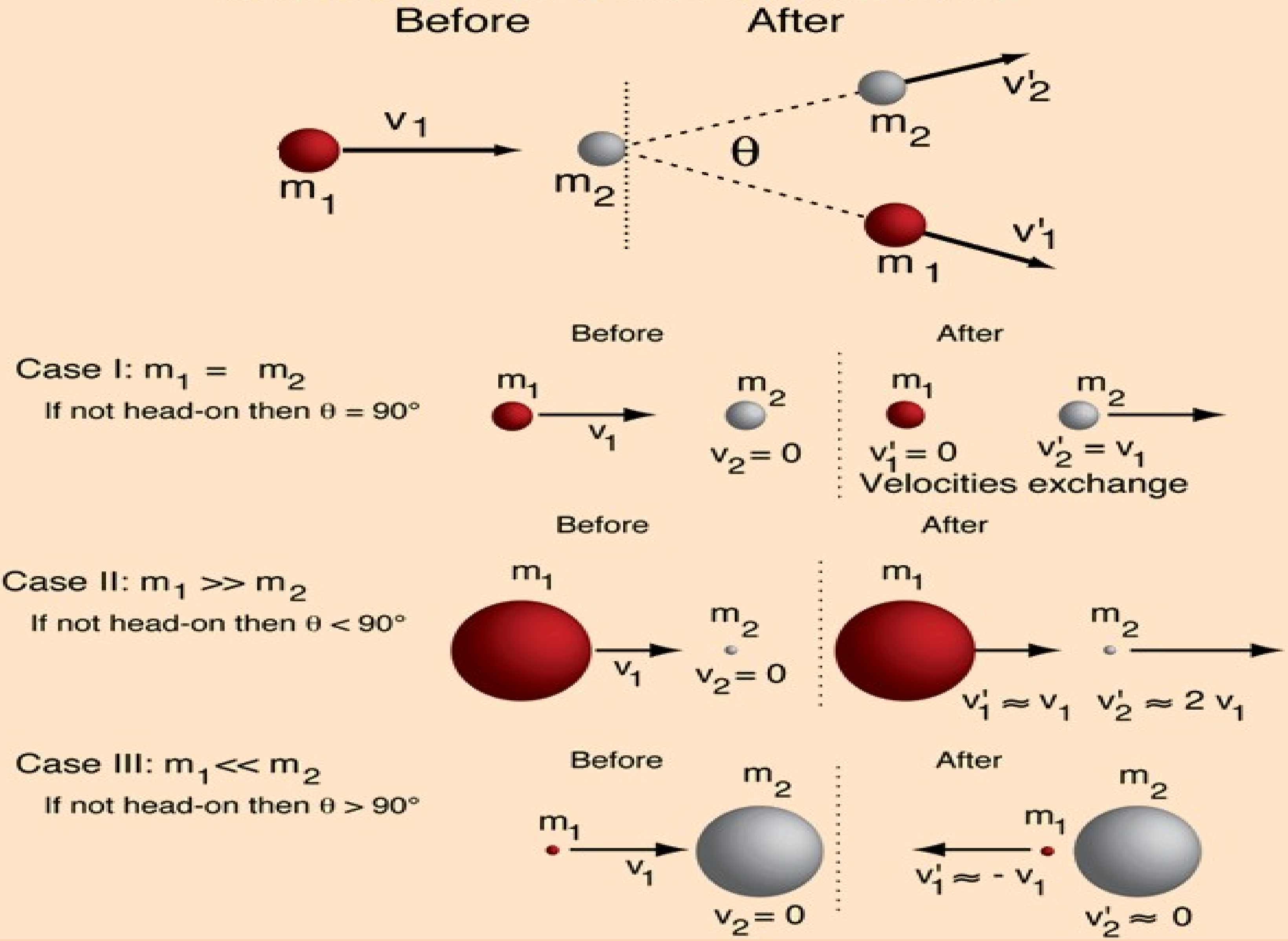
Elastic Collision, Massive Target

In a head-on elastic collision between a small projectile and a much more massive target, the projectile will bounce back with essentially the same speed and the massive target will be given a very small velocity. One example is a ball bouncing back from the Earth when we throw it down.



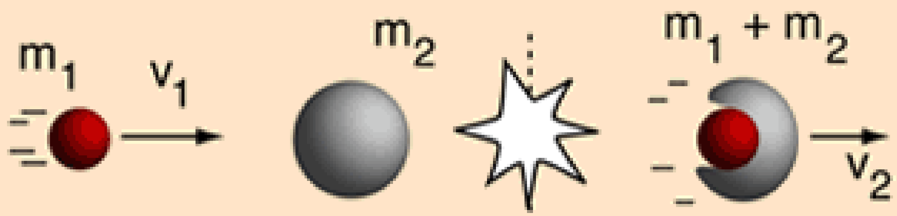
In the case of a non-headon elastic collision, the angle of the projectiles path after the collision will be more than 90 degrees away from the targets motion.

Head-on Elastic Collisions



Inelastic Collisions

Perfectly elastic collisions are those in which no kinetic energy is lost in the collision. Macroscopic collisions are generally inelastic and do not conserve kinetic energy, though of course the total energy is conserved as required by the general principle of conservation of energy. The extreme inelastic collision is one in which the colliding objects stick together after the collision, and this case may be analyzed in general terms:



	Before	After
Momentum	$m_1 v_1$	$(m_1 + m_2)v_2$
Kinetic energy	$\frac{1}{2} m_1 v_1^2$	$\frac{1}{2}(m_1 + m_2)v_2^2$

From conservation of momentum:

$$m_1 v_1 = (m_1 + m_2)v_2 \quad \Rightarrow \quad v_2 = \frac{m_1}{m_1 + m_2} v_1$$

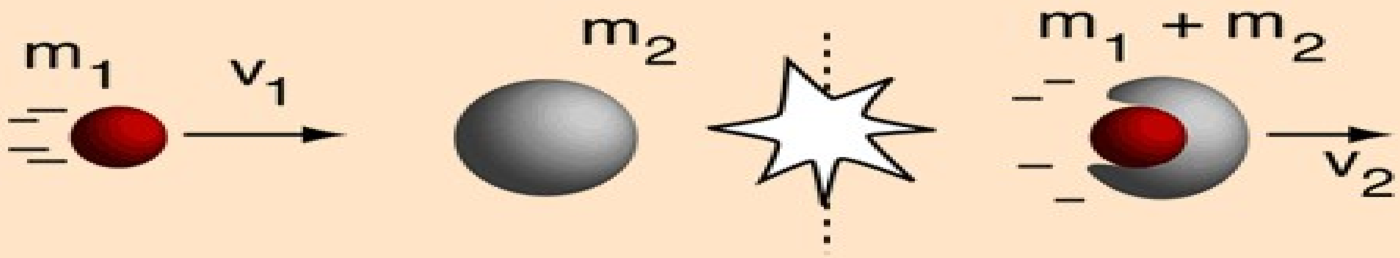
Ratio of kinetic energies before and after collision:

$$\frac{KE_f}{KE_i} = \frac{m_1}{m_1 + m_2}$$

Fraction of kinetic energy lost in the collision:

$$\frac{KE_i - KE_f}{KE_i} = \frac{m_2}{m_1 + m_2}$$

K.E. Lost in Inelastic Collision



From conservation of momentum:

$$m_1 v_1 = (m_1 + m_2)v_2 \Rightarrow v_2 = \frac{m_1}{m_1 + m_2} v_1$$

The ratio of kinetic energies before and after is:

$$\frac{KE_f}{KE_i} = \frac{\frac{1}{2}(m_1 + m_2)\left[\frac{m_1}{m_1 + m_2} v_1\right]^2}{\frac{1}{2} m_1 v_1^2} = \frac{m_1}{m_1 + m_2}$$

The fraction of kinetic energy lost is:

$$\frac{KE_i - KE_f}{KE_i} = \frac{\left[1 - \frac{m_1}{m_1 + m_2}\right] KE_i}{KE_i} = \frac{m_2}{m_1 + m_2}$$

Impulse of Force

The product of average force and the time it is exerted is called the impulse of force. From Newton's second law

$$F_{average} = ma_{average} = m\frac{\Delta v}{\Delta t}$$

the impulse of force can be extracted and found to be equal to the change in momentum of an object provided the mass is constant:

$$Impulse = F_{average}\Delta t = m\Delta v$$

The main utility of the concept is in the study of the average impact force during collisions. For collisions, the mass and change in velocity are often readily measured, but the force during the collision is not. If the time of collision can be measured, then the average force of impact can be calculated.

Minimizing Impact Force

The process of minimizing an impact force can be approached from the definition of the impulse of force:

Extend time of collision

$$Impulse = F_{average}\Delta t = m\,\Delta v$$

Reduce average impact force

For a given change in momentum, the impulse stays constant.

If an impact stops a moving object, then the change in momentum is a fixed quantity, and extending the time of the collision will decrease the impact force by the same factor. This principle is applied in many common-sense situations:

- If you jump to the ground from any height, you bend your knees upon impact, extending the time of collision and lessening the impact force.
- A boxer moves away from a punch, extending the time of impact and lessening the force.
- Automobiles are made to collapse upon impact, extending the time of collision and lessening the impact force.

Alternatively, the same scenario can be examined with the aid of the work-energy principle.

Extend distance of collision

$$F_{avg}d = -\frac{1}{2}mv^2$$

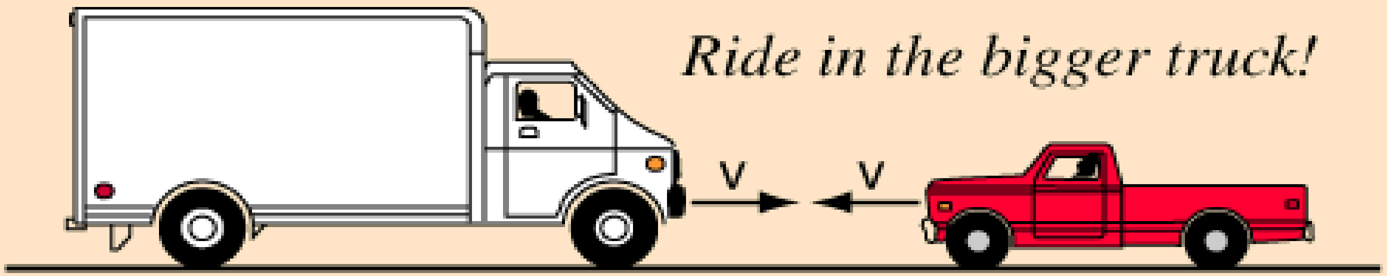
Reduce average impact force

For a given change in kinetic energy, the work required stays constant.

Car crash example

An impact which stops a moving object must do enough work to take away its kinetic energy, so extending the distance moved during the collision reduces the impact force.

Truck Collision



In a head-on collision the forces on the two vehicles are constrained to be the same by Newton's third law. But from both Newton's second law and the work-energy principle it becomes evident that it is safer to be in the bigger truck.

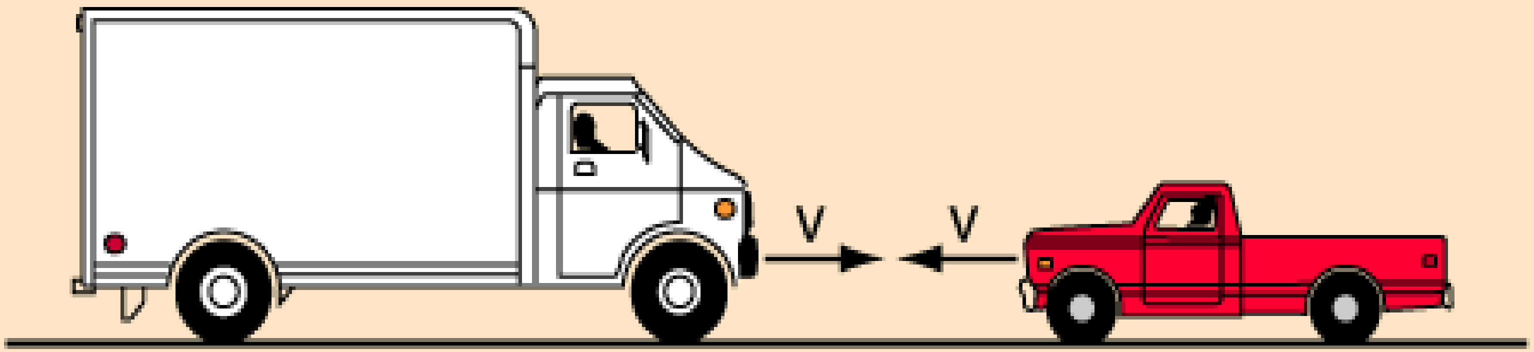
$$m\,\Delta v_{\text{big truck}} = m\,\Delta v_{\text{little truck}}$$

$$F_{\text{big truck}} = F_{\text{little truck}}$$

$$m\,a_{\text{big truck}} = m\,a_{\text{little truck}}$$

The change in velocity of the driver will be the same as the truck in which he/she is riding. A greater change in velocity implies a greater change in kinetic energy and therefore more work done on the driver.

Truck Collision



In a head-on collision:

Which truck will experience the greatest force?

Which truck will experience the greatest impulse?

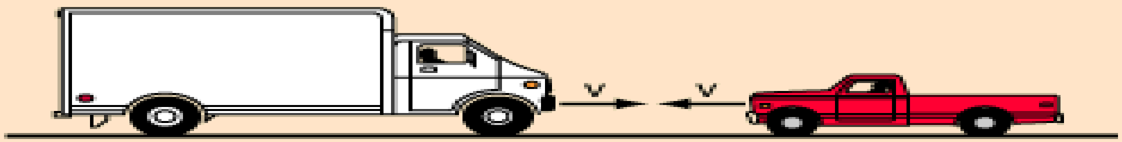
Which truck will experience the greatest change in momentum?

Which truck will experience the greatest change in velocity?

Which truck will experience the greatest acceleration?

Which truck would you rather be in during the collision?

Comparison of the collision variables for the two trucks:



Force	F	$=$	F
Impulse	F_t	$=$	F_t
Change in momentum	$M\Delta v$	$=$	$m\Delta V$
Acceleration	$M a$	$=$	$m A$

In a head-on collision:

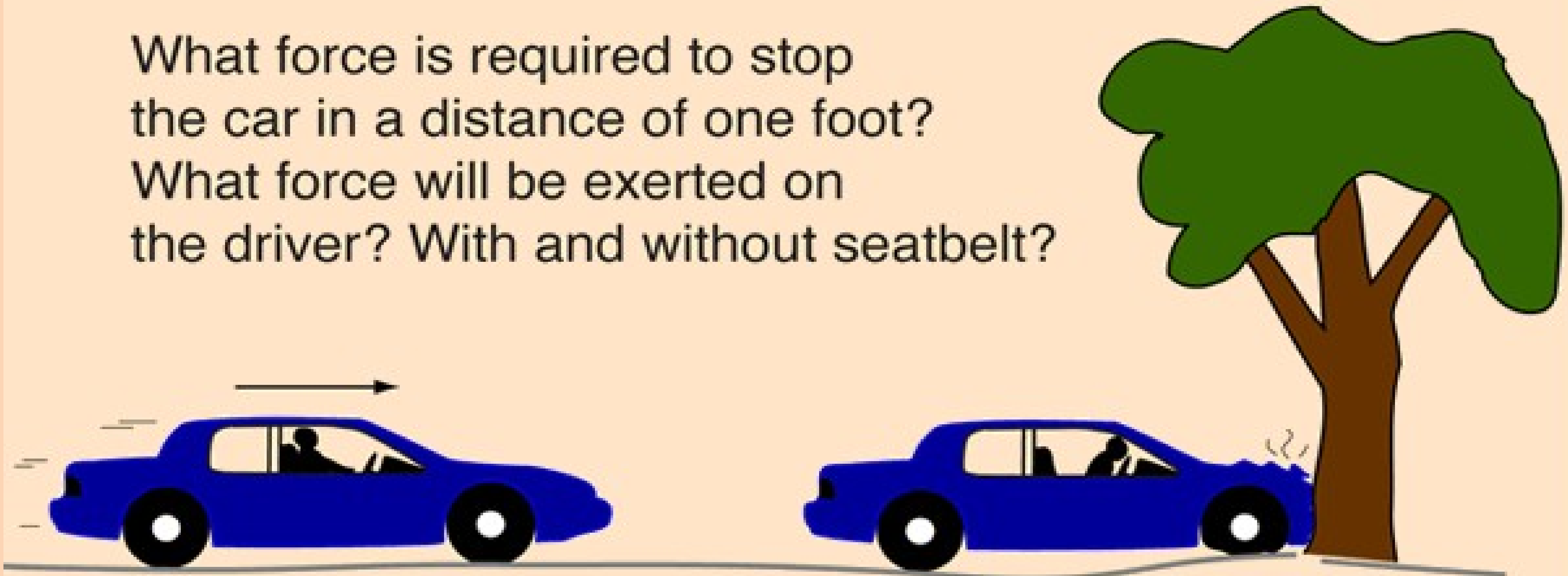
Newton's third law dictates that the forces on the trucks are equal but opposite in direction.

Impulse is force multiplied by time, and time of contact is the same for both, so the impulse is the same in magnitude for the two trucks. Change in momentum is equal to impulse, so changes in momenta are equal. With equal change in momentum and smaller mass, the change in velocity is larger for the smaller truck. Since acceleration is change in velocity over change in time, the acceleration is greater for the smaller truck.

Ride in the bigger truck! There are good physical reasons!

Forces in Car Crashes

What force is required to stop the car in a distance of one foot? What force will be exerted on the driver? With and without seatbelt?



Car collapses one foot upon impact.

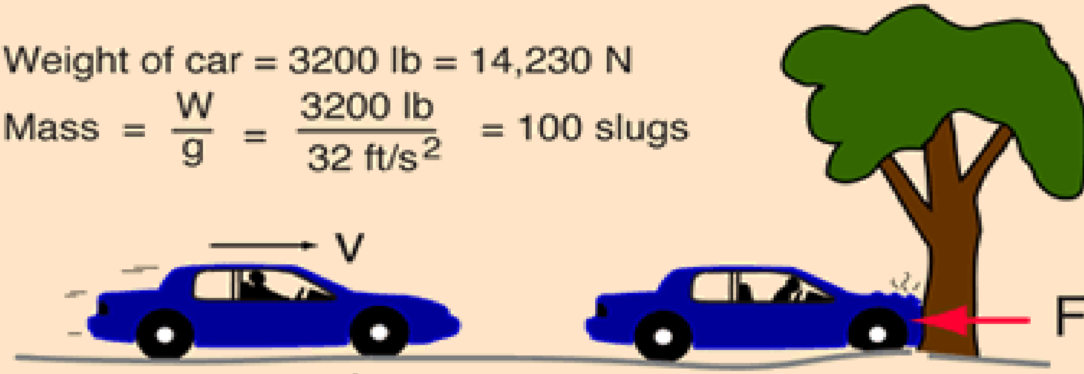
Initial kinetic energy $\frac{1}{2}mv^2$

Work required to stop the car $F_{avg}d = -\frac{1}{2}mv^2$

Example of Force on Car

Weight of car = 3200 lb = 14,230 N

$$\text{Mass} = \frac{W}{g} = \frac{3200\ \text{lb}}{32\ \text{ft/s}^2} = 100\ \text{slugs}$$



What effect would it have on the impact force if the car were more rigid, collapsing only 6 inches?

$$F_{avg} d = -\frac{1}{2} m v^2$$ Work required to stop the car

$$KE_{initial} = \frac{1}{2} m v^2$$

d

$$F_{avg} = \frac{-\frac{1}{2} m v^2}{d}$$

Velocity = 30 mi/hr = 44 ft/s

$$KE = \frac{1}{2}(100\ \text{slugs})(44\ \text{ft/s})^2$$

KE = 96,800 ft lb

d = 1 foot after impact

$$F_{avg} = \frac{96{,}800\ \text{ft lb}}{1\ \text{ft}} = 96{,}800\ \text{lb}$$

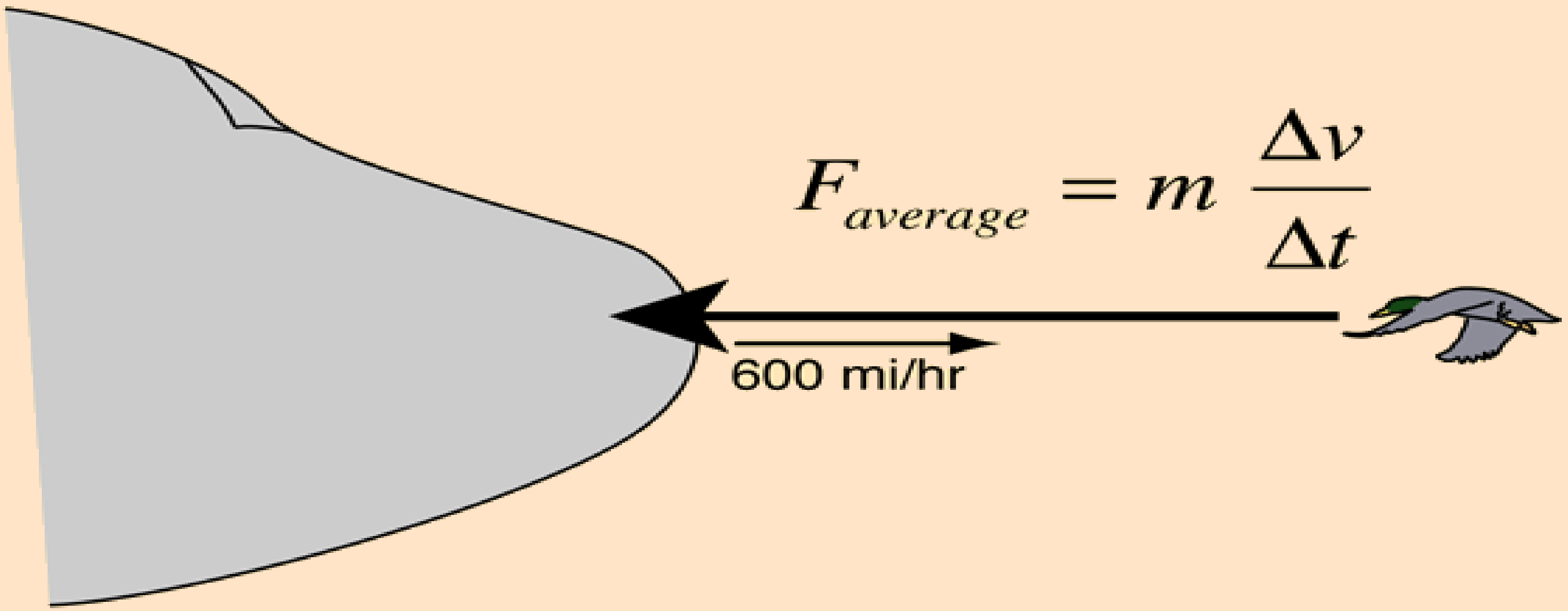
$$= 48.4\ \text{tons!}$$

(This initial example is cast in U.S. common units because most U.S. readers can make comparisons to known forces more easily in those terms. The calculation provides the results in SI units as well.)

