ÇANAKKALE ONSEKIZ MART UNIVERSITY

DEPARTMENT OF PHYSICS

GENERAL PHYSICS LABORATORY MANUAL AND WORKBOOK

EXPERIMENT 5
COLLISIONS AND CONSERVATION OF LINEAR MOMENTUM

PURPOSE

The purpose of this experiment is to verify the linear conservation of momentum in different types of collisions in an isolated system, to study the motion of the center of mass for the collision of a two-puck system and to investigate the conservation of kinetic energy in elastic and completely inelastic collisions.

THEORY

The linear momentum $\vec{P}$ of an object is defined as the product of its mass and velocity,

$$\vec{P} = m\vec{v} \tag{5.1}$$

Therefore, an object at rest will have zero linear momentum. From the above definition, it is clear that an object that has a constant mass will have a constant momentum, unless its velocity changes. Linear momentum will be referred to as momentum from here on. The velocity of an object changes only when a net external force $\vec{F}_{ext}$ is applied. This means that the momentum of an object will change only then that object experiences a net external force. This fact can be seen from Newton's second law. For an object with constant mass, we have Newton's second law.

$$\vec{F}_{ext} = m\vec{a} = m\frac{d\vec{v}}{dt} \tag{5.2}$$

this can be written as,

$$\vec{F}_{ext} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{P}}{dt} \tag{5.3}$$

This equation means that if no net force acts on an object then its momentum will be constant in time. That is if $\vec{F}_{ext}$=0, then

$$\frac{d\vec{P}}{dt} = 0 \tag{5.4}$$

or

$$\vec{P} = constant \tag{5.5}$$

Here, 'constant' means that the momentum does not change with time., i.e. the object will have the same momentum at all times. This can be generalized to a system of N-particles with masses

$m_1, m_2, \ldots\ldots, m_N$. the total momentum of the system of particles at any instant of time is by definition:

$$\vec{P}_{tot} = \vec{P}_1 + \vec{P}_2 + \ldots + \vec{P}_N \qquad (5.6)$$

Where $\vec{P}_1 = m_1\vec{v}_1, \vec{P}_2 = m_2\vec{v}_2\ldots$ etc. The sum in Equation 5.6 is a vector, not an algebraic sum. In this case, Equation 5.3 is generalized to,

$$\vec{F}_{ext} = \frac{d\vec{P}_{tot}}{dt} = \frac{d}{dt}\left(\vec{P}_1 + \vec{P}_2 + \ldots + \vec{P}_N\right) \qquad (5.7)$$

where $\vec{F}_{ext}$ means a net external force to the system of particles, i.e. any force other than the force that the particles of the system exert on each other. This force might be the friction force, the gravitational force… etc. Therefore, when no such net external force acts on the system of particles the total momentum of the system will be conserved.

$$\frac{d\vec{P}_{tot}}{dt} = \frac{d}{dt}\left(\vec{P}_1 + \vec{P}_2 + \ldots + \vec{P}_N\right) = 0 \qquad (5.8)$$

or

$$\vec{P}_{tot} = \vec{P}_1 + \vec{P}_2 + \ldots + \vec{P}_N = constant \qquad (5.9)$$

Recall that the above sum is vectorial. For a system of particles experiencing no net external force, the total momentum will be the same at any instant in time, regardless of any collisions among the particles.

In this experiment, we are going to investigate the conservation of momentum of the system of two pucks moving on an air table. The leveled-off air table, which can be accepted to be almost frictionless, it produces no net external forces on the pucks on it. Therefore, we expect the total momentum of the pucks to be conserved. The pucks will be aimed to collide, and their momentum before and after the collision will be measured and compared. The data points that we will have on the datasheet are shown in Figure 5.1.

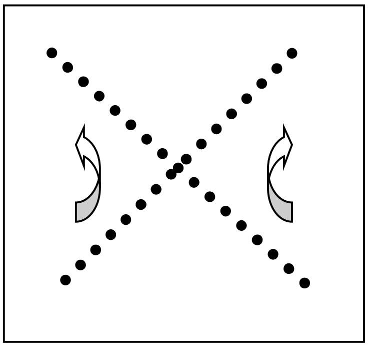


FIGURE 5.1. The data points in the elastic collision of the two pucks on a horizontal air table.

