

# ÇANAKKALE ONSEKIZ MART UNIVERSITY 

## DEPARTMENT OF PHYSICS

## GENERAL PHYSICS LABORATORY MANUAL AND WORKBOOK

## EXPERIMENT 6 ROTATIONAL MOTION

## PURPOSE

The purpose of this experiment is to investigate the dynamics of a disk that rotates about an axis passing through its center and to calculate the angular acceleration, angular velocity, and the moment of inertia of a disk, and to investigate the conservation of mechanical energy.

## THEORY

So far, we have studied the kinematics and the dynamics of an object having linear motion. In this experiment, we are going to study the rotational motion of a rigid body. A rigid body is an idealized model for an object that has a definite and unchanged shape and size. To study the rotational motion of a rigid body, we first need to introduce some new physical quantities and concepts that are essential to the description and understanding of this type of motion.

(a)

(b)

FIGURE 6.1. The same net force F is acting on two objects with masses $m$ and $M(M>m)$. The object in (a) will have a larger change in its motion than that in (b).

The first quantity that we are going to introduce is the moment of inertia of a rigid body. To understand this, we go back to linear motion to make an analogy. We know that in linear motion if the same force was applied to the two objects with masses $m$ and $M(M>m)$ (see Fig. 6-1), the one with the smaller mass would have greater acceleration. In other words, the force would cause a greater change in the motion of the object with the smaller mass compared to that with the larger mass. So, the mass of an object is a measure of the inertia (resistance) that an object exhibits against changes in its motion, and the larger this mass is, the larger is the inertia. This is why the mass appearing in Newton's second law $F=m a$ is usually referred to as the inertial mass.


FIGURE 6-2. The same person (i.e. the same force) trying to rotate a pencil (a) and a heavy metallic rod (b). It is easier to rotate the pencil than the rod.

Consider now the situation described in Figure 6.2, where a pencil and a heavy metallic rod are put into rotation by the same person, i.e. by the same force. Obviously, it will be easier to rotate the pencil than it is to rotate the rod. In physical terms, we say that the moment of inertia of the rod is larger than that of the pencil. You might, in the point of view of the above examples, rush to the conclusion that the moment of inertia of a rigid body is determined solely by its mass. To see that this is not true, consider the situation described in Figure 6.3. The same person rotates the two rods, with the same mass M , about two different axes.


FIGURE 6.3. Two identical metallic rods were rotated around two different axes by the same person (force). It is easier to rotate the rod in (a) than it is in (b).

It is harder to rotate the rod in Figure 6.3 b . That is, it has a larger moment of inertia compared to the other one, although they both have the same mass $M$. Therefore, the moment of inertia depends on the mass distribution with respect to the axis of rotation in addition to its mass. Figure 6.4 gives the moment of inertia of some symmetrical homogeneous rigid bodies.


FIGURE 6.4. The moment of inertia of some symmetrical homogeneous objects about different axes.

We know that according to Newton's second law of motion when a net force acts on an object that is at rest, it puts it into motion. So, can any force put a rigid body into rotation? To answer this question, see Figure 6.5 and its description.

(a)

(b)

FIGURE 6.5. A force F acting tangentially on a disk, that can rotate freely about an axis passing through its center as in (a), can rotate it. The same force acting as in (b) can not cause rotation.

The net tangential force on the disk in Figure 6.5a causes it to rotate about the axis passing through its center. On the other hand, when the same force acts on the same disk so that its line of action passes through its center as in Figure 6.5b, obviously it can not rotate the disk. From the above observation, we see that a net force is not sufficient to cause a rigid body to rotate; the point where the force acts is also important. In physical terms, we say that a rigid body will experience a change in its rotational motion if a net torque of that force about the rotation axis (or point) exists. The torque of a force about a point or an axis is defined as:
$\Gamma=\mathbf{r} \mathbf{x}$
Where $r$ is the vector from the axis of rotation to the point where the force is applied (see Fig 6.6). The torque is a vector quantity.