Airplane & Duck Estimates

For the airplane and duck force estimate, the mass of the duck is determined, but the change in velocity and time of collision must be estimated in order to estimate the average impact force.

The change in velocity of the duck is estimated to be 600 mi/hr = 880 ft/s by assuming a head on collision, assuming that the duck is riding with the airliner after the collision, and assuming that the duck's velocity is negligible compared to that of the airliner, the "hovering duck" approximation.



$$F_{average} = m \frac{\Delta v}{\Delta t}$$

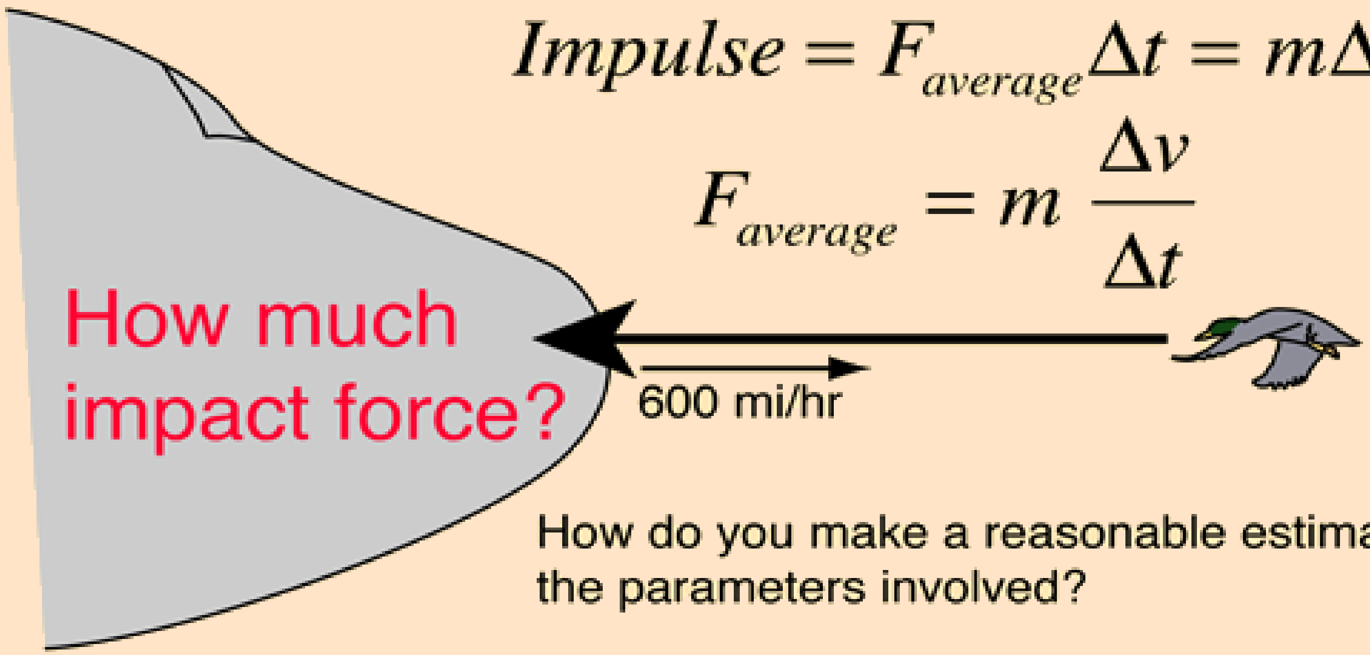
The time of collision is assumed to be the time of transit of the duck's dimension of 1 foot, so 1/880 second.

Airplane and Duck

Estimate the average impact force between an airliner traveling at 600 mi/hr and a 1 pound duck whose length is 1 foot. This is an example of the use of impulse of force.

$$Impulse = F_{average}\Delta t = m\Delta v$$

$$F_{average} = m\,\frac{\Delta v}{\Delta t}$$



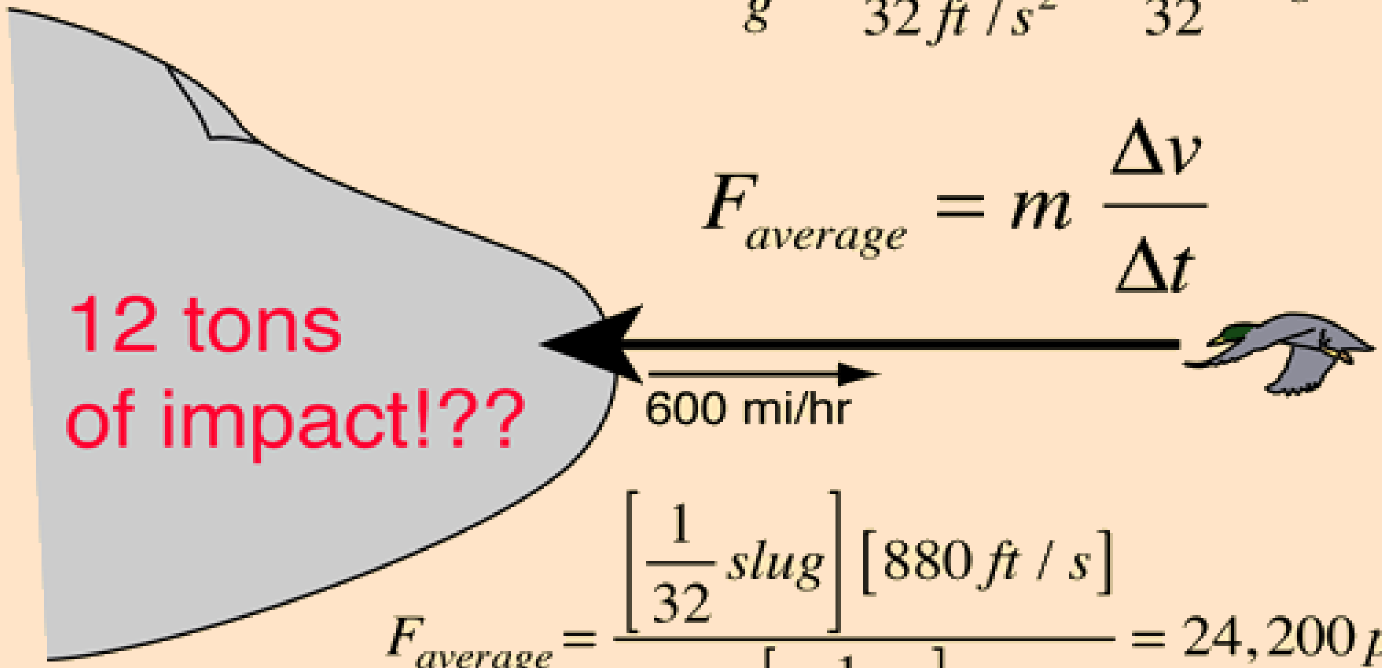
How do you make a reasonable estimate of the parameters involved?

Airplane & Duck Force Estimate

For the airplane and duck force estimate, the mass of the duck is needed, but the weight in the U. S. Common system of units is given. The mass is

$$m = \frac{W}{g} = \frac{1\,lb}{32\,ft/s^2} = \frac{1}{32}\,slug$$

$$F_{average} = m\,\frac{\Delta v}{\Delta t}$$

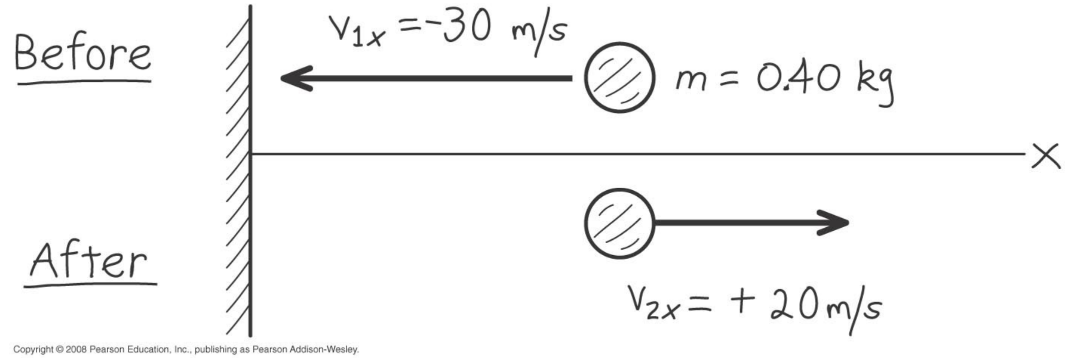


$$F_{average} = \frac{\left[\frac{1}{32}\,slug\right]\left[880\,ft/s\right]}{\left[\frac{1}{880}\,s\right]} = 24,200\,pounds$$

$$F_{average} = 12.1\,tons = 107,650\,N$$

Q8.1

A ball (mass 0.40 kg) is initially moving to the left at 30 m/s. After hitting the wall, the ball is moving to the right at 20 m/s. What is the impulse of the net force on the ball during its collision with the wall?



A. 20 kg • m/s to the right

B. 20 kg • m/s to the left

C. 4.0 kg • m/s to the right

D. 4.0 kg • m/s to the left

E. none of the above

A8.1

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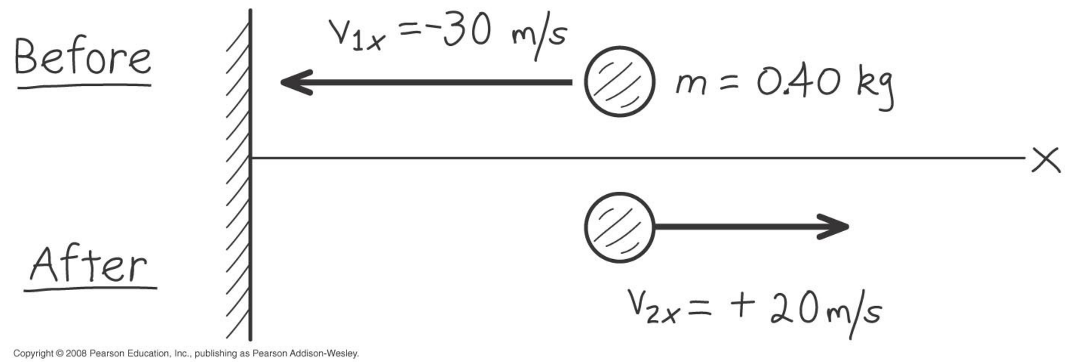

A. 20 kg • m/s to the right ✓

B. 20 kg • m/s to the left

C. 4.0 kg • m/s to the right

D. 4.0 kg • m/s to the left

E. none of the above

Q8.2

You are testing a new car using crash test dummies. Consider two ways to slow the car from 90 km/h (56 mi/h) to a complete stop:

(i) You let the car slam into a wall, bringing it to a sudden stop.

(ii) You let the car plow into a giant tub of gelatin so that it comes to a gradual halt.

In which case is there a greater *impulse* of the net force on the car?

A. in case (i)

B. in case (ii)

C. The impulse is the same in both cases.

D. not enough information given to decide

A8.2

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A. in case (i)

B. in case (ii)

✓ C. The impulse is the same in both cases.

D. not enough information given to decide

Q8.3

A 3.00-kg rifle fires a 0.00500-kg bullet at a speed of 300 m/s. Which force is greater in magnitude:
(i) the force that the *rifle* exerts on the *bullet*; or
(ii) the force that the *bullet* exerts on the *rifle?*

A. the force that the rifle exerts on the bullet

B. the force that the bullet exerts on the rifle

C. both forces have the same magnitude

D. not enough information given to decide

A8.3

A 3.00-kg rifle fires a 0.00500-kg bullet at a speed of 300 m/s. Which force is greater in magnitude:
(i) the force that the *rifle* exerts on the *bullet*; or
(ii) the force that the *bullet* exerts on the *rifle?*

A. the force that the rifle exerts on the bullet

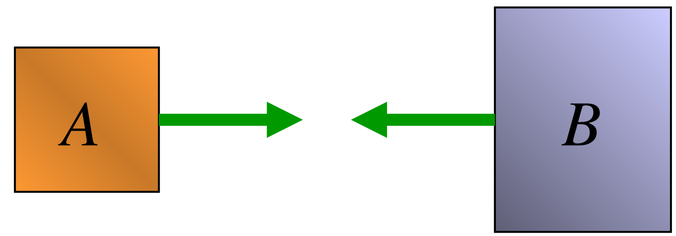
B. the force that the bullet exerts on the rifle

✓ C. both forces have the same magnitude

D. not enough information given to decide

Q8.4

Two objects with different masses collide and *stick* to each other. Compared to *before* the collision, the system of two objects *after* the collision has



A. the same total momentum and the same total kinetic energy.

B. the same total momentum but less total kinetic energy.

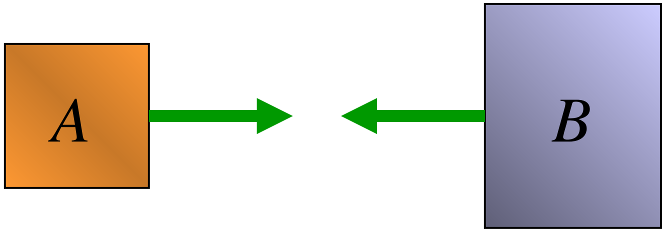
C. less total momentum but the same total kinetic energy.

D. less total momentum and less total kinetic energy.

E. not enough information given to decide

A8.4

Two objects with different masses collide and *stick* to each other. Compared to *before* the collision, the system of two objects *after* the collision has



A. the same total momentum and the same total kinetic energy.

✓ B. the same total momentum but less total kinetic energy.

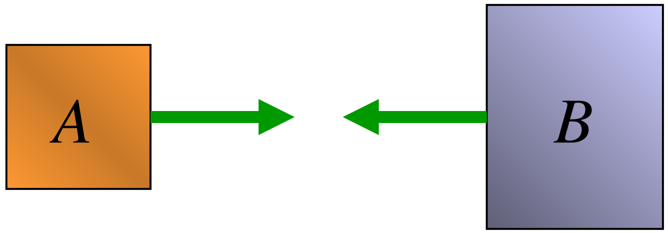
C. less total momentum but the same total kinetic energy.

D. less total momentum and less total kinetic energy.

E. not enough information given to decide

Q8.5

Two objects with different masses collide and *bounce off* each other. Compared to *before* the collision, the system of two objects *after* the collision has



A. the same total momentum and the same total kinetic energy.

B. the same total momentum but less total kinetic energy.

C. less total momentum but the same total kinetic energy.

D. less total momentum and less total kinetic energy.

E. not enough information given to decide

A8.5

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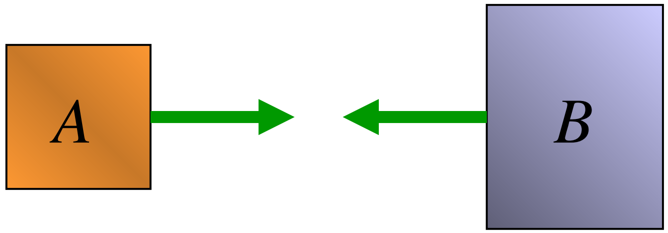

A. the same total momentum and the same total kinetic energy.

B. the same total momentum but less total kinetic energy.

C. less total momentum but the same total kinetic energy.

D. less total momentum and less total kinetic energy.

✓ E. not enough information given to decide

Q8.6

Block *A* has mass 1.00 kg and block *B* has mass 3.00 kg. The blocks collide and stick together on a level, frictionless surface. After the collision, the kinetic energy (KE) of block *A* is

A. 1/9 the KE of block *B*.

B. 1/3 the KE of block *B*.

C. 3 times the KE of block *B*.

D. 9 times the KE of block *B*.

E. the same as the KE of block *B*.

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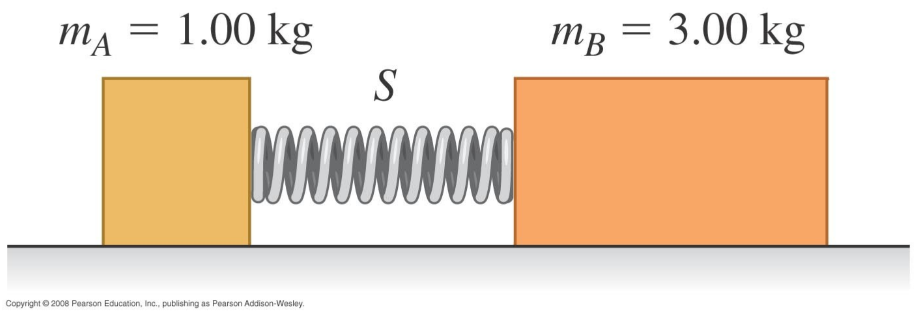
C. 3 times the KE of block *B*.

D. 9 times the KE of block *B*.

E. the same as the KE of block *B*.

Q8.7

Block *A* on the left has mass 1.00 kg. Block *B* on the right has mass 3.00 kg. The blocks are forced together, compressing the spring. Then the system is released from rest on a level, frictionless surface. After the blocks are released, the kinetic energy (KE) of block *A* is



A. 1/9 the KE of block *B*.

B. 1/3 the KE of block *B*.

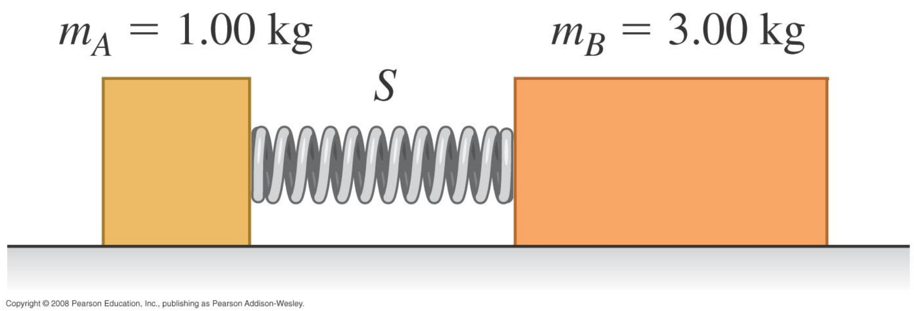
C. 3 times the KE of block *B*.

D. 9 times the KE of block *B*.

E. the same as the KE of block *B*.

A8.7

Block *A* on the left has mass 1.00 kg. Block *B* on the right has mass 3.00 kg. The blocks are forced together, compressing the spring. Then the system is released from rest on a level, frictionless surface. After the blocks are released, the kinetic energy (KE) of block *A* is



A. 1/9 the KE of block *B*.

B. 1/3 the KE of block *B*.

C. 3 times the KE of block *B*.

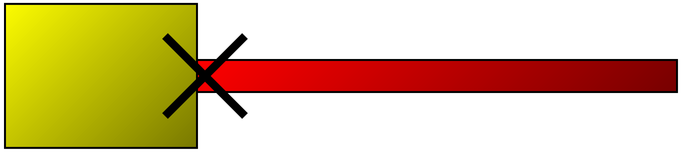
D. 9 times the KE of block *B*.