The velocities of the two pucks before the collision will be $\vec{v}_A$ and $\vec{v}_B$, and after the collision $\vec{v}_A'$ and $\vec{v}_B'$. Since the system is an isolated system, the total momentum will be conserved;

$$\vec{P}_{tot} = constant \qquad (5.10)$$

so,

$$\vec{P}_A + \vec{P}_B = \vec{P}_A' + \vec{P}_B' \qquad (5.11)$$

where $\vec{P}_A = m_A \vec{v}_A$ … etc. Since the masses of the pucks are identical, the above relation reduces to

$$\vec{v}_A + \vec{v}_B = \vec{v}_A' + \vec{v}_B' \qquad (5.12)$$

In a completely inelastic collision, momentum will be conserved too, since the system is still an isolated one. The two pucks in this collision will stick together and form a single object with mass = 2m that moves with velocity $\vec{v}'$. The dots on the datasheet will look like those in Figure 5.2 below.

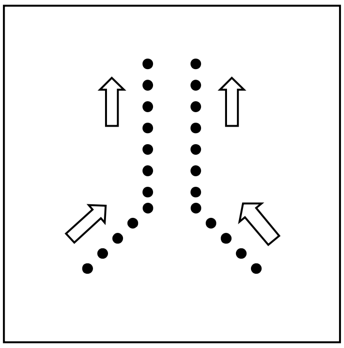


FIGURE 5.2. The data points in the completely inelastic collision of two pucks on a horizontal air table.

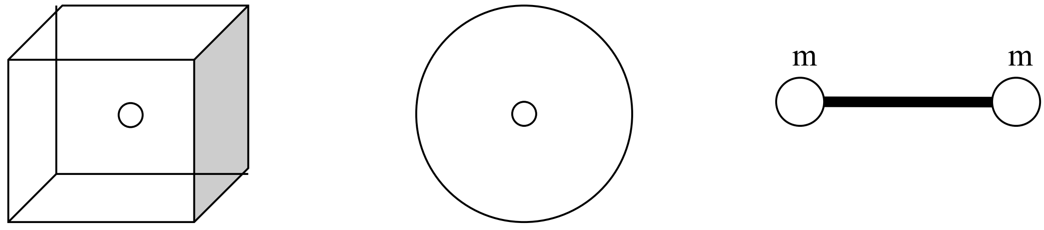
For this case, the conservation of momentum is

$$\vec{P}_A + \vec{P}_B = \vec{P}' \qquad (5.13)$$

or

$$\vec{v}_A + \vec{v}_B = 2\vec{v}' \qquad (5.14)$$

Another concept that we are going to introduce and study in this experiment is the center of mass (CM). Intuitively, you can guess that the CM of a homogeneous cube (Figure 5.3a) or a sphere (Figure 5.3b.) will be at the geometrical center of these symmetrical objects. Also, you can guess the CM of the dumbbells shown in Figure 5.3c is at the midpoint point of its rod. Thus, the CM of two identical homogeneous spheres is at exactly the midpoint of the line joining their centers (Figure 5.3d.). If one of the spheres is heavier, the CM will be shifted towards the heavier sphere as shown in Figure 4.3e. The amount of the shift is determined by how much the mass M is larger than m. From these examples, it is clear that it is possible to guess the position of the CM of some symmetrical mass distributions. For example, the CM of a two-puck system of this experiment will be at the midpoint of the line joining their centers.



(a) (b) (c)

FIGURE 5.3. The center of mass of some symmetrical homogeneous objects.