FIGURE 6.6. The angle $\theta$ in the equation $\Gamma=\mathrm{rFsin} \theta$.
From Equation 6.1, and the definition of the vector (cross) product,

$$
\begin{equation*}
\Gamma=\mathrm{rFsin} \theta \tag{6.2}
\end{equation*}
$$

where $\theta$ is the angle between $r$ and $F$ (see Fig. 6.6). The above equations explain why the net force in Figure $6.5 a$ causes rotation, while that in Fig. $6.5 b$ does not. The angle $\theta$ in the latter case is zero.

The net force acting on an object gives it a net linear acceleration; net torque acting on a rigid body gives it an angular acceleration $\alpha$ defined by

$$
\begin{equation*}
\Gamma=\mathrm{I} \boldsymbol{\alpha} \tag{6.3}
\end{equation*}
$$

where $I$ is the moment of inertia of the rigid body about the axis of rotation. Now, let us define angular velocity and angular acceleration. Consider a rigid disk that is rotating about an axis passing through its center as shown in Fig. 6.7 below.


FIGURE 6.7. The change in the distance $d s$ and the angle $d \varphi$ in an infinitesimal time interval $d t$ for a rigid disk rotating about an axis passing through its center, O .

Consider a point on the rim of the disk, and take the origin of the coordinates to be at the center of the disk. In a time interval $d t$, such a point will travel a distance $d s$ along the rim of the disk, and the radius R will sweep an angle $d \varphi$. The magnitude of the linear velocity of the point on the rim of the disk will therefore be

$$
\begin{equation*}
v=\frac{d s}{d t} \tag{6.4}
\end{equation*}
$$

Consequently, the linear acceleration, which is the rate of change of the velocity, will be

$$
\begin{equation*}
a=\frac{d v}{d t}=\frac{d^{2} s}{d t^{2}} \tag{6.5}
\end{equation*}
$$

The angular velocity of the rotating disk is defined as the rate of change of the angle $\varphi$ (the angle measured in radians). So

$$
\begin{equation*}
\omega=\frac{d \phi}{d t} \tag{6.6}
\end{equation*}
$$

and the angular acceleration is

$$
\begin{equation*}
\alpha=\frac{d \omega}{d t}=\frac{d^{2} \phi}{d t^{2}} \tag{6.7}
\end{equation*}
$$

In the above geometry, the length of the arc element $d s$ is $d s=R d \varphi$ ( $\varphi$ in radians), therefore with constant R;

$$
\begin{equation*}
v=\frac{d s}{d t}=R \frac{d \phi}{d t} \tag{6.8}
\end{equation*}
$$

or

$$
\begin{equation*}
\mathrm{v}=\mathrm{R} \omega \tag{6.8}
\end{equation*}
$$

Consequently,

$$
\begin{equation*}
a=R \alpha \tag{6.9}
\end{equation*}
$$

The above two equations define the dimensions (i.e. units of $\omega$ ve $\alpha$ ). Since in the SI system of units $v$ is given in $\mathrm{m} / \mathrm{s}$ and R in m , then from Eq. (6.8) $\omega$ has the unit $1 / \mathrm{s}$. Similarly, $\alpha$ has the unit of $1 / \mathrm{s}^{2}$.

From the above developments, it is clear that there is a close analogy between the kinematical and dynamical quantities in linear and rotational motions. Table 6.1 summarizes this analogy.

TABLE 6.1. Analogy between kinematical and dynamical quantities in linear and rotational motions.

| Linear Motion | Rotational Motion |
| :---: | :---: |
| v | $\Omega$ |
| a | A |
| m | I |
| $\mathbf{F}=\mathrm{ma}$ | $\Gamma=\mathrm{I} \boldsymbol{\alpha}$ |
| $\mathrm{v}(\mathrm{t})=\mathrm{v}_{\mathrm{o}}+\mathrm{at}$ | $\omega(\mathrm{t})=\omega_{\mathrm{o}}+\mathrm{t}$ |
| $K . . E=\frac{1}{2} m v^{2}$ | $K . E=\frac{1}{2} I \omega^{2}$ |



FIGURE 6.8. The experimental setup. Puck $m$ performs linear motion downwards causing the puck $M$ to rotate.