E. the same as the KE of block *B*.

Q8.9

A yellow block and a red rod are joined together. Each object is of uniform density. The center of mass of the *combined* object is at the position shown by the black “X.”

Which has the *greater mass*, the yellow block or the red rod?



A. the yellow block

B. the red rod

C. they both have the same mass

D. not enough information given to decide

A8.9

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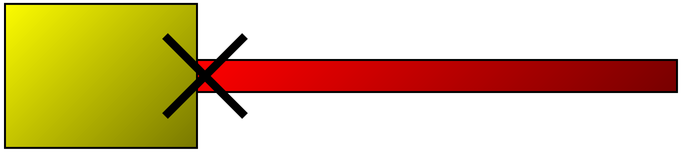


A. the yellow block

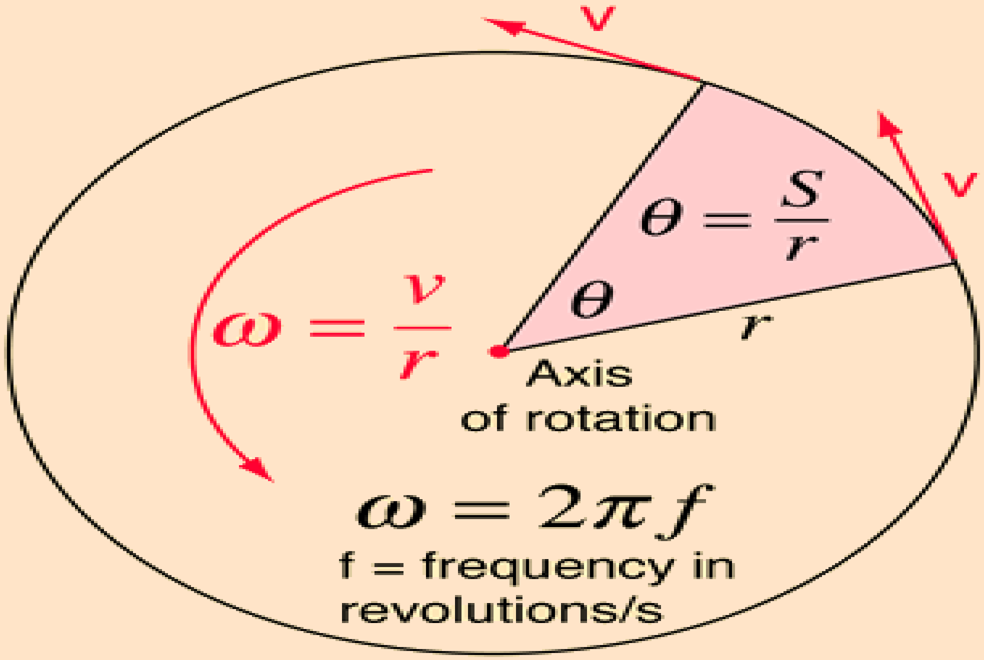
B. the red rod

C. they both have the same mass

D. not enough information given to decide

Rotational Motion

Angular Velocity



For an object rotating about an axis, every point on the object has the same angular velocity. The tangential velocity of any point is proportional to its distance from the axis of rotation. Angular velocity has the units rad/s.

$$v = \omega r \quad or \quad \omega = \frac{v}{r}$$

Angular velocity is the rate of change of angular displacement and can be described by the relationship

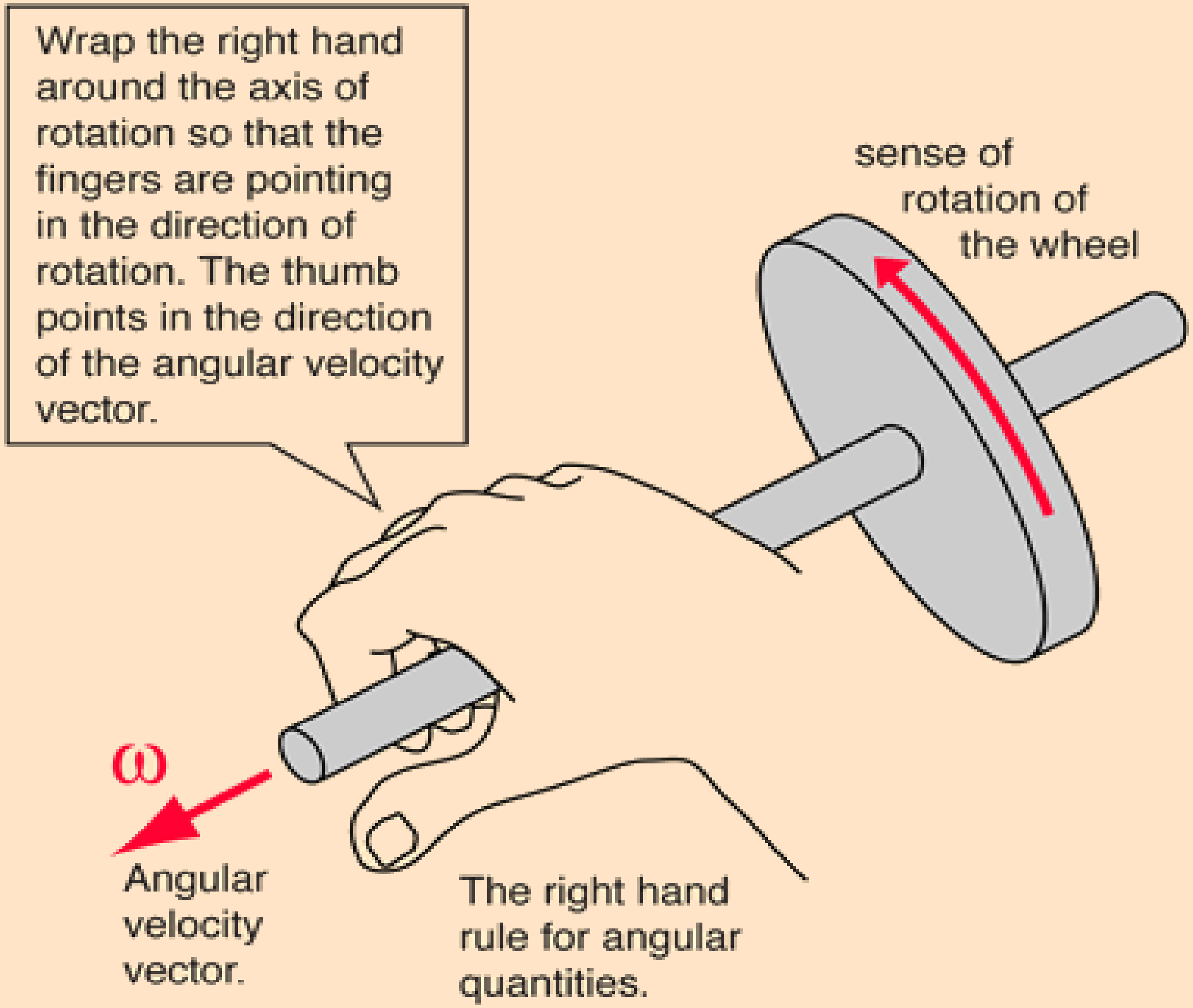
$$\omega_{average} = \frac{\Delta\theta}{\Delta t}$$

Angular velocity can be considered to be a vector quantity, with direction along the axis of rotation in the right-hand rule sense.

and if v is constant, the angle can be calculated from

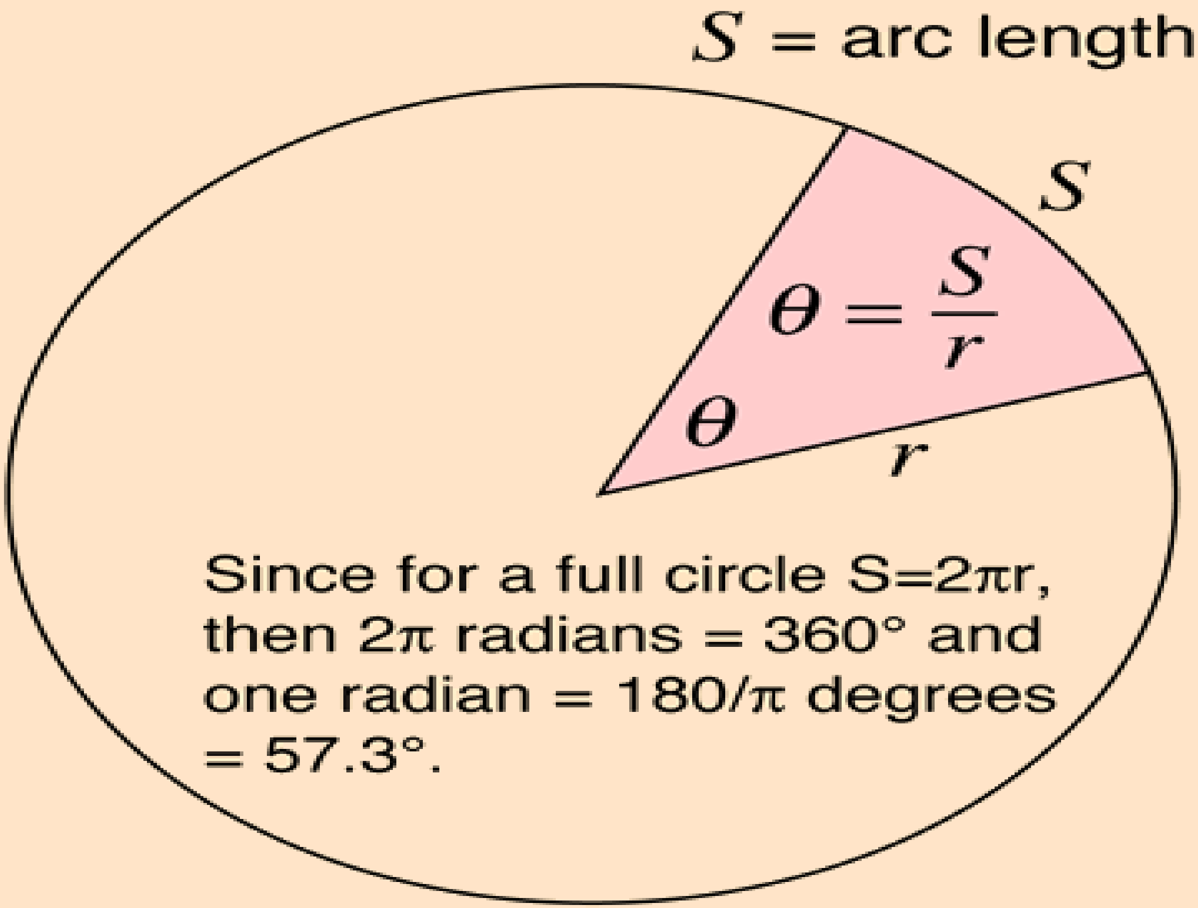
$$\theta = \theta_0 + \omega t$$

Rotation Vectors



Angular motion has direction associated with it and is inherently a vector process. But a point on a rotating wheel is continuously changing direction and it is inconvenient to track that direction. The only fixed, unique direction for a rotating wheel is the axis of rotation, so it is logical to choose this axis direction as the direction of the angular velocity. Left with two choices about direction, it is customary to use the right hand rule to specify the direction of angular quantities.

Basic Rotational Quantities



The angular displancment is defined by:

$$\theta = \frac{S}{r}$$

For a circular path it follows that the angular velocity is

$$\omega = \frac{v}{r}$$

and the angular acceleration is

$$\alpha = \frac{a_t}{r}$$

where the acceleration here is the tangential acceleration.

In addition to any tangential acceleration, there is always the centripetal acceleration:

$$a_c = \frac{v^2}{r}$$

Description of Rotation

Rotation is described in terms of angular displacement , time , angular velocity, and angular acceleration . Angular velocity is the rate of change of angular displacement and angular acceleration is the rate of change of angular velocity. The averages of velocity and acceleration are defined by the relationships:

Average angular velocity: $\overline{\omega} = \dfrac{\Delta\theta}{\Delta t}$

Average angular acceleration: $\overline{\alpha} = \dfrac{\Delta\omega}{\Delta t}$

where the Greek letter delta indicates the change in the quantity following it.

1. $\theta = \overline{\omega} t$ $\qquad \overline{\omega} = \dfrac{\omega_0 + \omega}{2}$

2. $\omega = \omega_0 + \alpha t$

3. $\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2$

4. $\omega^2 = \omega_0^2 + 2\alpha\theta$

Equations for constant angular acceleration.

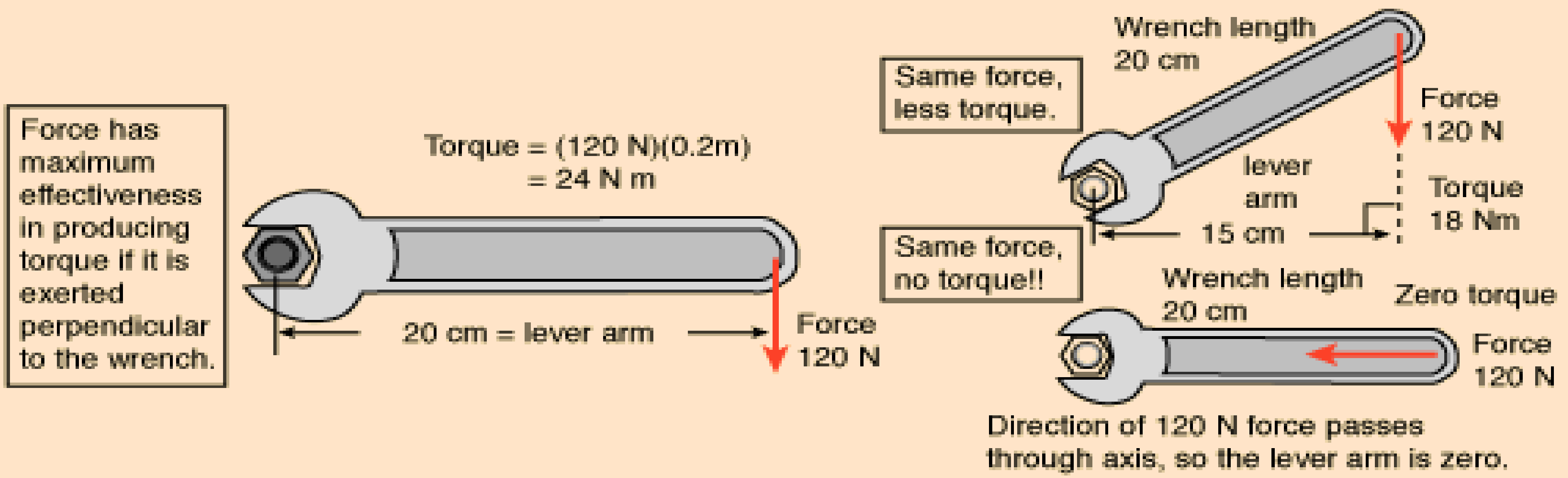
A bar above any quantity indicates the average value of that quantity. If α is constant, equations 1,2, and 3 represent a complete description of the rotation. Equation 4 is obtained by a combination of the others.

Torque

A torque is an influence which tends to change the rotational motion of an object. One way to quantify a torque is

Torque = Force applied x lever arm

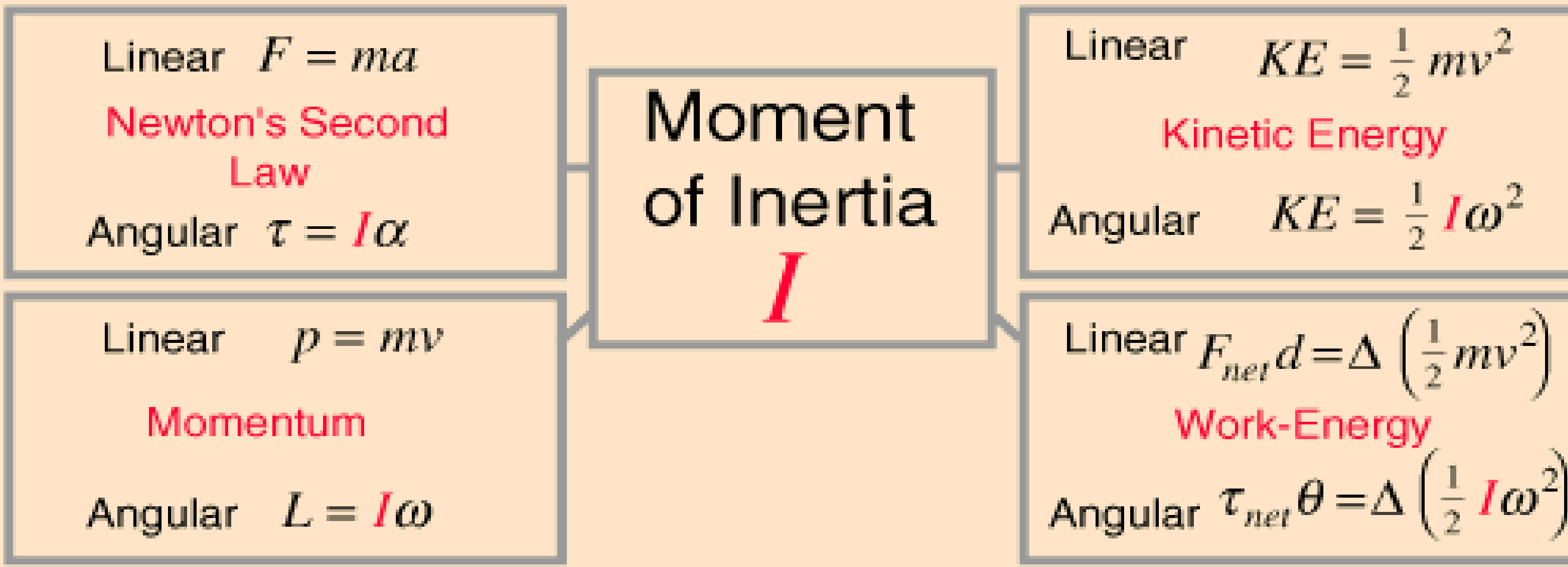
The lever arm is defined as the perpendicular distance from the axis of rotation to the line of action of the force.



Three examples of torque exerted on a wrench of length 20 cm.

Moment of Inertia

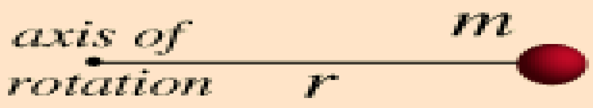
Moment of inertia is the name given to rotational inertia, the rotational analog of mass for linear motion. It appears in the relationships for the dynamics of rotational motion. The moment of inertia must be specified with respect to a chosen axis of rotation. For a point mass the moment of inertia is just the mass times the square of perpendicular distance to the rotation axis, $I = mr^2$. That point mass relationship becomes the basis for all other moments of inertia since any object can be built up from a collection of point masses.



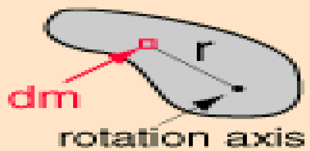
Moment of Inertia, General Form

Since the moment of inertia of an ordinary object involves a continuous distribution of mass at a continually varying distance from any rotation axis, the calculation of moments of inertia generally involves calculus, the discipline of mathematics which can handle such continuous variables. Since the moment of inertia of a point mass is defined by

$$I = mr^2$$



then the moment of inertia contribution by an infinitesmal mass element dm has the same form. This kind of mass element is called a differential element of mass and its moment of inertia is given by



$$dI = r^2 dm$$

The "d" preceding any quantity denotes a vanishingly small or "differential" amount of it.

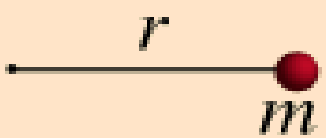
Note that the differential element of moment of inertia dI must always be defined with respect to a specific rotation axis. The sum over all these mass elements is called an integral over the mass.

$$I = \int dI = \int_0^M r^2 dm$$

Usually, the mass element dm will be expressed in terms of the geometry of the object, so that the integration can be carried out over the object as a whole (for example, over a long uniform rod).

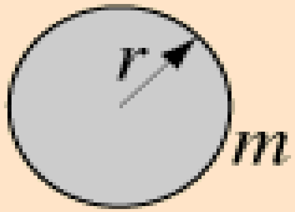
Moment of Inertia Examples

Moment of inertia is defined with respect to a specific rotation axis. The moment of inertia of a point mass with respect to an axis is defined as the product of the mass times the distance from the axis squared. The moment of inertia of any extended object is built up from that basic definition. The general form of the moment of inertia involves an integral.



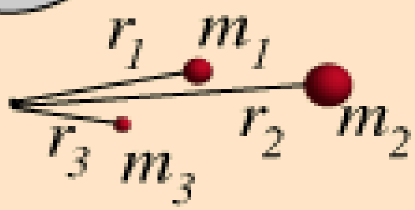
$$I = mr^2$$

For a point mass the moment of inertia is just the mass times the radius from the axis squared. For a collection of point masses (below) the moment of inertia is just the sum for the masses.