For more arbitrary mass distributions, the CM can be defined more formally. The position vector $\vec{R}$ of the center of mass of a system of N particles with masses $m_1, m_2, \ldots, m_N$, whose position vectors are $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N$, respectively, is defined as,

$$\vec{R} = \frac{m_1\vec{r}_1+m_2\vec{r}_2+\ldots+m_N\vec{r}_N}{m_1+m_2+\ldots+m_N} \qquad (5.15)$$

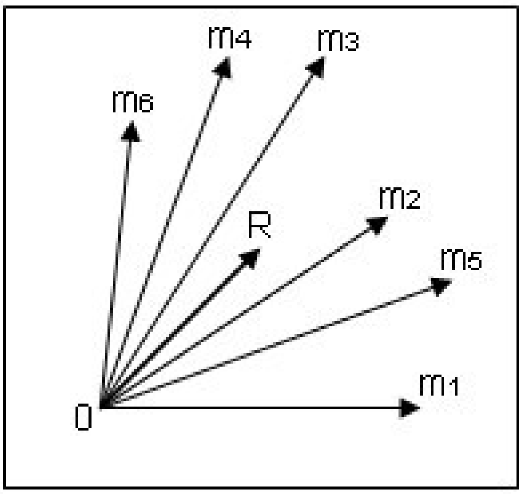


FIGURE 5.4. The center of mass of a distribution of masses.

As the particle change their positions in time, the position of the CM will also change and this rate of change of the position vector of the CM is the velocity of the CM.

$$\vec{V}_{CM} = \frac{d\vec{R}}{dt} \qquad (5.16)$$

For particles with constant masses, taking the time derivative of both sides of Equation 5.15

$$R = \frac{m_1\dot{\vec{r}}_1+m_2\dot{\vec{r}}_2+\ldots+m_N\dot{\vec{r}}_N}{m_1+m_2+\ldots+m_N} \qquad (5.17)$$

or

$$\vec{V}_{CM} = \frac{m_1\vec{v}_1+m_2\vec{v}_2+\ldots+m_N\vec{v}_N}{m_1+m_2+\ldots+m_N} \qquad (5.18)$$

For the two-puck system,

$$R = \frac{m\vec{r}_A + m\vec{r}_B}{m+m} \qquad (5.19)$$

$$R = \frac{\vec{r}_A + \vec{r}_B}{2} \qquad (5.20)$$

where we have canceled out the masses (Equation 5.20), since the pucks have identical masses. The velocity of CM is,

$$\vec{V}_{CM} = \frac{\vec{v}_A + \vec{v}_B}{2} \qquad (5.21)$$

The above equation has important implications. First, note that the numerator of the right-hand side is a constant in the case of the two-puck system of the horizontal air table, since the momentum is conserved. This means that the velocity of the CM is constant. In other words, the CM moves with constant velocity. So, for an isolated system for which the total momentum is conserved, the CM of the system always moves in a straight line and with constant velocity. Moreover, for this case, Equation 5.21 implies that the velocity of the CM is half the vectorial sum of the velocities of the pucks at any instant in time. Therefore, for this two-puck system before and after the collision,

$$\vec{V}_{CM} = \vec{V}_{CM}^{'} \qquad (5.22)$$

or

$$\vec{V}_{CM} = \frac{\vec{v}_A + \vec{v}_B}{2} = \vec{V}_{CM}^{'} = \frac{\vec{v}_A^{'} + \vec{v}_B^{'}}{2} \qquad (5.23)$$

In this experiment, we are also going to investigate the conservation of the kinetic energy of the pucks for the case of the collision. Remember the definition of the kinetic energy K,

$$K = \frac{1}{2} m v^2 \qquad (5.24)$$

Therefore, the total kinetic energy of the two-puck system before an elastic collision will be

$$K = \frac{1}{2} m {v_A}^2 + \frac{1}{2} m {v_B}^2 \qquad (5.25)$$

and after the collision

$$K^{'} = \frac{1}{2} m {v_A^{'}}^2 + \frac{1}{2} m {v_B^{'}}^2 \qquad (5.26)$$

In the completely inelastic collision, where the two pucks stick together to form a single object with mass 2m and velocity $v^{'}$, the total kinetic energy after the collision is

$$K^{'} = \frac{1}{2}(2m) v^{'2} = m v^{'2} \qquad (5.27)$$

Since the kinetic energy is a scalar quantity, the sums in Equations 5.25 and 5.26 are scalar sums. On the other hand, while the kinetic energy is almost conserved in an elastic collision, it is not conserved in a completely inelastic collision. The fractional loss of the kinetic energy is defined as

$$Fractional\ loss\ = \frac{K-K'}{K}$$

and using this, we can define the percentage fractional loss in kinetic energies

$$Percentage\ loss\ = \frac{K-K'}{K}.100\%$$

EQUIPMENT

An air table, Velcro bands (To stick the pucks together in the completely inelastic collision), ruler.