In this experiment, we are going to have the setup shown in Figure 6.8. A solid disk of mass $M$ that can rotate freely about an axis passing through its center is mounted to the upper
edge of the inclined air table. A cord is wrapped around this disk and the free end of the cord is attached to a puck of mass $m$ as shown in the figure (In this experiment only one puck will be used). As the system is released from rest, the suspending puck $m$ will accelerate down the plane of the inclined air table putting the disk $M$ into rotation. The tension $T$ in the cord will act as the force that torque causes the disk to rotate.

The forces acting on the suspended puck are shown in Figure 6.8. Applying Newton's second law, we have

$$
\begin{equation*}
\mathrm{mgsin} \Phi-\mathrm{T}=\mathrm{ma} \tag{6.10}
\end{equation*}
$$

Where $\Phi$ is the angle of inclination of the air table, and a is the linear acceleration of the puck. The angular acceleration of the rotating disk is related to the above linear acceleration through Eq. (6.9);

$$
a=R \alpha
$$

where R is the radius of the rotating disk. The torque caused by the tension $T$ is given by

$$
\begin{equation*}
\Gamma=\mathrm{RT}=\mathrm{I} \alpha \tag{6.11}
\end{equation*}
$$

where $I$ is the inertia of the disk. If the system is released from rest, then at a later time $t$, the linear velocity of the puck will be given by

$$
\mathrm{v}=a t
$$

Similarly, the angular velocity of the rotating puck will be

$$
\begin{equation*}
\omega=\alpha \mathrm{t} \tag{6.13}
\end{equation*}
$$

The puck has kinetic energy due to its translational (linear) motion and the disk has kinetic energy due to its rotational motion. The total kinetic energy of the system at any instant of time will be

$$
\begin{equation*}
K=\frac{1}{2} m v^{2}+\frac{1}{2} I \omega^{2} \tag{6.14}
\end{equation*}
$$

As the puck falls down the plane, its potential energy changes into translational and rotational kinetic energies of the puck and the disk, respectively. Conservation of energy -with the friction neglected- requires that $\Delta K=-\Delta \mathrm{U}$, where $\Delta$ means the change in energy. Therefore, at any instant of time after the system is put into motion, we have

$$
\begin{equation*}
\operatorname{mg}(\Delta \mathrm{d}) \sin \Phi=\frac{1}{2} m v^{2}+\frac{1}{2} I \omega^{2} \tag{6.15}
\end{equation*}
$$

Here, $\Delta \mathrm{d}$ is the change in the distance of the puck, traveling down the inclined plane.

## EQUIPMENT

An air table, a wooden block, a rotating disk, a cord, and a ruler, millimetric graph paper.

## PROCEDURE

This experiment is carried out on an inclined air table. Therefore, first level off the air table, then bring it into the inclined position using the wooden block.