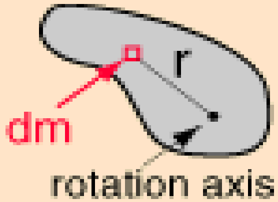
$$I = kmr^2$$

For an object with an axis of symmetry, the moment of inertia is some fraction of that which it would have if all the mass were at the radius r.



$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots$$

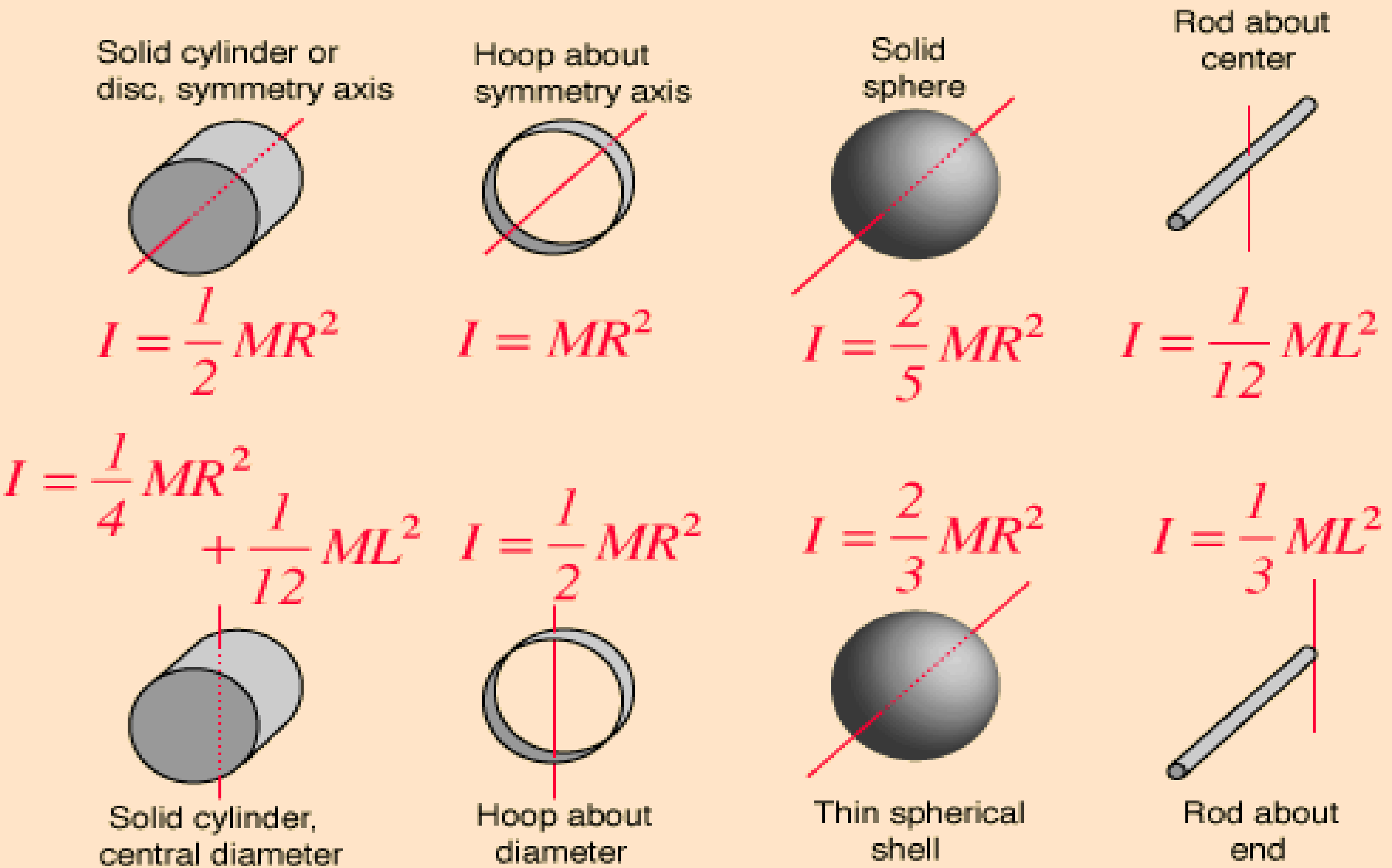
Sum of the point mass moments of inertia.



$$I = \int_0^M r^2 \, dm$$

Continuous mass distributions require an infinite sum of all the point mass moments which make up the whole. This is accomplished by an integration over all the mass.

Common Moments of Inertia



Rotational-Linear Parallels

	Linear Motion	Rotational Motion	
Position	x	θ	Angular position
Velocity	v	ω	Angular velocity
Acceleration	a	α	Angular acceleration
Motion equations	$x = \bar{v}t$	$\theta = \bar{\omega}t$	Motion equations
	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$	
	$x = v_0 t + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	
	$v^2 = v_0^2 + 2ax$	$\omega^2 = \omega_0^2 + 2\alpha\theta$	
Mass (linear inertia)	m	I	Moment of inertia
Newton's second law	$F = ma$	$\tau = I\alpha$	Newton's second law
Momentum	$p = mv$	$L = I\omega$	Angular momentum
Work	Fd	$\tau\theta$	Work
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	Kinetic energy
Power	Fv	$\tau\omega$	Power

Equilibrium

Conditions for Equilibrium

An object at equilibrium has no net influences to cause it to move, either in translation (linear motion) or rotation. The basic conditions for equilibrium are:

1. Net force = 0

$$\sum_i F_i = 0$$

x and y components of force may be separately set = 0.

Forces left = forces right
Forces up = forces down.

2. Net torque = 0

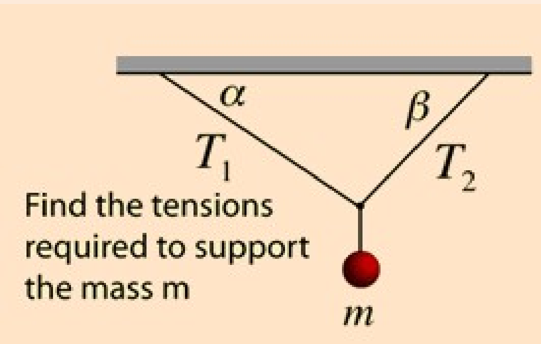
$$\sum_i \tau_i = 0$$

The axis may be chosen for advantage to eliminate some unknown forces..

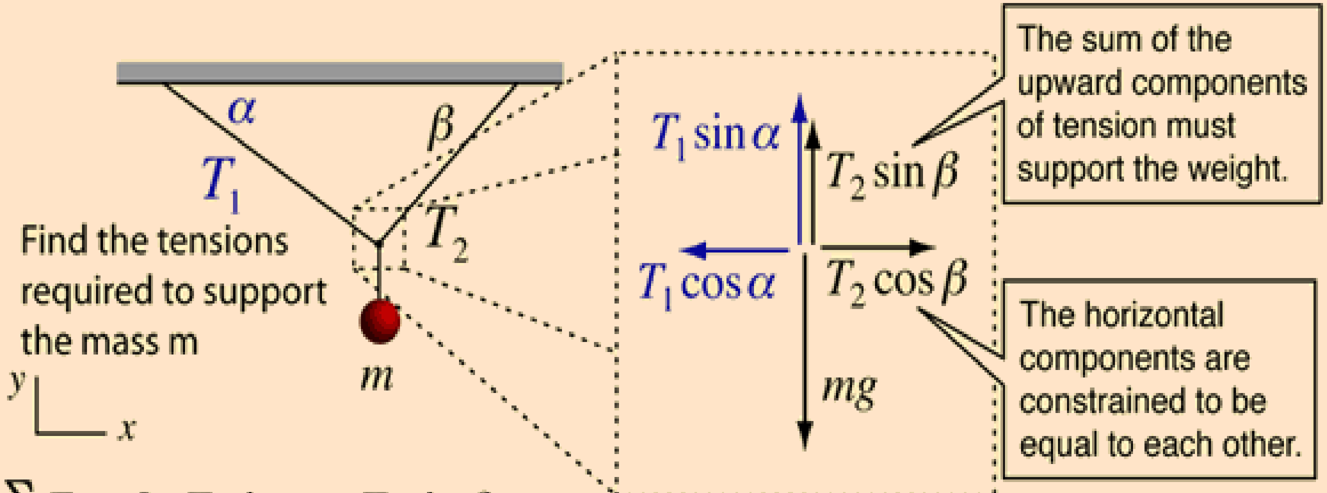
The sum of the clockwise torques is equal to the sum of the counterclockwise torques.

The conditions for equilibrium are basic to the design of any load-bearing structure such as a bridge or a building since such structures must be able to maintain equilibrium under load. They are also important for the study of machines, since one must first establish equilibrium and then apply extra force or torque to produce the desired movement of the machine. The conditions of equilibrium are used to analyze the "simple machines" which are the building blocks for more complex machines.

Force Equilibrium Example



Force equilibrium problems like this can be analyzed by drawing a free-body diagram of the point of attachment of the mass m, since it must be at equilibrium. The tensions should be resolved into horizontal and vertical components to apply the force equilibrium condition.



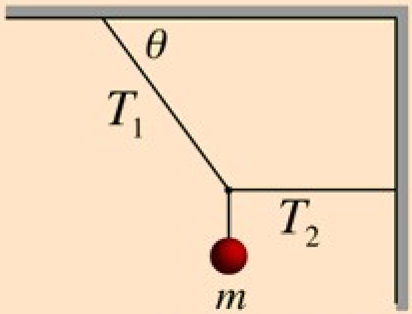
$$\sum F_y = 0 : T_1 \sin\alpha + T_2 \sin\beta = mg$$

$$\sum F_x = 0 : T_1 \cos\alpha = T_2 \cos\beta$$

$$\therefore\ T_2 = T_1 \frac{\cos\alpha}{\cos\beta} \qquad T_1 = \frac{mg}{\sin\alpha + \frac{\cos\alpha}{\cos\beta}\sin\beta}$$

Force Equilibrium Example

Force equilibrium problems like this can be analyzed by drawing a free body diagram of the point of attachment of the mass m, since it must be at equilibrium. The tensions should be resolved into horizontal and vertical components to apply the force equilibrium condition.

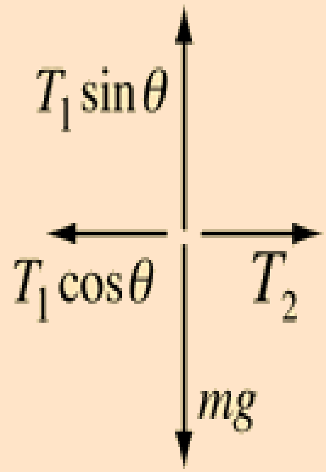


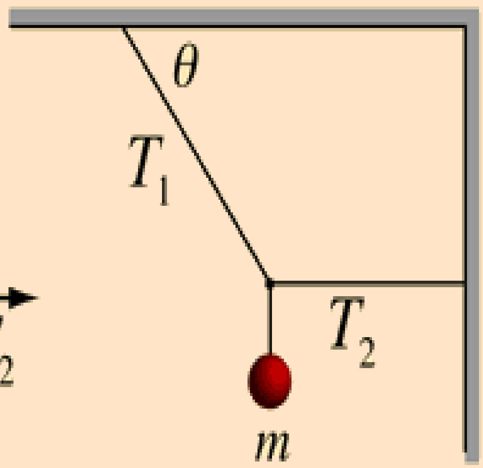
Find the required tensions if one cable is horizontal.

$$\sum F_y = 0 : T_1 \sin\theta = mg$$

$$\sum F_x = 0 : T_1 \cos\theta = T_2$$

$$T_1 = \frac{mg}{\sin\theta}$$

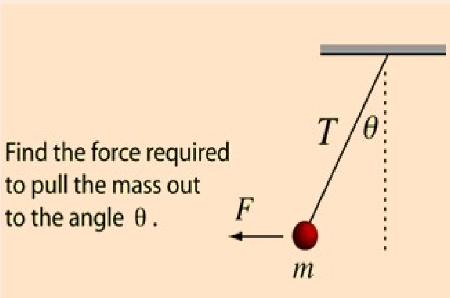


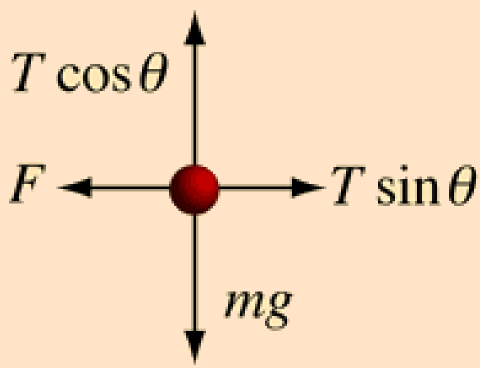


Find the required tensions if one cable is horizontal.

Force Equilibrium Example

Force equilibrium problems like this can be analyzed by drawing a free body diagram of the point of attachment of the mass m, since it must be at equilibrium. The tensions should be resolved into horizontal and vertical components to apply the force equilibrium condition.

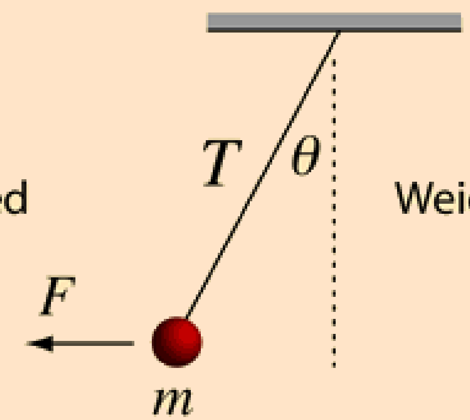




$$\sum F_y = 0 : T\cos\theta = mg \quad \therefore\ T = \frac{mg}{\cos\theta}$$

$$\sum F_x = 0 : T\sin\theta = F \ \therefore F = mg\tan\theta$$

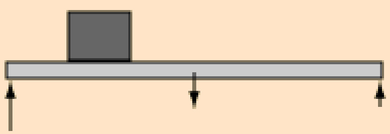
Find the force required
to pull the mass out
to the angle θ .



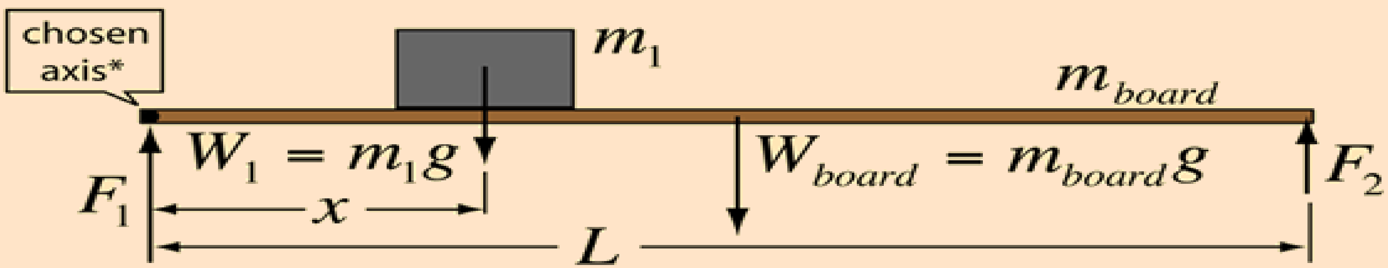
Weight = mg

Supporting an Extended Load

For an extended system to be at equilibrium, the sum of the forces must be equal to zero and the sum of torques about any axis must equal zero. It is logical to choose one of the ends as the axis since that eliminates one of the unknown forces (lever arm = zero). Considering a long uniform wooden board, note that the mass of the board can be considered to be concentrated at its center of mass for the purposes of calculating torque. For a uniform board, the center of mass is at its geometrical center, so the lever arm with respect to either end of the board will be L/2. Choosing the left end as the axis, the torque and force equilibrium equations are shown below.



Supporting an extended object.



$$W_1 x + W_{board} \frac{L}{2} = F_2 L$$

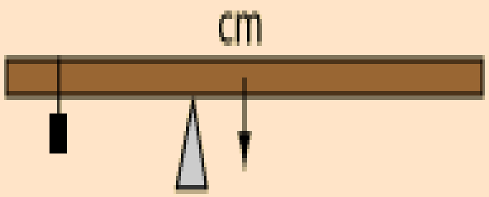
(from $\tau_{clockwise} = \tau_{counterclockwise}$)

$$F_1 + F_2 = W_1 + W_{board}$$

(from forces up = forces down)

Determining the Mass of an Extended Oject

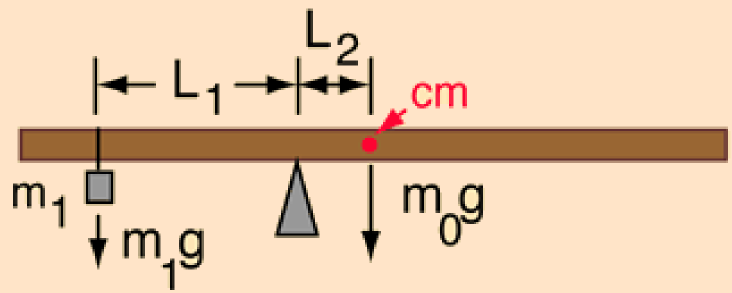
The mass of an extended object can be found by using the conditions for equilibrium of torques. If the object is first balanced to find its center of mass, then the entire weight of the object can be considered to act at that center of mass. If the object is then shifted a measured distance away from the center of mass and again balanced by hanging a known mass on the other side of the pivot point, the unknown mass of the object can be determined by balancing the torques.



Mass of an extended object.

$$\tau_{counterclockwise} = \tau_{clockwise}$$

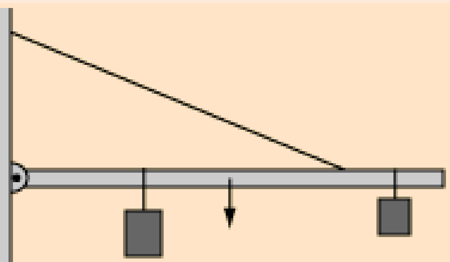
$$m_1 g L_1 = m_0 g L_2$$



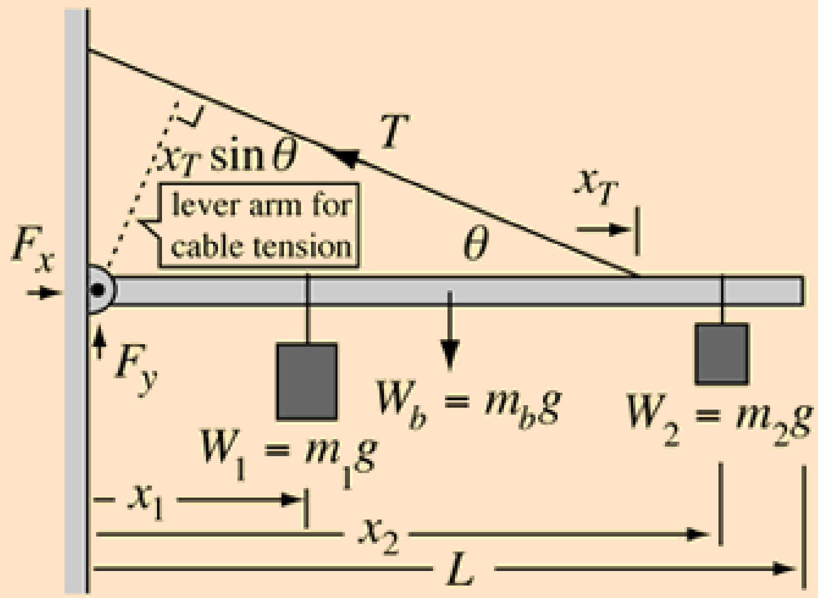
Mass of an extended object.

Support of a Boom

The support of a pivoted, uniform boom with a cable is a standard exercise in equilibrium of torques. Using a pivot at the wall, which is assumed to exert no torque, the torque equation is that shown below. The forces exerted on the boom are then obtainable from the force equation shown. The lever arm for the cable tension T must be obtained from the triangle as shown since the cable is not perpendicular to the boom.



Balancing the boom.



$$\tau_{clockwise} = \tau_{counterclockwise}$$

$$W_2 x_2 + W_1 x_1 + W_b \frac{L}{2} = T x_T \sin\theta$$

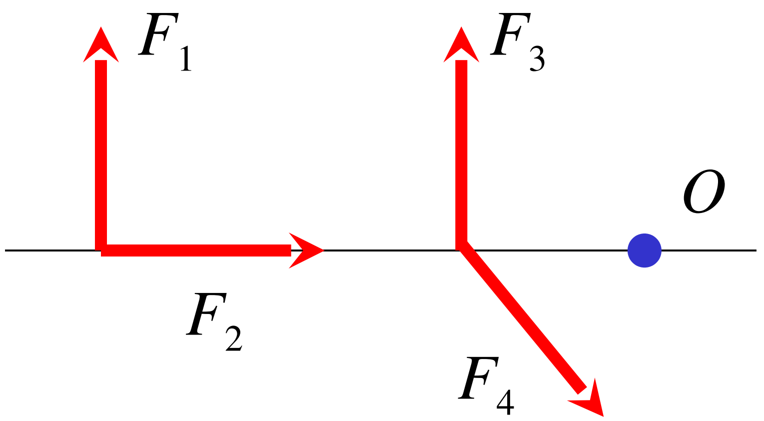
$$F_x = T\cos\theta$$

$$F_y = W_1 + W_2 + W_b - T\sin\theta$$

Q10.1

The four forces shown all have the same magnitude: $F_1 = F_2 = F_3 = F_4$.