PROCEDURE

This experiment consists of two parts. Part A is the elastic collision and Part B is the completely inelastic collision. The experiment will be carried out on a horizontal air table. Therefore, before you start the experiment, level off the air table as described in the first part of this manual.

PART A: ELASTIC COLLISION

1. Activate only the pump switch (P) and project the two pucks diagonally towards each other across the air table so that they collide somewhere in the middle of the air table. Repeat this procedure several times until you get a satisfactory collision. Don't project the two pucks too slowly, or too fast, but push them so that they move with a moderate speed. Now, choose a proper spark timer frequency (20 Hz, for example), and then project the pucks across the air table while activating the switch (P), and once you release them, activate the spark timer switch (S). Keep both switches pressed until the two pucks complete their motion.
2. Remove the datasheet and examine the dots produced. They should look like those in Figure 4.1a. Number the dots for each puck as 0, 1, 2, … etc. Have your data approved by your instructor before you go on.
3. Find the velocity of each puck before and after the collision by measuring the length of two or three intervals of each trajectory and dividing by time. Label the two trajectories of the pucks by A and B before the collision and A' and B' after collision.
4. Find the vectorial sums $\vec{v}_A + \vec{v}_B$ and $\vec{v}'_A + \vec{v}'_B$. To find $\vec{v}_A + \vec{v}_B$, for example, extend the trajectories A and B until they intersect. Then, so starting from the intersection point, draw vectors along the directions of $\vec{v}_A$ and $\vec{v}_B$, and with lengths proportional to the magnitude of these velocities. You can, for instance, draw a 1 cm vector to represent a velocity of 10 cm.s^{-1}, (see Figure 4.5). Then, completing the parallelograms find the sum of these velocities. Repeat the same procedure to find $\vec{v}'_A + \vec{v}'_{\boldsymbol{B}}$.

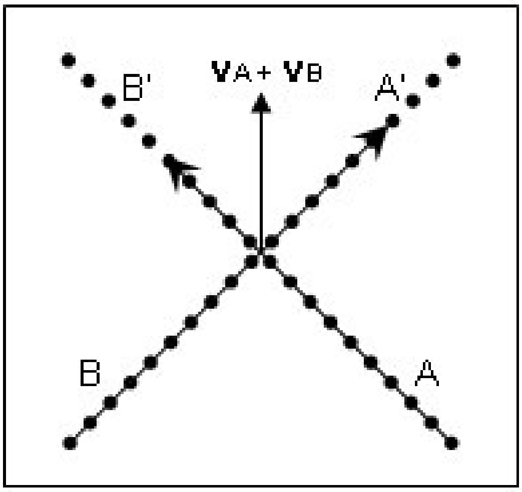


FIGURE 5.5. Vector sum $\vec{v}_A + \vec{v}_B$

5. Identify the dots produced at the same instant of time before and after the collision, and joining these determine the position of the CM along the line which joins every pair of dots. By doing this, you will get a record of the position of the CM.
6. Using the record you obtained for the CM in step 5, find its velocities before and after the collision.
7. Find the total kinetic energy of the two pucks before and after the collision, and compare them.

PART B: COMPLETELY INELASTIC COLLISION

1. Wrap the Velcro band firmly around the two pucks (see Figure 5.6); make sure that the rims of the bands are not in contact with the surface of the datasheet. Activate only the pump switch (P), and project the two pucks towards each other across the air table so that they collide and stick together somewhere in the middle of the air table. Make sure that the pucks do not rotate after the collision. Repeat several times until you get a convenient collision.
2. Now, activating the pump (P), project the pucks toward each other, and at the moment you release them activate the spark timer switch (S). Keep both switches pressed until the pucks complete their motion. The dots on the datasheet will look like those in Figure 5.2. Find the velocities of the pucks before the collision and the common velocity $\vec{v}\,'$ of the two pucks, that are stuck together, after the collision.
3. Applying the method described in step 5 and 6 of part A, calculate the vector sum $\vec{v}_A + \vec{v}_B$ and verify the conservation of momentum.
4. Find the total kinetic energy of the pucks before and after the collision, and calculate the fractional loss, and thus the percentage fractional loss.