1. Attach one end of the cord to the rotating disk $M$. Then wrap the cord several times around the rim of the disk. Attach the other free end of the cord to the suspended puck. As only one puck is going to be used in this experiment, keep the other puck still at the lower corner of the air table by folding a piece of paper and putting it under the puck.
2. Adjust the suspended puck so that the rope is stretched, then pressing only the footswitch ( P ), allow it to fall down the inclined plane. Observe the rotational motion of the disk. Repeat several times until you get a convenient motion.
3. Now, adjust the suspended puck again as in step 2 , and choose a suitable spark timer frequency ( 20 or 10 Hz ). Put the footswitches S and P on top of each other and press them simultaneously. Keep the switches depressed until the suspended puck reaches the bottom of the inclined plane.
4. Remove the datasheet, and observe the data produced on it. Is the trajectory of the puck a straight line? Are the dots evenly spaced or does the distance between successive dots increase with time? What kind of trajectory and inter-dot spacing was expected? Does the data you obtained agree with what you have expected? In the view of the above questions and your answers to them, assess the data that you have got. If you feel that your data is inconvenient, repeat the experiment. Consult your instructor if you have any further problems.
5. Starting from the first dot, number the first five dots as $0,1,2, \ldots$ etc. Measure and record the time and the distance of each dot from dot 0 along with the corresponding errors. Record your data in Table 6-2.
6. Using the data in Table 6-2, plot $x$ vs $t^{2}$ on a graph. Take the positive $x$-direction in the direction of motion. Draw both best and worst lines and find the slope of the graph and from this find the linear acceleration $a$ of the suspended puck.
7. Measure the radius of the rotating puck, and using Eq. (6.9) find the angular acceleration $\alpha$ of the rotating disk.
8. Measure the mass $m$ of the suspended puck, and using Eq. (6.10) find the tension $T$ in the cord. Using Eq. (6.11) then find the torque of this tension, also calculate the moment of inertia $\boldsymbol{I}$ of the disk using $\Gamma=\mathrm{RT}=\mathrm{I} \alpha$.

| INSTRUCTOR'S NAME \& TITLE: |  | SIGNATURE: | EXAM DATE: |  |  |
| :---: | :---: | :--- | :--- | :--- | :---: |
| STUDENT'S NAME/SURNAME: <br> STUDENT ID \#: |  |  |  |  |  |
| Question |  |  |  | SIGNATURE: | EXAM <br> DURATION: |
| Point |  |  |  |  |  |
| Prog. <br> Outcomes | PO1,2 |  |  |  |  |
| TOTAL POINT: |  |  |  |  |  |

## RESULTS AND DISCUSSION

1. Write your data in Table 6.2.

TABLE 6.2

| Dot number | $x \pm \Delta x(\mathrm{~cm})$ | $t \pm \Delta t(\mathrm{~s})$ | $t^{2} \pm \Delta t^{2}\left(\mathrm{~s}^{2}\right)$ |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

2. Plot $x$ vs $t^{2}$ graph, calculate the linear acceleration of the suspending puck.

$$
\mathrm{a} \pm \Delta \mathrm{a}=
$$

$\qquad$
3. Measure and record the radius of the rotating disc.

$$
\mathrm{R} \pm \Delta \mathrm{R}=
$$

$\qquad$
4. Calculate the angular acceleration $\alpha$ of the rotating disk. Show your calculations and report $\alpha$.

$$
\alpha=
$$

$\qquad$
5. Find the tension T in the cord, the torque $\Gamma$ and the moment of inertia $I$ of the rotating disc. Show your calculations.

$$
\begin{aligned}
& \text { T=. } \\
& \Gamma= \\
& \text { I=. }
\end{aligned}
$$

6. Find linear velocity v , and the angular velocity $\omega$. Show your calculations.
```
v =.
\omega=.
```

7. Find the translational and rotational kinetic energies of the suspending and rotating pucks. Show your calculations.
8. Verify the conservation of energy.
9. Write down the comments related to the experiment, and/or elaborate on and discuss any points.

## Laboratory and Examination Rules:

1- The most important thing in the laboratory is your safety. The dangers mostly result from a lack of knowledge of the equipment and procedures.
2- Personal safety rules must be obeyed with extreme discipline.
3- When you enter the laboratory never play with the equipment until it has been explained and the instructor has given permission.
4- Keep your experimental equipment and tabletop clean.
5- Report any accident to your instructor immediately.
6- Most of the equipment used in the laboratory is expensive and some of them are delicate. Even after you are familiar with the equipment, always have your experimental setup checked and approved by the instructor before putting it into operation.
7- If any of the equipment is broken or does not function properly, report it to the instructor.
8- Read and study the experiments before you come to the laboratory.
9- It is forbidden to share the questions and answers of this assignment on the internet or in another environment.
10- Students must answer the assignment by themselves. It is forbidden to do the assignment with others or to get help from others.
11- Write your name and surname and student number and sign your signature on the top right of each answer page.