Which force produces the *greatest torque* about the point *O* (marked by the blue dot)?



A. F_1

B. F_2

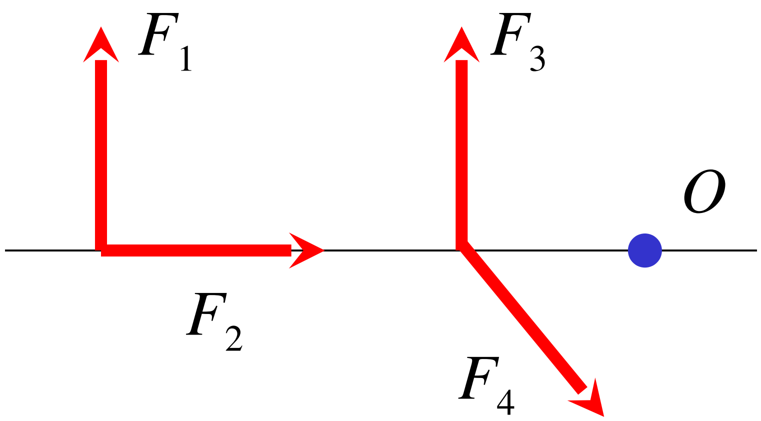
C. F_3

D. F_4

E. not enough information given to decide

A10.1

The four forces shown all have the same magnitude: $F_1 = F_2 = F_3 = F_4$.



Which force produces the *greatest torque* about the point O (marked by the blue dot)?

A. F_1 ✓

B. F_2

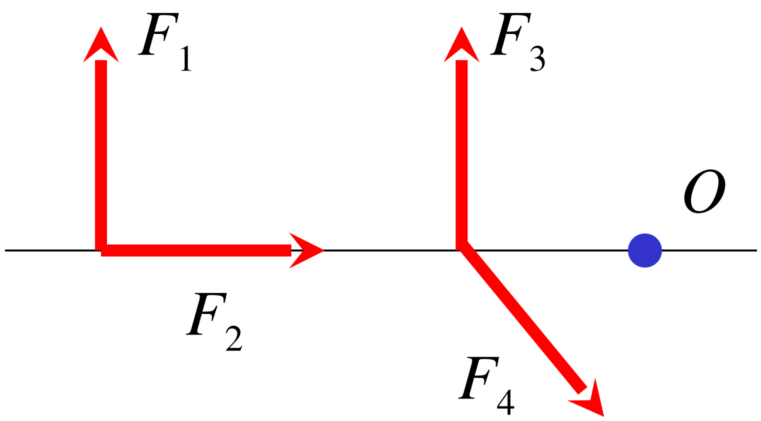
C. F_3

D. F_4

E. not enough information given to decide

Q10.2

Which of the four forces shown here produces a torque about *O* that is directed *out of* the plane of the drawing?



A. F_1

B. F_2

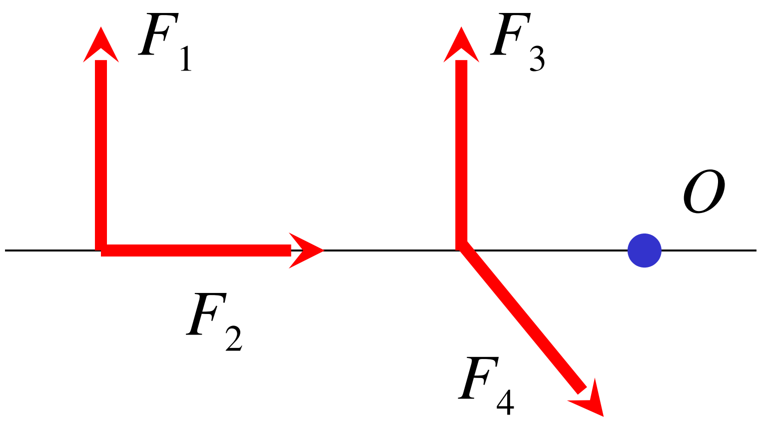
C. F_3

D. F_4

E. more than one of these

A10.2

Which of the four forces shown here produces a torque about *O* that is directed *out of* the plane of the drawing?



A. F_1

B. F_2

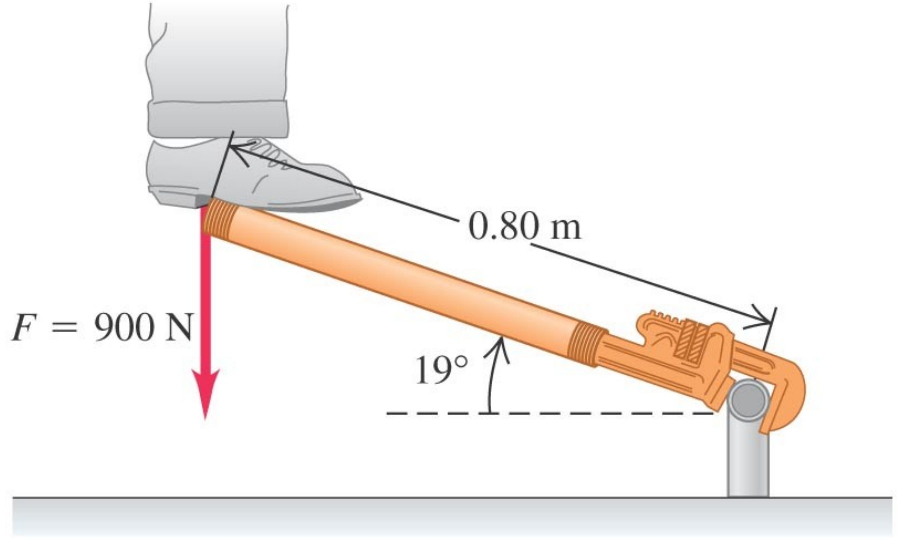
C. F_3

D. F_4

E. more than one of these

Q10.3

A plumber pushes straight down on the end of a long wrench as shown. What is the magnitude of the torque he applies about the pipe at lower right?



A. (0.80 m)(900 N)sin 19°

B. (0.80 m)(900 N)cos 19°

C. (0.80 m)(900 N)tan 19°

D. none of the above

A10.3

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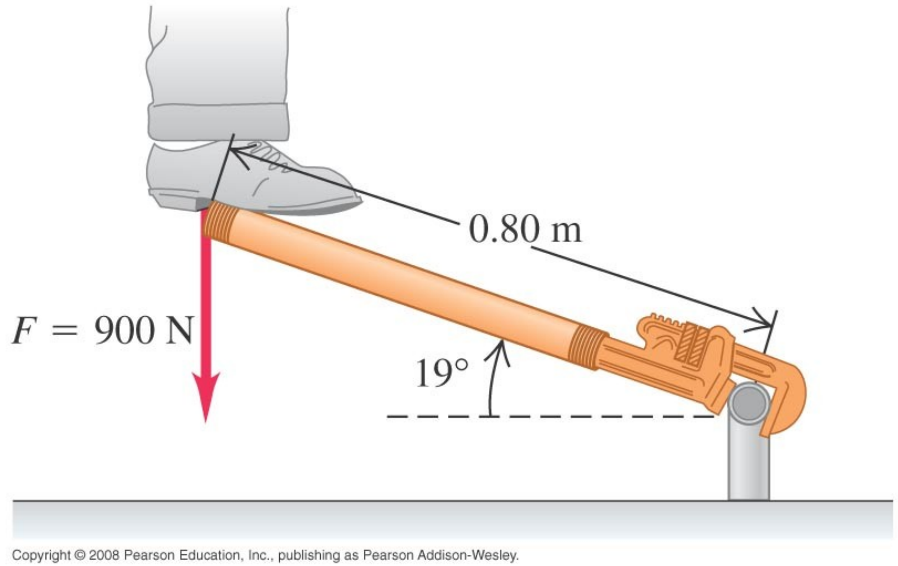

A. (0.80 m)(900 N)sin 19°

✔ B. (0.80 m)(900 N)cos 19°

C. (0.80 m)(900 N)tan 19°

D. none of the above

Q10.4

A force $\vec{F} = \left(4\hat{\boldsymbol{i}} + 3\hat{\boldsymbol{j}}\right)\text{N}$ acts an object at a point located at the position $\vec{r} = \left(6\hat{\boldsymbol{k}}\right)\text{m}$.

What is the torque that this force applies about the origin?

A. zero

B. $\left(24\hat{\boldsymbol{i}} + 18\hat{\boldsymbol{j}}\right)\text{N}\cdot\text{m}$

C. $\left(-24\hat{\boldsymbol{i}} - 18\hat{\boldsymbol{j}}\right)\text{N}\cdot\text{m}$

D. $\left(-18\hat{\boldsymbol{i}} + 24\hat{\boldsymbol{j}}\right)\text{N}\cdot\text{m}$

E. $\left(-18\hat{\boldsymbol{i}} - 24\hat{\boldsymbol{j}}\right)\text{N}\cdot\text{m}$

A10.4

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What is the torque that this force applies about the origin?

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B. $\left(24\hat{i} + 18\hat{j}\right)\mathrm{N}\cdot\mathrm{m}$

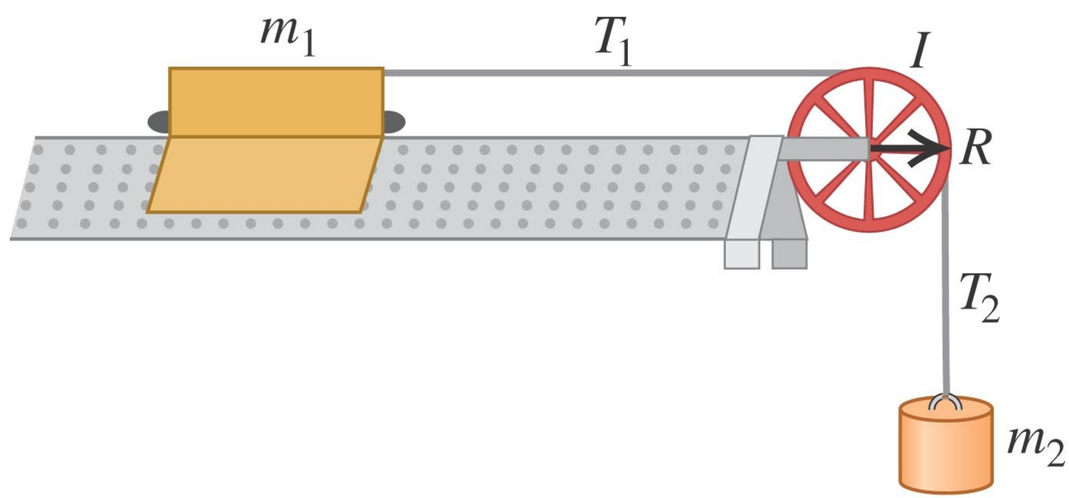
C. $\left(-24\hat{i} - 18\hat{j}\right)\mathrm{N}\cdot\mathrm{m}$

✓ D. $\left(-18\hat{i} + 24\hat{j}\right)\mathrm{N}\cdot\mathrm{m}$

E. $\left(-18\hat{i} - 24\hat{j}\right)\mathrm{N}\cdot\mathrm{m}$

Q10.5

A glider of mass m_1 on a frictionless horizontal track is connected to an object of mass m_2 by a massless string. The glider accelerates to the right, the object accelerates downward, and the string rotates the pulley. What is the relationship among T_1 (the tension in the horizontal part of the string), T_2 (the tension in the vertical part of the string), and the weight m_2g of the object?



A. $m_2g = T_2 = T_1$

B. $m_2g > T_2 = T_1$

C. $m_2g > T_2 > T_1$

D. $m_2g = T_2 > T_1$

E. none of the above

A10.5

A glider of mass m_1 on a frictionless horizontal track is connected to an object of mass m_2 by a massless string. The glider accelerates to the right, the object accelerates downward, and the string rotates the pulley. What is the relationship among T_1 (the tension in the horizontal part of the string), T_2 (the tension in the vertical part of the string), and the weight m_2g of the object?

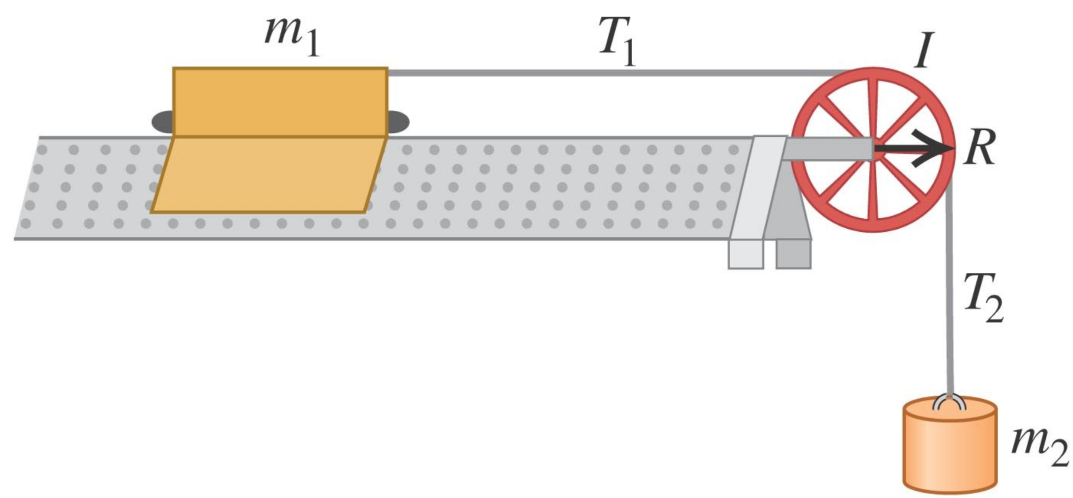
A. $m_2g = T_2 = T_1$

B. $m_2g > T_2 = T_1$

C. $m_2g > T_2 > T_1$

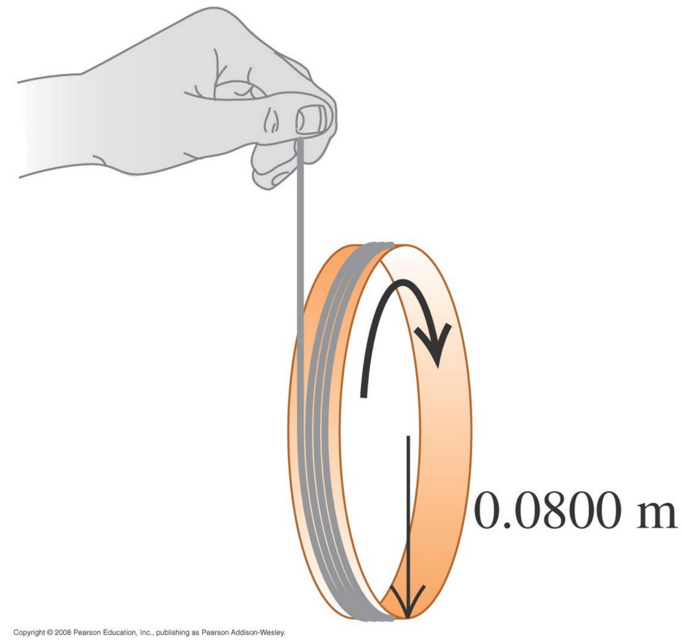
D. $m_2g = T_2 > T_1$

E. none of the above



Q10.6

A lightweight string is wrapped several times around the rim of a small hoop. If the free end of the string is held in place and the hoop is released from rest, the string unwinds and the hoop descends. How does the tension in the string (T) compare to the weight of the hoop (w)?



A. $T = w$

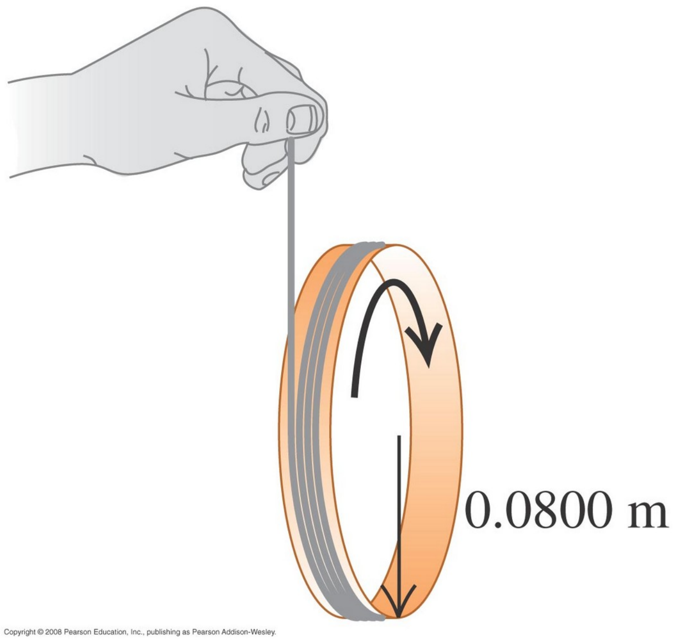
B. $T > w$

C. $T < w$

D. not enough information given to decide

A10.6

A lightweight string is wrapped several times around the rim of a small hoop. If the free end of the string is held in place and the hoop is released from rest, the string unwinds and the hoop descends. How does the tension in the string (T) compare to the weight of the hoop (w)?



A. $T = w$

B. $T > w$

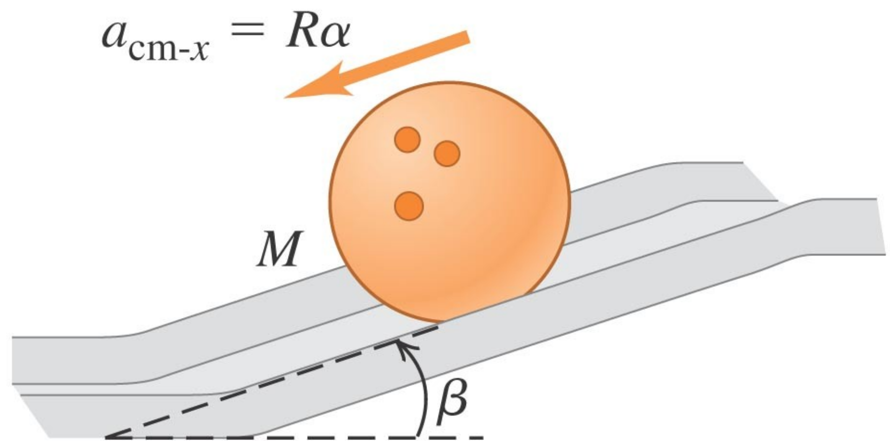
✓ C. $T < w$

D. not enough information given to decide

Q10.7

A solid bowling ball rolls down a ramp.

Which of the following forces exerts a torque on the bowling ball about its center?



A. the weight of the ball

B. the normal force exerted by the ramp

C. the friction force exerted by the ramp

D. more than one of the above

E. The answer depends on whether the ball rolls without slipping.

A10.7

A solid bowling ball rolls down a ramp.