ENGINEERING FACULTY
DEPARTMENT OF ENVIRONMENTAL ENGINEERING
Year: 2021-2022 Semester: Fall
Course Name: ENV-1021 – **General Physics Laboratory**
EXPERIMENT 5: COLLISIONS AND CONSERVATION OF LINEAR MOMENTUM

INSTRUCTOR'S NAME & TITLE:						SIGNATURE:		EXAM DATE:
STUDENT'S NAME/SURNAME: STUDENT ID #:						SIGNATURE:		EXAM DURATION:
Question								TOTAL POINT:
Point								
Prog. Outcomes	**PO1,2**							

Group:

RESULTS AND DISCUSSION

PART A: ELASTIC COLLISION

1. Write down to velocities of the two pucks before and after the collision.

$\vec{v}_A$= …………………… $\vec{v}_B$=……………………

$\vec{v}'_A$=………………….. $\vec{v}'_B$=……………………

2. Find the sums $\vec{v}_A + \vec{v}_{\boldsymbol{B}}$ and $\vec{v}'_A + \vec{v}'_{\boldsymbol{B}}$, and discuss the conservation of momentum.

$|\vec{v}_A + \vec{v}_{\boldsymbol{B}}|$ =…………………………..

$|\vec{v}'_A + \vec{v}'_{\boldsymbol{B}}|$ =…………………………..

3. Find the velocity of the CM before and after the collision.

$\vec{V}_{CM}$=…………………… $\vec{V}'_{CM}$=……………………

4. What kind of motion does the CM have? Compare the results you reported above for $\vec{V}_{CM}$ and $\vec{V}'_{CM}$ with $\frac{\vec{v}_A+\vec{v}_B}{2}$ and $\frac{\vec{v}'_A+\vec{v}'_B}{2}$, respectively.

………………………………………………………………………………………………
………………………………………………………………………………………………

5. Find the total kinetic energy of the two pucks before and after the collision. Is the kinetic energy conserved?

K =………………….. K'=………………………

NAME SURNAME:
STUDENT ID #:
Signature:

PART B: COMPLETELY INELASTIC COLLISION

6. Write down the velocities of the two pucks before the collision and the common velocity of the pucks, that are stuck together, after the collision.

$\vec{v}_A$= …………………. $\vec{v}_B$=………………….
$\vec{v}'$=…………………..

7. Find the sum $\vec{v}_A + \vec{\boldsymbol{v}}_{\boldsymbol{B}}$ and verify the conservation of linear momentum.

$|\vec{v}_A + \vec{\boldsymbol{v}}_{\boldsymbol{B}}|$=……………..

8. Find the total kinetic energy of the pucks before and after the collision. Is the kinetic energy conserved? If not, find the percentage fractional loss of energy.
……………………………………………………………………………………………
……………………………………………………………………………………………

9. Write down the comments related to the experiment, and/or elaborate on and discuss any points.
……………………………………………………………………………………………
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Laboratory and Examination Rules:

1- The most important thing in the laboratory is your safety. The dangers mostly result from a lack of knowledge of the equipment and procedures.
2- Personal safety rules must be obeyed with extreme discipline.
3- When you enter the laboratory never play with the equipment until it has been explained and the instructor has given permission.
4- Keep your experimental equipment and tabletop clean.
5- Report any accident to your instructor immediately.
6- Most of the equipment used in the laboratory is expensive and some of them are delicate. Even after you are familiar with the equipment, always have your experimental setup checked and approved by the instructor before putting it into operation.
7- If any of the equipment is broken or does not function properly, report it to the instructor.
8- Read and study the experiments before you come to the laboratory.
9- It is forbidden to share the questions and answers of this assignment on the internet or in another environment.
10- Students must answer the assignment by themselves. It is forbidden to do the assignment with others or to get help from others.
11- **Write your name and surname and student number and sign your signature on the top right of each answer page.**