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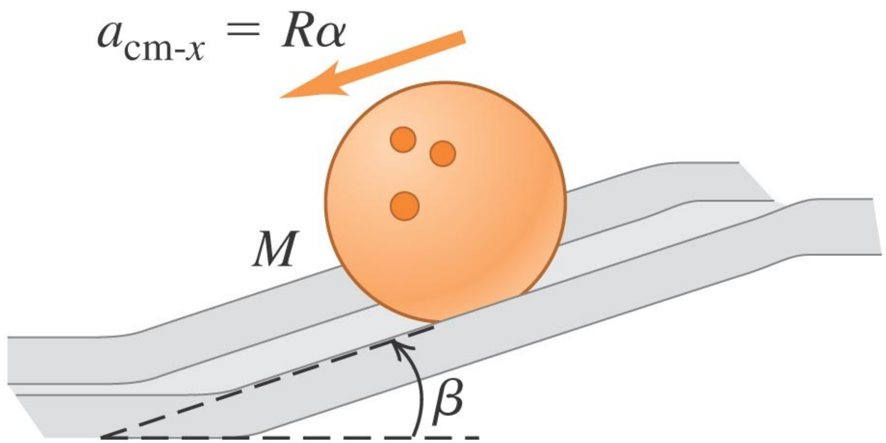


A. the weight of the ball

B. the normal force exerted by the ramp

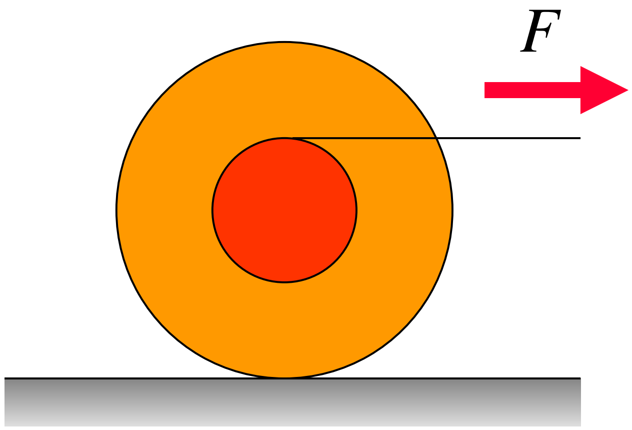
✔ C. the friction force exerted by the ramp

D. more than one of the above

E. The answer depends on whether the ball rolls without slipping.

Q10.8

A yo-yo is placed on a horizontal surface as shown. There is sufficient friction for the yo-yo to roll without slipping. If the string is pulled to the right as shown,



A. the yo-yo rolls to the right.

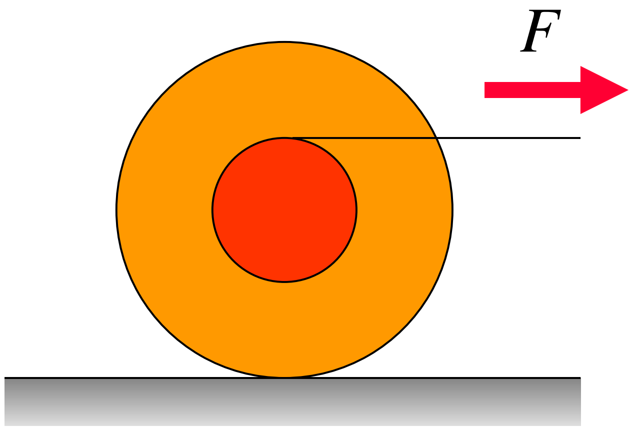
B. the yo-yo rolls to the left.

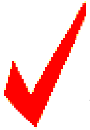
C. the yo-yo remains at rest.

D. The answer depends on the magnitude F of the pulling force compared to the magnitude of the friction force.

A10.8

A yo-yo is placed on a horizontal surface as shown. There is sufficient friction for the yo-yo to roll without slipping. If the string is pulled to the right as shown,



A. the yo-yo rolls to the right.

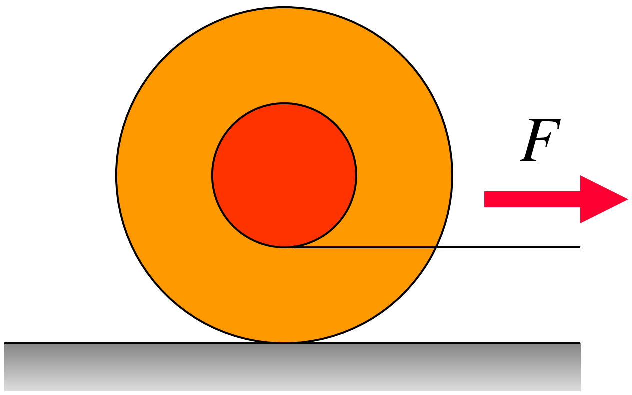
B. the yo-yo rolls to the left.

C. the yo-yo remains at rest.

D. The answer depends on the magnitude *F* of the pulling force compared to the magnitude of the friction force.

Q10.9

A yo-yo is placed on a horizontal surface as shown. There is sufficient friction for the yo-yo to roll without slipping. If the string is pulled to the right as shown,



A. the yo-yo rolls to the right.

B. the yo-yo rolls to the left.

C. the yo-yo remains at rest.

D. The answer depends on the magnitude F of the pulling force compared to the magnitude of the friction force.

A10.9

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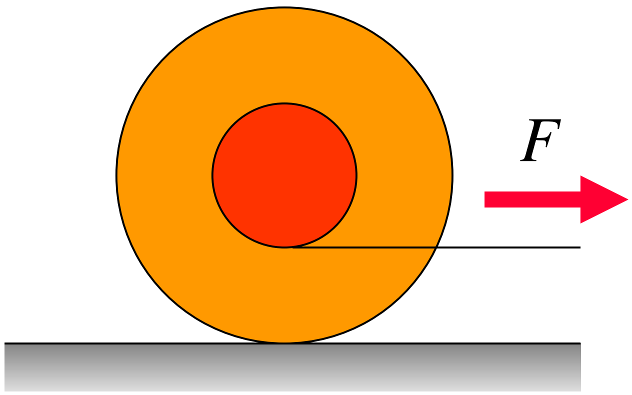


A. the yo-yo rolls to the right.

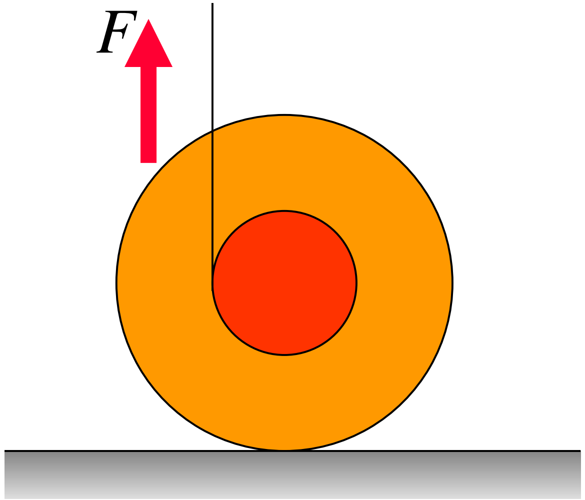
B. the yo-yo rolls to the left.

C. the yo-yo remains at rest.

D. The answer depends on the magnitude F of the pulling force compared to the magnitude of the friction force.

Q10.10

A yo-yo is placed on a horizontal surface as shown. There is sufficient friction for the yo-yo to roll without slipping. If the string is pulled straight up as shown,



A. the yo-yo rolls to the right.

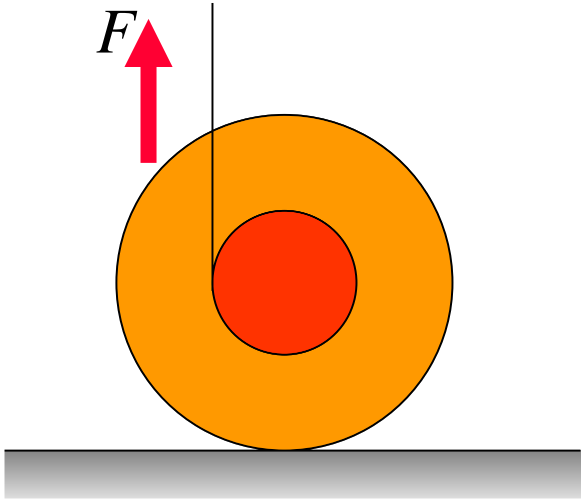
B. the yo-yo rolls to the left.

C. the yo-yo remains at rest.

D. The answer depends on the magnitude F of the pulling force compared to the magnitude of the friction force.

A10.10

A yo-yo is placed on a horizontal surface as shown. There is sufficient friction for the yo-yo to roll without slipping. If the string is pulled straight up as shown,



A. the yo-yo rolls to the right.

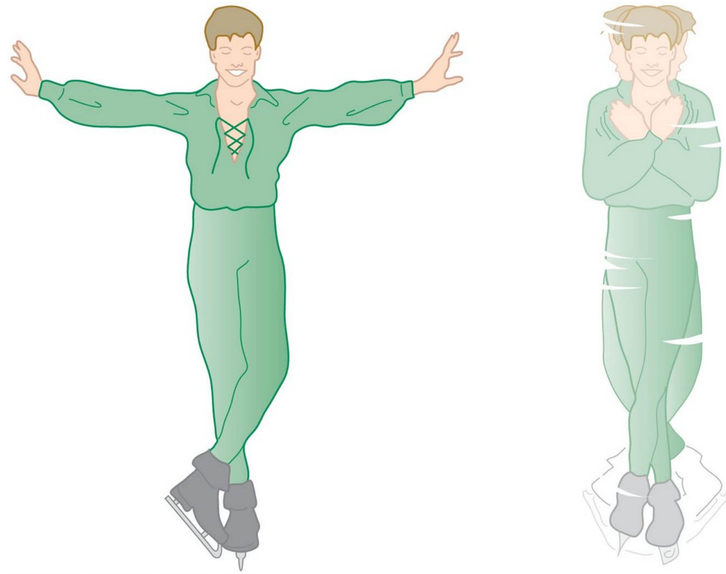
B. the yo-yo rolls to the left.

C. the yo-yo remains at rest.

D. The answer depends on the magnitude *F* of the pulling force compared to the magnitude of the friction force.

Q10.11

A spinning figure skater pulls his arms in as he rotates on the ice. As he pulls his arms in, what happens to his angular momentum L and kinetic energy K?



A. L and K both increase.

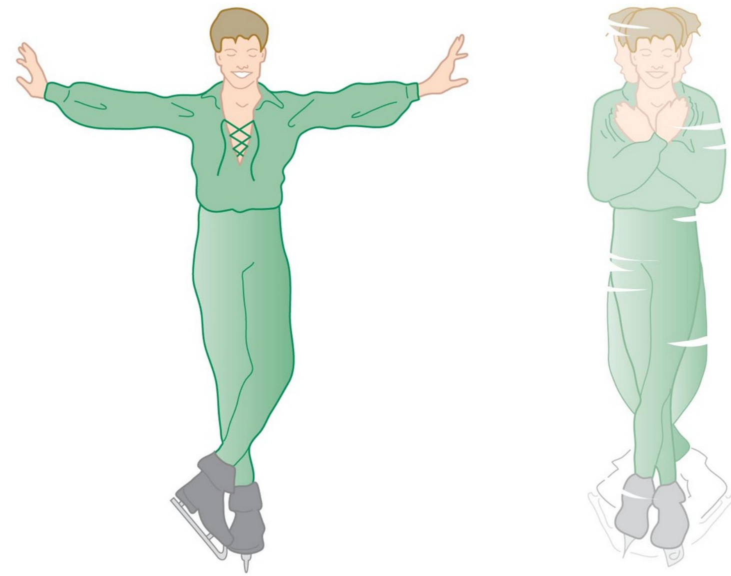
B. L stays the same, K increases.

C. L increases, K stays the same.

D. L and K both stay the same.

A10.11

A spinning figure skater pulls his arms in as he rotates on the ice. As he pulls his arms in, what happens to his angular momentum *L* and kinetic energy *K*?



A. *L* and *K* both increase.

B. *L* stays the same, *K* increases.

C. *L* increases, *K* stays the same.

D. *L* and *K* both stay the same.

Directions of Angular Quantities

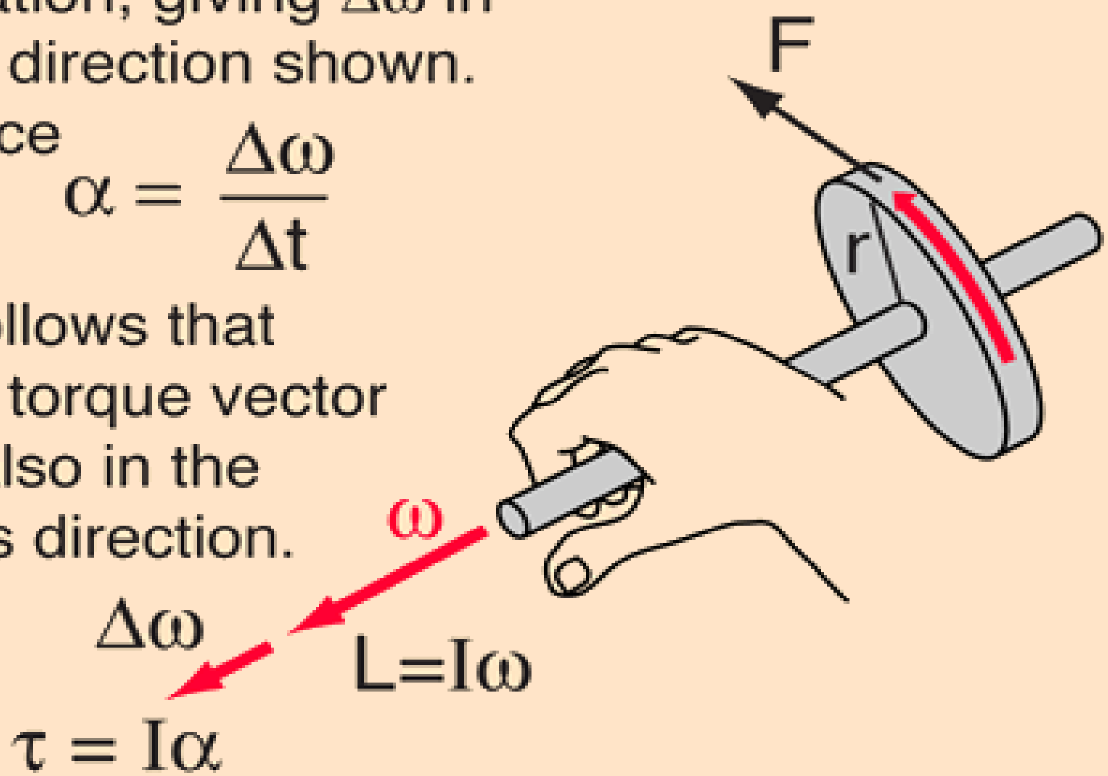
In this case the torque

$$\tau = Fr = I\alpha$$

acts to speed up the rotation, giving $\Delta\omega$ in the direction shown. Since

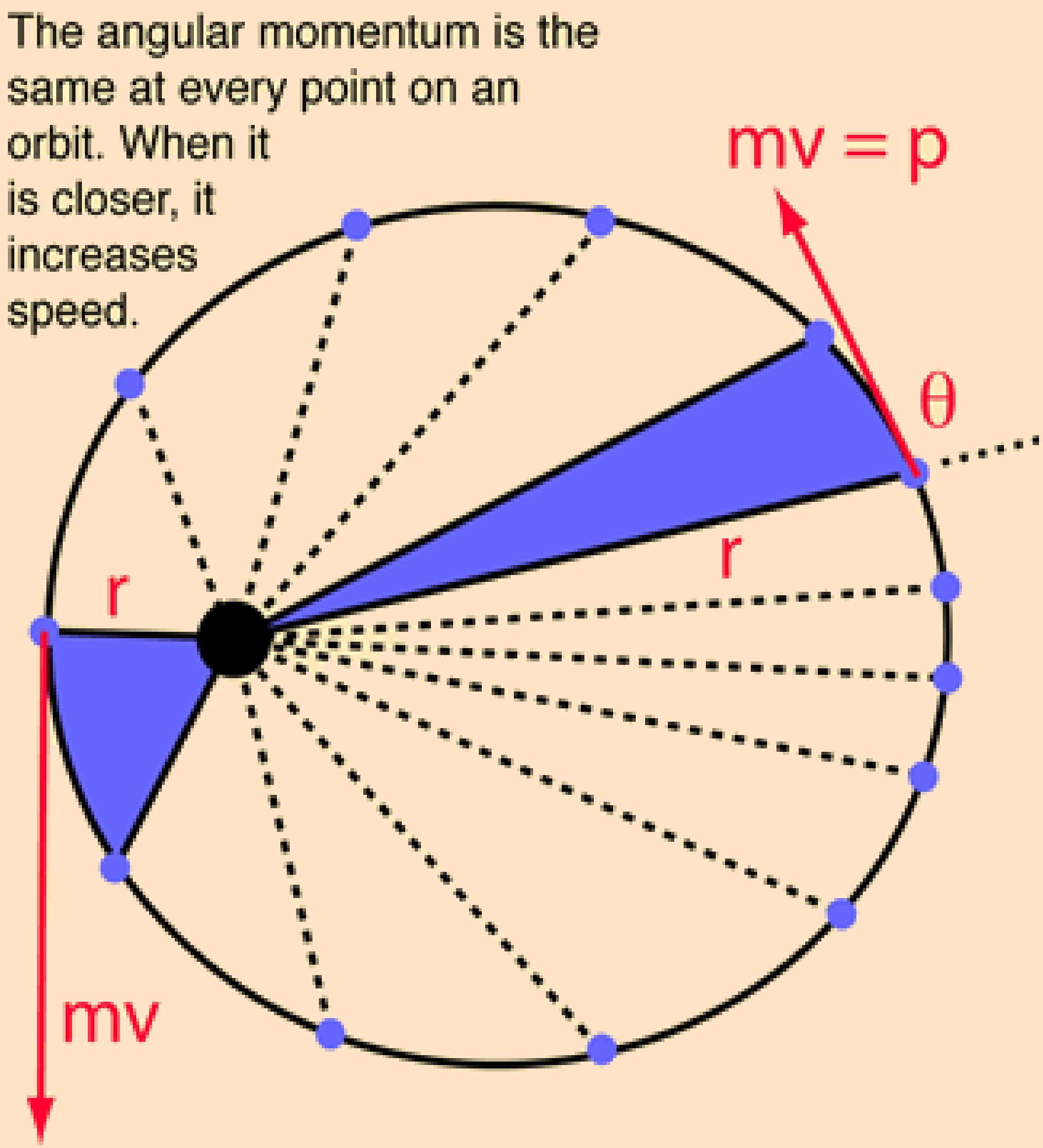
$$\alpha = \frac{\Delta\omega}{\Delta t}$$

it follows that the torque vector is also in the axis direction.



As an example of the directions of angular quantities, consider a vector angular velocity as shown. If a force acts tangential to the wheel to speed it up, it follows that the change in angular velocity and therefore the angular acceleration are in the direction of the axis. Newton's 2nd law for rotation implies that the torque is also in the axis direction. The angular momentum will also be in this direction, so in this example, all of these angular quantities act along the axis of rotation as shown.

Angular Momentum of a Particle



The angular momentum of a particle of mass m with respect to a chosen origin is given by

$$L = mvr \sin\theta$$

or more formally by the vector product

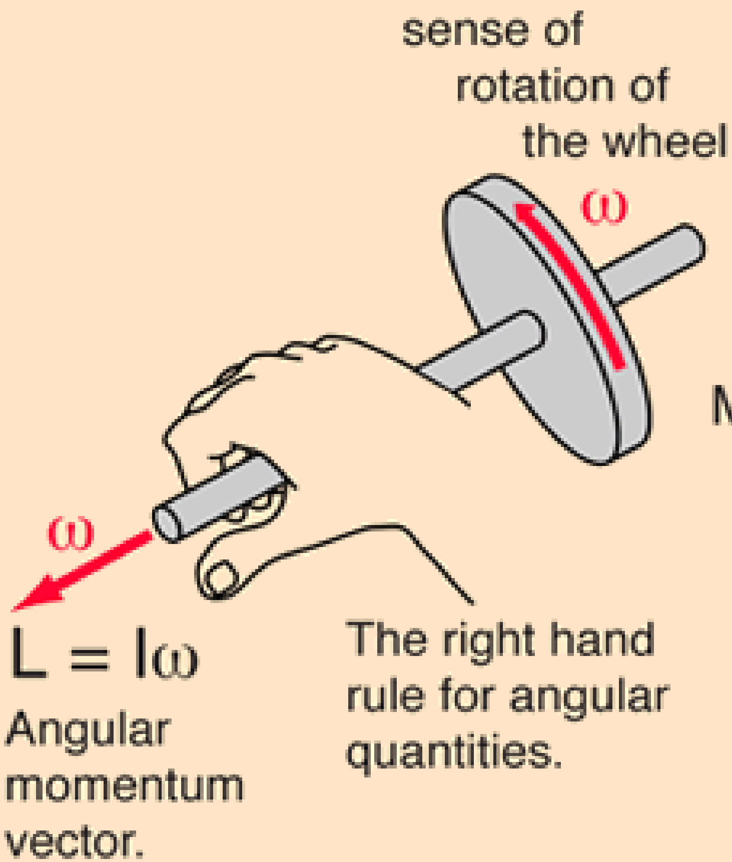
$$L = r \times p$$

The direction is given by the right hand rule which would give L the direction out of the diagram. For an orbit, angular momentum is conserved, and this leads to one of Kepler's laws. For a circular orbit, L becomes

$$L = mvr$$

Angular Momentum

The angular momentum of a rigid object is defined as the product of the moment of inertia and the angular velocity. It is analogous to linear momentum and is subject to the fundamental constraints of the conservation of angular momentum principle if there is no external torque on the object. Angular momentum is a vector quantity. It is derivable from the expression for the angular momentum of a particle

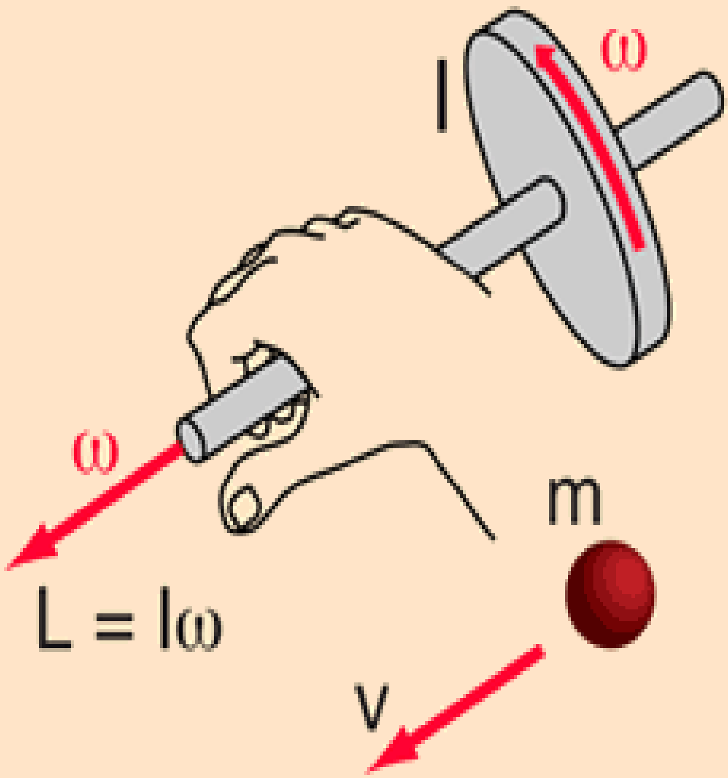


Angular Momentum = Moment of Inertia x Angular Velocity

$$L = I \times \omega$$

Angular and Linear Momentum

Angular momentum and linear momentum are examples of the parallels between linear and rotational motion. They have the same form and are subject to the fundamental constraints of conservation laws, the conservation of momentum and the conservation of angular momentum .



Angular Momentum	=	Moment of Inertia	x	Angular Velocity
L	=	I	x	ω
Linear Momentum	=	Mass	x	Velocity
p	=	m	x	v

The x implies simple multiplication here.

An Isolated System

An isolated system implies a collection of matter which does not interact with the rest of the universe at all - and as far as we know there are really no such systems. There is no shield against gravity, and the electromagnetic force is infinite in range. But in order to focus on basic principles, it is useful to postulate such a system to clarify the nature of physical laws. In particular, the conservation laws can be presumed to be exact when referring to an isolated system:

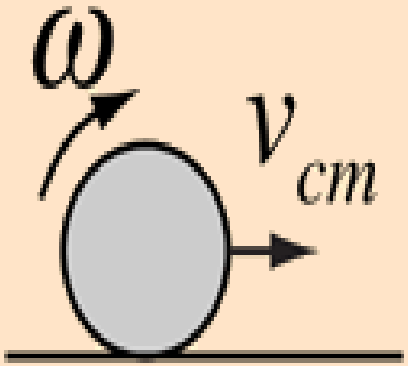
Conservation of Energy: the total energy of the system is constant.

Conservation of Momentum: the mass times the velocity of the center of mass is constant.

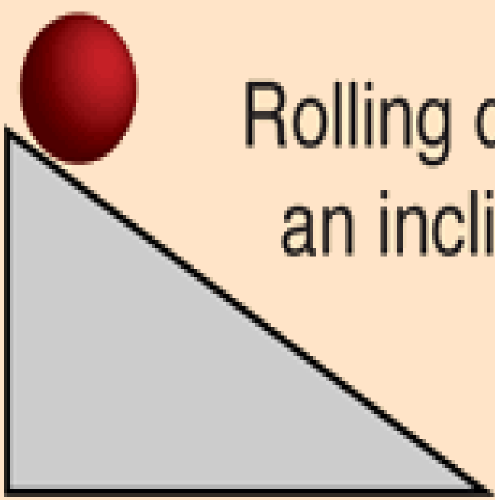
Conservation of Angular Momentum: The total angular momentum of the system is constant.

Newton's Third Law: No net force can be generated within the system since all internal forces occur in opposing pairs. The acceleration of the center of mass is zero.

Rolling Objects



Kinetic energy of rolling



Rolling down an incline

In describing the motion of rolling objects, it must be kept in mind that the kinetic energy is divided between linear kinetic energy and rotational kinetic energy. Another key is that for rolling without slipping, the linear velocity of the center of mass is equal to the angular velocity times the radius.

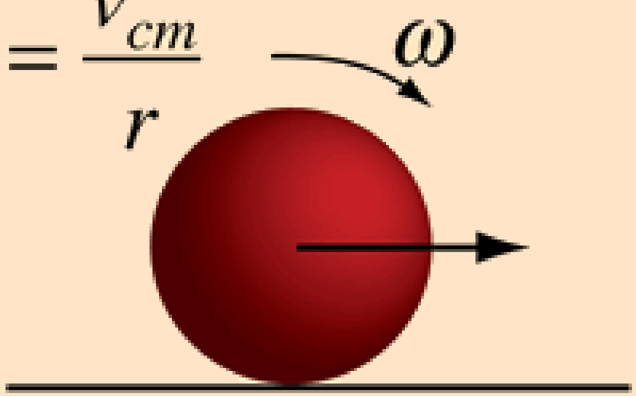
Kinetic Energy of Rolling Object

If an object is rolling without slipping, then its kinetic energy can be expressed as the sum of the translational kinetic energy of its center of mass plus the rotational kinetic energy about the center of mass. The angular velocity is of course related to the linear velocity of the center of mass, so the energy can be expressed in terms of either of them as the problem dictates, such as in the rolling of an object down an incline. Note that the moment of inertia used must be the moment of inertia about the center of mass. If it is known about some other axis, then the parallel axis theorem may be used to obtain the needed moment of inertia.

$$KE_{rolling} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm}\, \omega^2$$

If not slipping then

$$\omega = \frac{v_{cm}}{r}$$



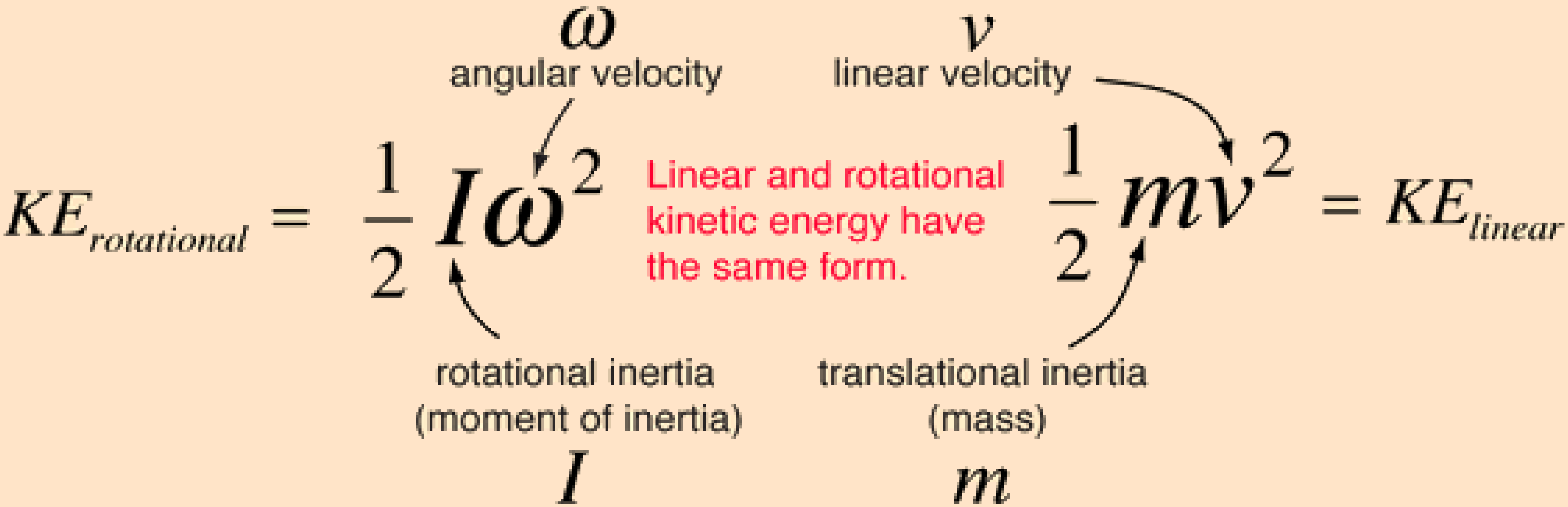
I_{cm} is related to the moment of inertia about the point of contact by the parallel axis theorem.

ω is related to v_{cm} if it is rolling without slipping.

Rotational Kinetic Energy

The kinetic energy of a rotating object is analogous to linear kinetic energy and can be expressed in terms of the moment of inertia and angular velocity. The total kinetic energy of an extended object can be expressed as the sum of the translational kinetic energy of the center of mass and the rotational kinetic energy about the center of mass. For a given fixed axis of rotation, the rotational kinetic energy can be expressed in the form

$$KE_{rotational} = \frac{1}{2} I \omega^2$$



Work-Energy Principle

$$W_{net} = \frac{1}{2}mv_{final}^2 - \frac{1}{2}mv_{initial}^2$$

The change in the kinetic energy of an object is equal to the net work done on the object.

Work-Energy Principle

The work-energy principle is a general principle which can be applied specifically to rotating objects. For pure rotation, the net work is equal to the change in rotational kinetic energy:

$$W_{net} = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$$

For a constant torque, the work can be expressed as

$$W = \tau\theta$$

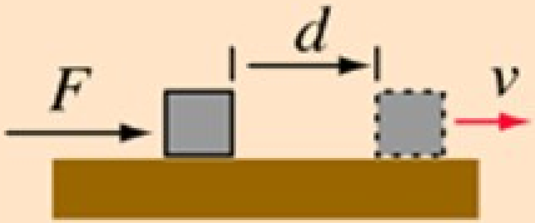
and for a net torque, Newton's 2nd law for rotation gives

$$W_{net} = \tau_{net}\,\theta = I\alpha\theta$$

Combining this last expression with the work-energy principle gives a useful relationship for describing rotational motion.

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

The expressions for rotational and linear kinetic energy can be developed in a parallel manner from the work-energy principle. Consider the following parallel between a constant torque exerted on a flywheel with moment of inertia I and a constant force exerted on a mass m, both starting from rest.



$$Work = Fd = mad = m\frac{v}{t}\frac{v}{2}t = \frac{1}{2}mv^2$$

$$Work = \tau\theta = I\alpha\theta = I\frac{\omega}{t}\frac{\omega}{2}t = \frac{1}{2}I\omega^2$$

For the linear case, starting from rest, the acceleration from Newton's second law is equal to the final velocity divided by the time and the average velocity is half the final velocity, showing that the work done on the block gives it a kinetic energy equal to the work done. For the rotational case, also starting from rest, the rotational work is $\tau\theta$ and the angular acceleration α given to the flywheel is obtained from Newton's second law for rotation. The angular acceleration is equal to the final angular velocity divided by the time and the average angular velocity is equal to half the final angular velocity. It follows that the rotational kinetic energy given to the flywheel is equal to the work done by the torque.

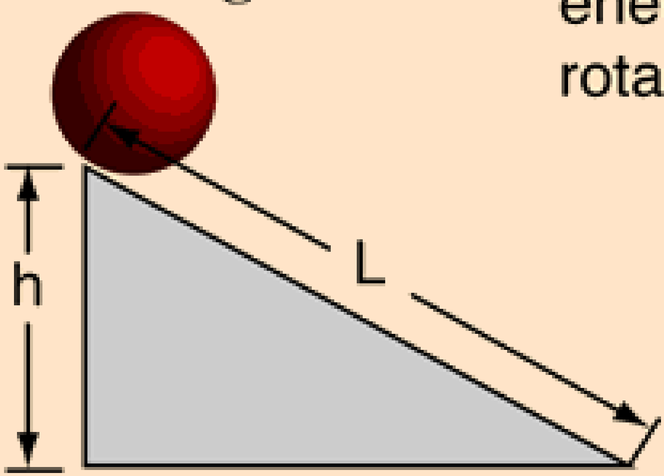
Rolling Down an Incline

In rolling without slipping through the distance L down the incline, the height of the rolling object changes by "h". Hence the gravitational potential energy changes by mgh. The velocity of the center of mass depends upon the particular form of the moment of inertia.

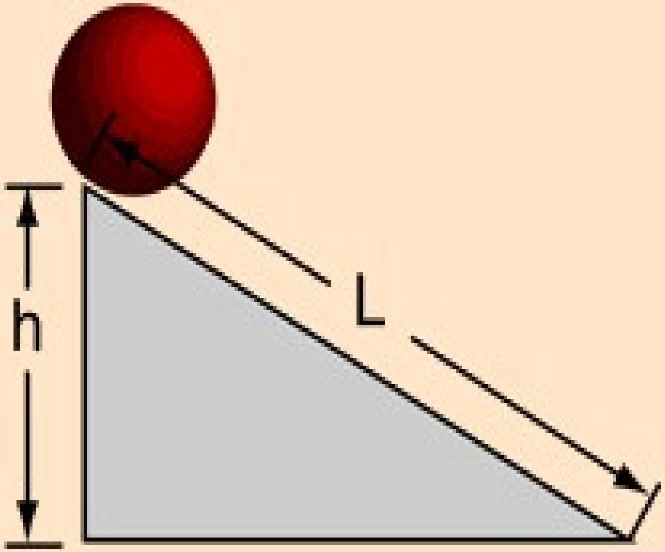
Moment of inertia of sphere about center of mass = $I = \frac{2}{5}mr^2$

PE = mgh

In rolling down the incline, the potential energy mgh is transformed into linear and rotational kinetic energy.



KE = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$



KE = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

The energy transformation equation is

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\left[\frac{v}{r}\right]^2$$

which gives a velocity at the bottom of the incline $v = \sqrt{\frac{10}{7}gh}$

Hoop and Cylinder Motion

Given a race between a thin hoop and a uniform cylinder down an incline. Which will win?

Do the relative masses of the hoop and cylinder affect the outcome?

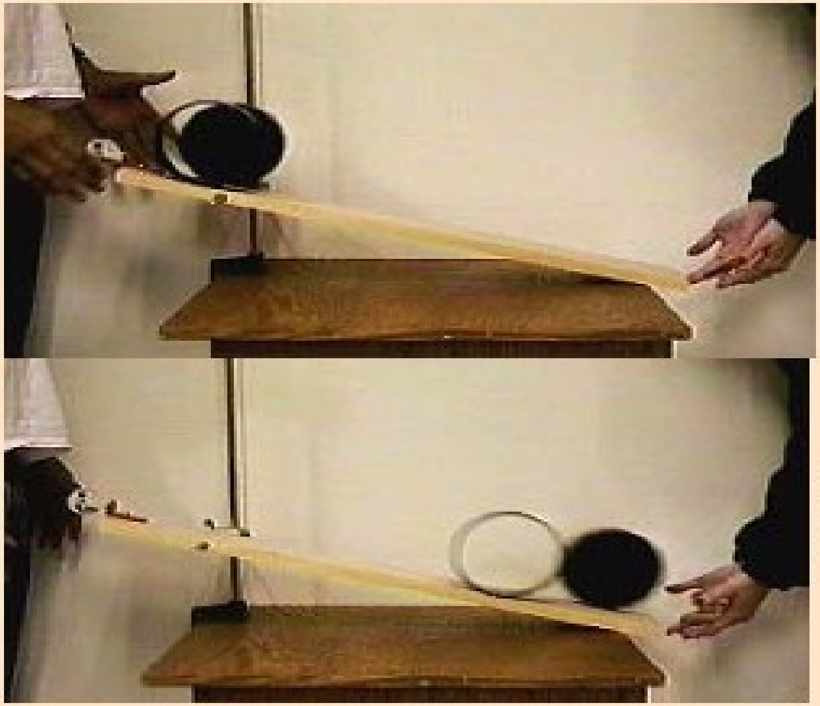
Do the relative radii of the hoop and cylinder affect the outcome?

Both start at the same height and have gravitational potential energy = mgh. Assume that they roll without slipping.

The analysis uses angular velocity and rotational kinetic energy. For rolling without slipping, the linear velocity and angular velocity are strictly proportional.

Given a race between a thin hoop and a uniform cylinder down an incline. Which will win?

Conservation of energy gives:



$$PE_{gravity} = KE_{translational} + KE_{rotational}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For rolling without slipping, $\omega = v/r$. The difference between the hoop and the cylinder comes from their different rotational inertia.

$$I_{hoop} = mr^2 \qquad I_{cylinder} = \frac{1}{2}mr^2$$

Solving for the velocity shows the cylinder to be the clear winner.

$$v_{hoop} = \sqrt{gh} \qquad v_{cylinder} = \sqrt{\frac{4}{3}gh}$$

Hoop and Cylinder Motion

Given a race between a thin hoop and a uniform cylinder down an incline, rolling without slipping. Which will win?

For the hoop:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left[mr^2\right]\frac{v^2}{r^2}$$

$$v_{hoop} = \sqrt{gh}$$

For the cylinder:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left[\frac{1}{2}mr^2\right]\frac{v^2}{r^2}$$

$$v_{cylinder} = \sqrt{\frac{4}{3}gh}$$

Sliding with no rotation on a frictionless incline.

For the hoop

$$mgh = \frac{1}{2}mv^2$$

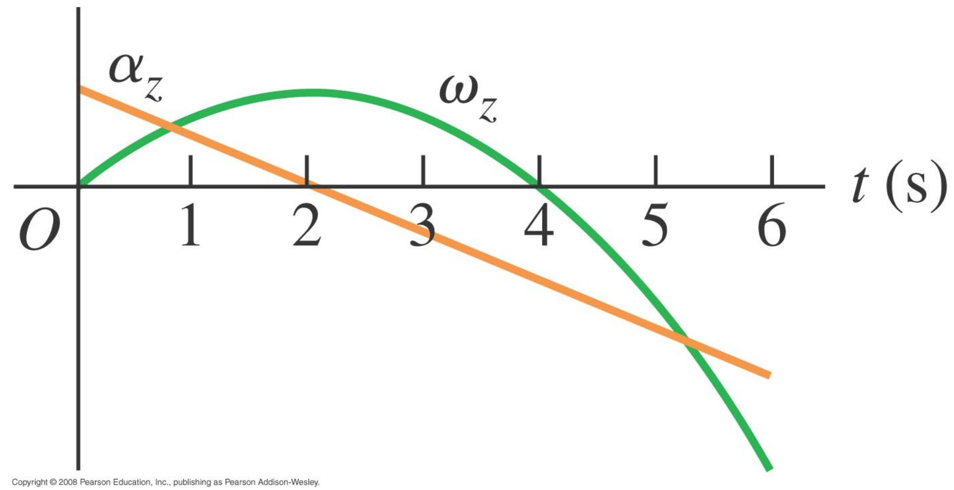
For the cylinder:

$$mgh = \frac{1}{2}mv^2$$

$$v_{frictionless} = \sqrt{2gh}$$

Q9.1

The graph shows the angular velocity and angular acceleration versus time for a rotating body. At which of the following times is the rotation speeding up at the greatest rate?



A. $t = 1$ s

B. $t = 2$ s

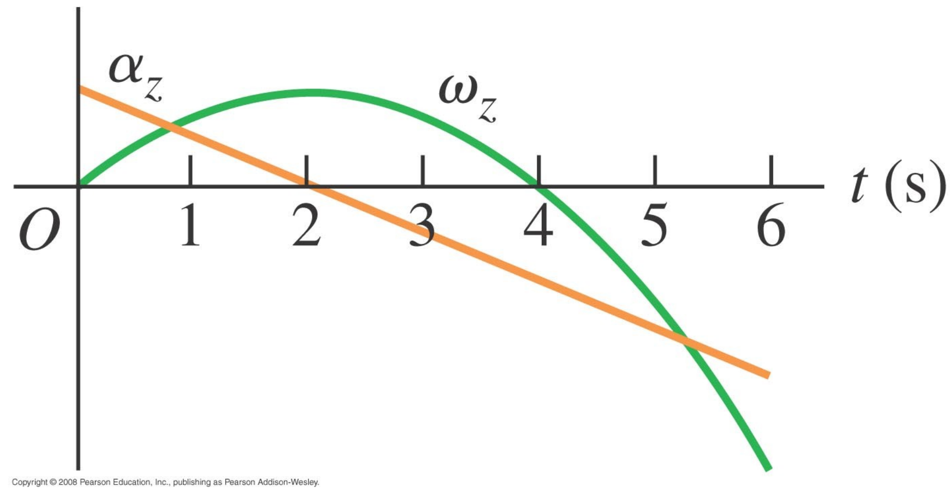
C. $t = 3$ s

D. $t = 4$ s

E. $t = 5$ s

A9.1

The graph shows the angular velocity and angular acceleration versus time for a rotating body. At which of the following times is the rotation speeding up at the greatest rate?



A. $t = 1$ s

B. $t = 2$ s

C. $t = 3$ s

D. $t = 4$ s

E. $t = 5$ s

Q9.2

A DVD is initially at rest so that the line PQ on the disc's surface is along the $+x$-axis. The disc begins to turn with a constant $\alpha_z = 5.0\ \text{rad/s}^2$.

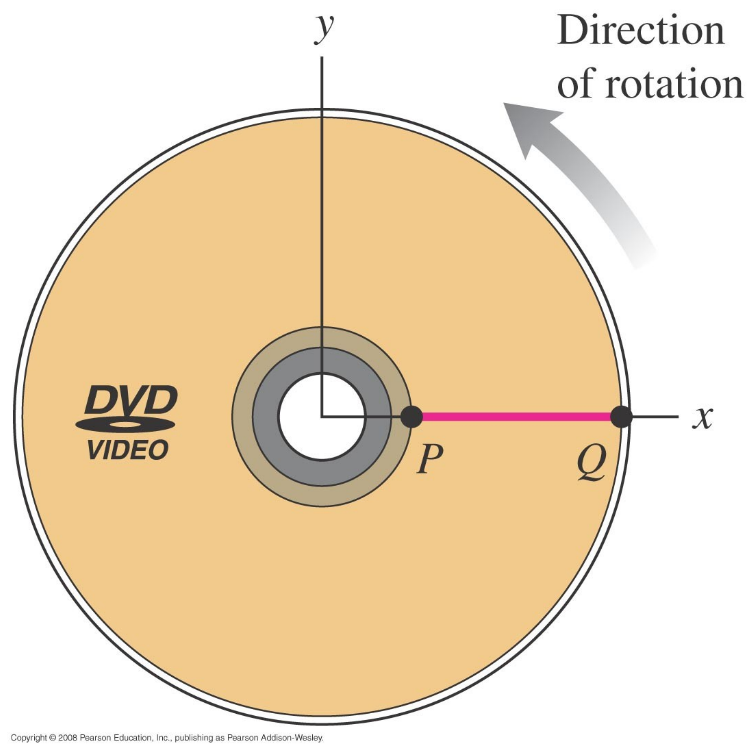
At $t = 0.40$ s, what is the angle between the line PQ and the $+x$-axis?

A. 0.40 rad

B. 0.80 rad

C. 1.0 rad

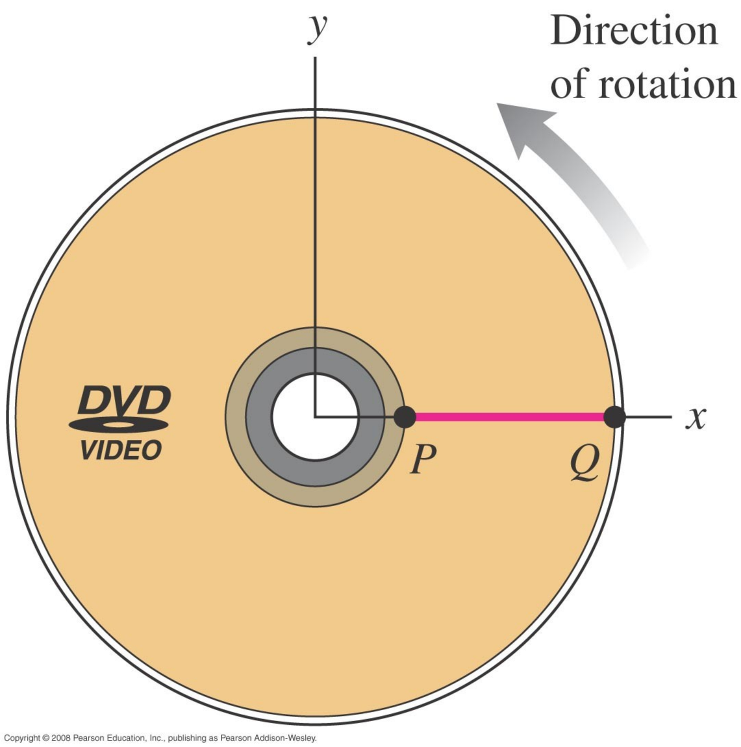
D. 2.0 rad



A9.2

A DVD is initially at rest so that the line *PQ* on the disc's surface is along the +*x*-axis. The disc begins to turn with a constant $\alpha_z = 5.0\ \text{rad/s}^2$.

At $t = 0.40$ s, what is the angle between the line *PQ* and the +*x*-axis?



✓ A. 0.40 rad

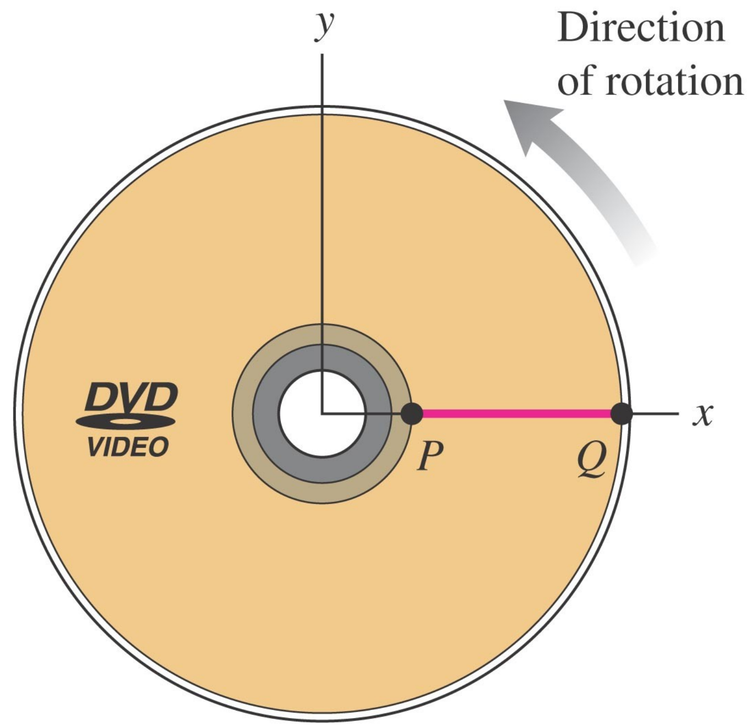
B. 0.80 rad

C. 1.0 rad

D. 2.0 rad

Q9.3

A DVD is rotating with an ever-increasing speed. How do the centripetal acceleration a_{rad} and tangential acceleration a_{tan} compare at points P and Q?



A. P and Q have the same a_{rad} and a_{tan}.

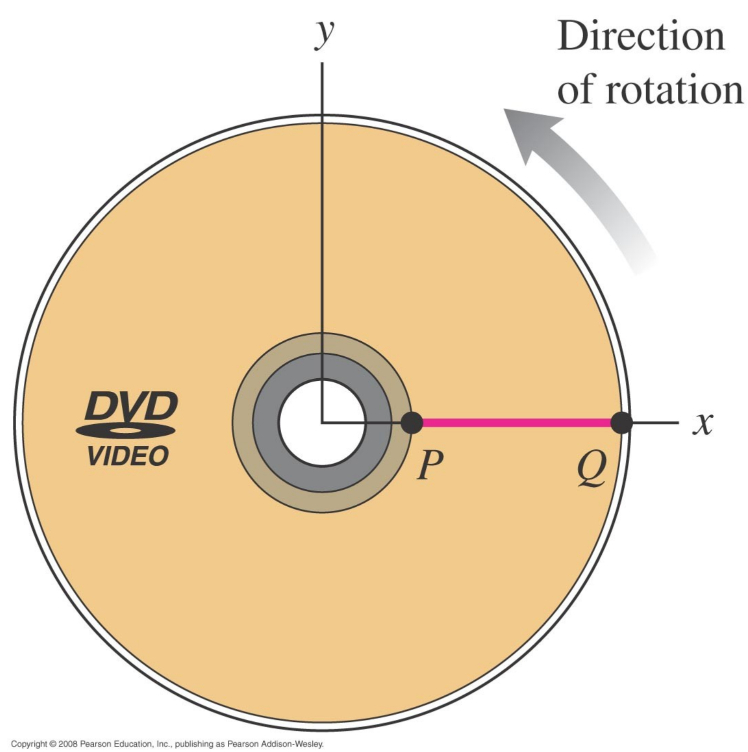
B. Q has a greater a_{rad} and a greater a_{tan} than P.

C. Q has a smaller a_{rad} and a greater a_{tan} than P.

D. P and Q have the same a_{rad}, but Q has a greater a_{tan} than P.

A9.3

A DVD is rotating with an ever-increasing speed. How do the centripetal acceleration a_{rad} and tangential acceleration a_{tan} compare at points P and Q?



A. P and Q have the same a_{rad} and a_{tan}.

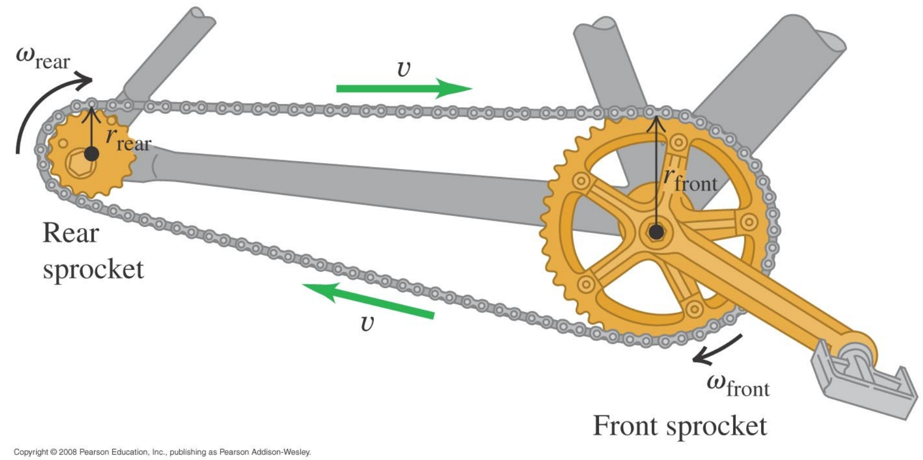
✓ B. Q has a greater a_{rad} and a greater a_{tan} than P.

C. Q has a smaller a_{rad} and a greater a_{tan} than P.

D. P and Q have the same a_{rad}, but Q has a greater a_{tan} than P.

Q9.4

Compared to a gear tooth on the rear sprocket (on the left, of small radius) of a bicycle, a gear tooth on the *front* sprocket (on the right, of large radius) has



A. a faster linear speed and a faster angular speed.

B. the same linear speed and a faster angular speed.

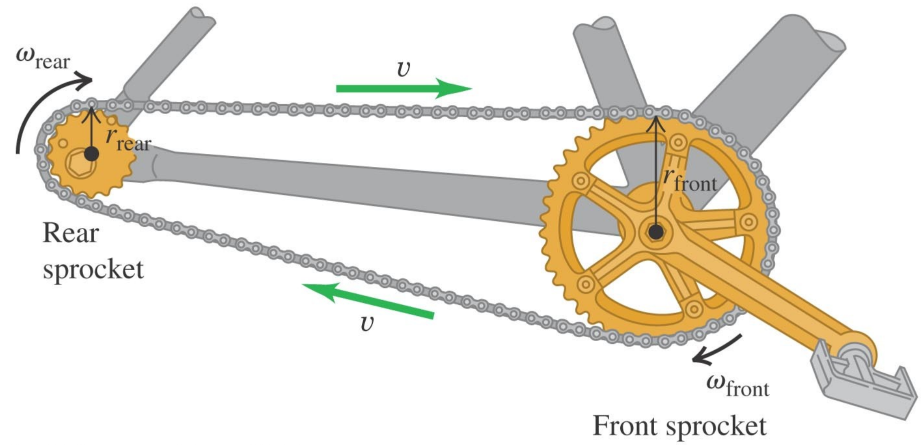
C. a slower linear speed and the same angular speed.

D. the same linear speed and a slower angular speed.

E. none of the above

A9.4

Compared to a gear tooth on the rear sprocket (on the left, of small radius) of a bicycle, a gear tooth on the *front* sprocket (on the right, of large radius) has



A. a faster linear speed and a faster angular speed.

B. the same linear speed and a faster angular speed.

C. a slower linear speed and the same angular speed.

✓ D. the same linear speed and a slower angular speed.

E. none of the above

Q9.5

You want to double the radius of a rotating solid sphere while keeping its kinetic energy constant. (The mass does not change.) To do this, the final angular velocity of the sphere must be

A. 4 times its initial value.

B. twice its initial value.

C. the same as its initial value.

D. 1/2 of its initial value.

E. 1/4 of its initial value.

A9.5

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A. 4 times its initial value.

B. twice its initial value.

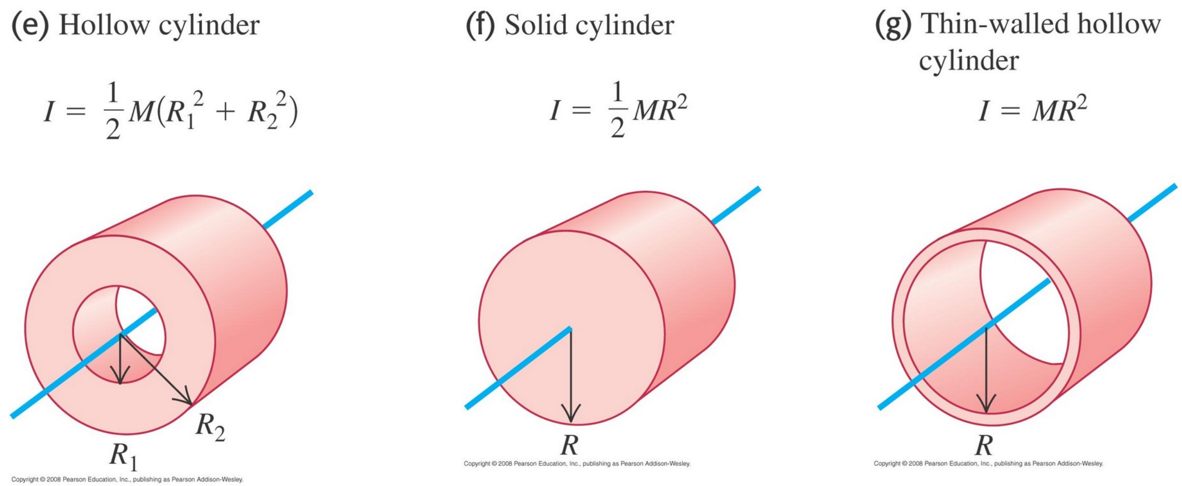
C. the same as its initial value.

D. 1/2 of its initial value.

E. 1/4 of its initial value.

Q9.6

The three objects shown here all have the same mass M and radius R. Each object is rotating about its axis of symmetry (shown in blue). All three objects have the *same* rotational kinetic energy. Which one is rotating *fastest*?



A. thin-walled hollow cylinder

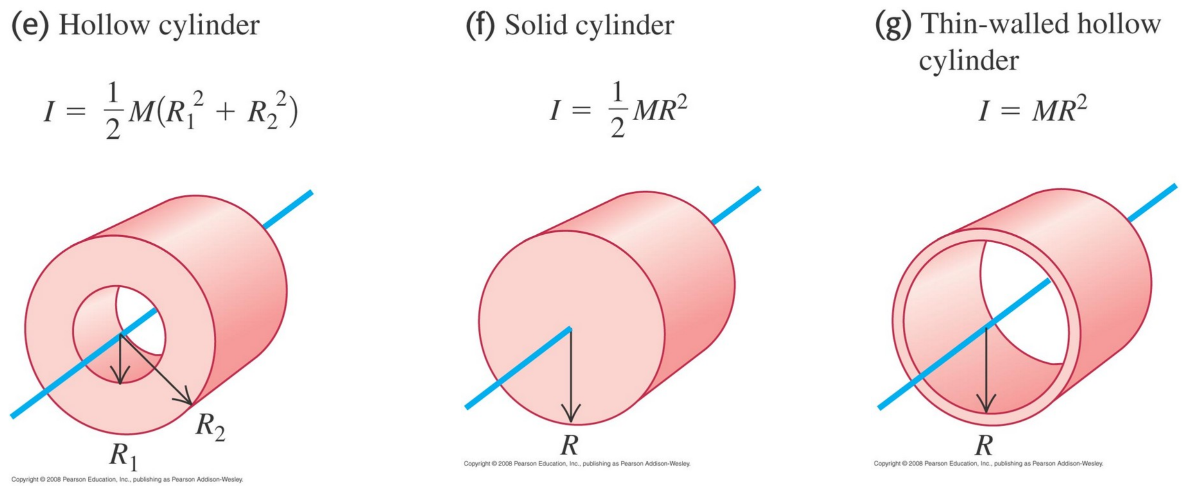
B. solid cylinder

C. thin-walled hollow sphere

D. two or more of these are tied for fastest

A9.6

The three objects shown here all have the same mass *M* and radius *R*. Each object is rotating about its axis of symmetry (shown in blue). All three objects have the *same* rotational kinetic energy. Which one is rotating *fastest*?



A. thin-walled hollow cylinder

B. solid cylinder

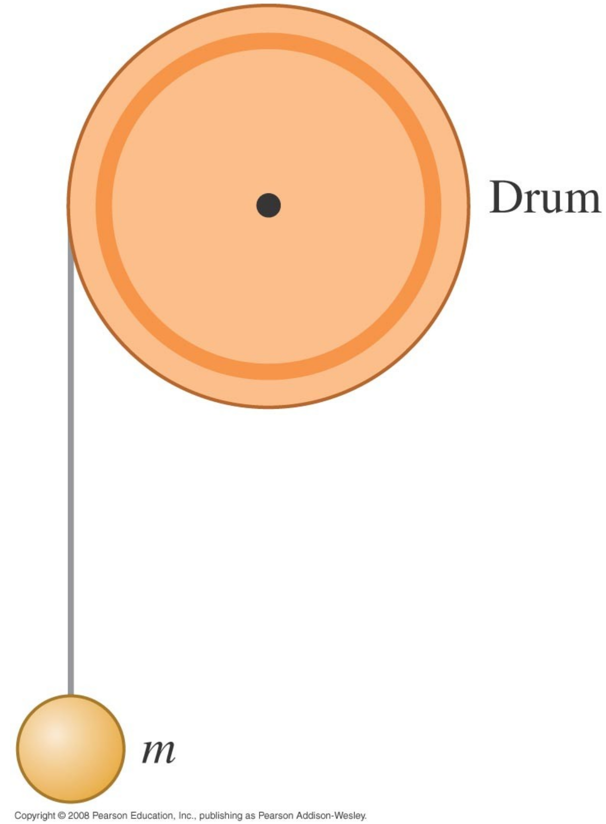
C. thin-walled hollow sphere

D. two or more of these are tied for fastest

Q9.7

A thin, very light wire is wrapped around a drum that is free to rotate. The free end of the wire is attached to a ball of mass m. The drum has the same mass m. Its radius is R and its moment of inertia is $I = (1/2)mR^2$. As the ball falls, the drum spins.

At an instant that the ball has translational kinetic energy K, the drum has rotational kinetic energy



A. K.

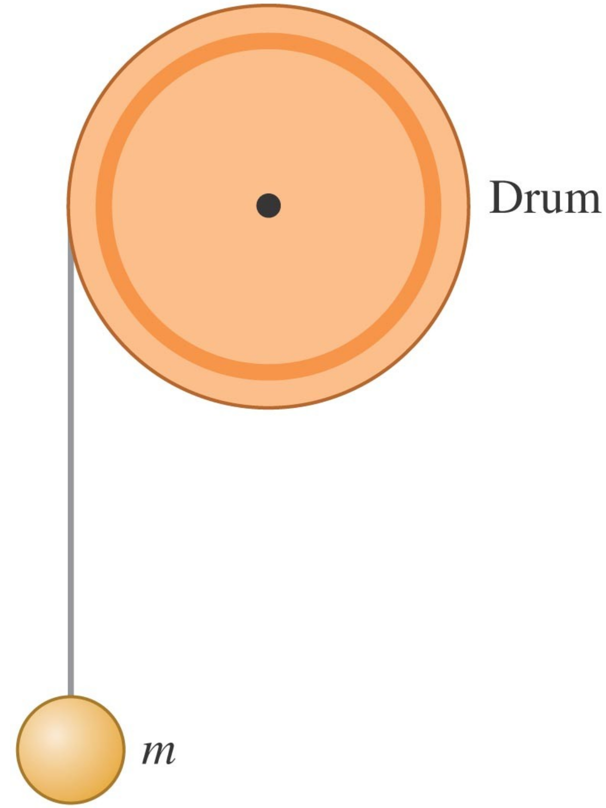
B. $2K$.

C. $K/2$.

D. none of these

A9.7

A thin, very light wire is wrapped around a drum that is free to rotate. The free end of the wire is attached to a ball of mass m. The drum has the same mass m. Its radius is R and its moment of inertia is $I = (1/2)mR^2$. As the ball falls, the drum spins.



At an instant that the ball has translational kinetic energy K, the drum has rotational kinetic energy

A. K.

B. $2K$.

C. $K/2$. ✓

D. none of these